Claim 1: Concepts and Procedures
Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.

Content Domain: **Ratios and Proportional Relationships**

Target A [m]: Understand ratio concepts and use ratio reasoning to solve problems. (DOK 1, 2)

Tasks for this target will require students to make sense of problems that use ratio and rate language and to find unit rates associated with given ratios. Students will be asked to display equivalent ratios in tables and as coordinate pairs, using information to compare ratios or find missing values.

Other tasks for this target ask students to find a percent as a rate per hundred. Problems involving rates, ratios, percents (finding the whole, given a part and the percent), and measurement conversions that use ratio reasoning will also be assessed in Claims 2–4.

Standards: 6.RP.A, 6.RP.A.1, 6.RP.A.2, 6.RP.A.3	**6.RP.A Understand ratio concepts and use ratio reasoning to solve problems.** **6.RP.A.1** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* **6.RP.A.2** Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."* **6.RP.A.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, "If it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?"* c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.
Related Below-Grade and Above-Grade Standards for Purposes of Planning for Vertical Scaling: 5.MD.A, 5.MD.A.1 7.RP.A, 7.RP.A.1, 7.RP.A.2, 7.RP.A.3	**Related Grade 5 Standards** **5.MD.A Convert like measurement units within a given measurement system.** **5.MD.A.1** Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real-world problems.

	Related Grade 7 Standards **7.RP.A Analyze proportional relationships and use them to solve real-world and mathematical problems.** **7.RP.A.1** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction* $^{1/2}/_{1/4}$ *miles per hour, equivalently 2 miles per hour.* **7.RP.A.2** Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price* p*, the relationship between the total cost and the number of items can be expressed as t = pn*. d. Explain what a point *(x, y)* on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r)* where *r* is the unit rate. **7.RP.A.3** Use proportional relationships to solve multi-step ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*
DOK Levels:	1, 2
Achievement Level Descriptors:	
RANGE Achievement Level Descriptor (Range ALD) Target A: Understand ratio concepts and use ratio reasoning to solve problems.	**Level 1** Students should be able to describe a ratio relationship between two whole number quantities, find missing values in tables that display a proportional relationship, and plot the pairs of values from a table on the coordinate plane. They should be able to find a percent as a rate per hundred and convert measurement units.
	Level 2 Students should be able to understand the concept of unit rate in straightforward, well-posed problems and solve straightforward, well-posed, one-step problems requiring ratio reasoning.
	Level 3 Students should be able to use ratio reasoning to solve and understand the concept of unit rates in unfamiliar or multi-step problems, including instances of unit pricing and constant speed, and solve percent problems by finding the whole, given a part and the percent. They should be able to describe a ratio relationship between any two number quantities (denominators less than or equal to 12).
	Level 4 Students should be able to solve unfamiliar or multi-step problems by finding the whole, given a part and the percent; explain ratio relationships between any two number quantities; and identify relationships between models or representations.

Evidence Required:	1. The student uses ratio language to describe a ratio relationship. 2. The student determines the unit rate associated with a real-world ratio. 3. The student finds missing values in tables of equivalent ratios. 4. The student plots coordinate pairs to represent equivalent ratios. 5. The student makes tables of equivalent ratios relating quantities with whole-number measurements. 6. The student solves real-world problems involving unit rate. 7. The student solves mathematical problems involving finding the whole, given a part and the percent. 8. The student solves real-world and mathematical problems involving finding a percent of a quantity as a rate per 100. 9. [Retired Evidence Required statement] 10. The student uses ratio reasoning to manipulate and transform units appropriately when multiplying or dividing quantities.
Allowable Response Types:	Multiple Choice, multiple correct response; Equation/Numeric; Fill-in Table; Graphing; Matching Tables
Allowable Stimulus Materials:	coordinate planes, tables, tape diagrams
Construct-Relevant Vocabulary:	ratio, unit rate, unit price, ordered pair
Allowable Tools:	Calculator (varies by task model)
Target-Specific Attributes:	Unit rates are limited to non-complex fractions.
Non-Targeted Constructs:	
Accessibility Guidance:	Item writers should consider the following Language and Visual Element/Design guidelines[1] when developing items. Language Key Considerations: • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level • Avoid ambiguous or obscure words, idioms, jargon, unusual names and references Visual Elements/Design Key Considerations: • Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context • Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary

[1] For more information, refer to the General Accessibility Guidelines at: http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf

	• Avoid crowding of details and graphics Items are selected for a student's test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.[2]
Development Notes:	Real-world problems involving rates, ratios, percentages (finding the whole, given a part and the percent), and measurement conversions that use ratio reasoning will also be assessed in Claim 2, Claim 3, and Claim 4, as appropriate.

[2] For more information about student accessibility resources and policies, refer to http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

Task Model 1 **Response Type: Multiple Choice, multiple correct response** **DOK Level 1** **6.RP.A.1** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* **Evidence Required:** 1. The student uses ratio language to describe a ratio relationship. **Tools:** None **Version 3 Update:** Added new example stem 2.	**Prompt Features:** The student is prompted to identify statements that use ratio language to describe a ratio relationship. **Stimulus Guidelines:** Context should be familiar to students 11 to 13 years old. **TM1** **Stimulus:** The student is presented with a ratio relationship between two whole-number quantities. **Example Stem:** A game has green and blue pieces. The ratio of green game pieces to total pieces is 5:12. Select **all** the statements about the game pieces that are correct. A. The ratio of green pieces to blue pieces is 7:5. B. The ratio of total pieces to blue pieces is 12:7. C. There must be 7 more blue pieces than green pieces. D. The ratio of total pieces to green pieces is 12:5. **Answer Choices:** Answer choices will be four statements describing the ratio relationship. At least two statements must be correct. **Rubric:** (1 point) Student selects all the correct statements (e.g., B and D). **Response Type:** Multiple Choice, multiple correct response **Example Stem 2:** A punch recipe calls for 3 cups of orange juice for every 2 cups of cranberry juice. Select **all** of the statements about the recipe that are correct. A. There are 3 cups of orange juice for every 5 cups of punch. B. The ratio of cranberry juice to orange juice is 2 to 3. C. The ratio of orange juice to cranberry juice is 2:1. D. The ratio of cranberry juice to punch is 2:5. **Answer Choices:** Answer choices will be four statements describing the ratio relationship. At least two statements must be correct. **Rubric:** (1 point) Student selects all the correct statements (e.g., A, B and D). **Response Type:** Multiple Choice, multiple correct response

Task Model 2 **Response Type: Equation / Numeric** **DOK Level 2** **6.RP.A.2** Understand the concept of a unit rate *a*/*b* associated with a ratio *a*:*b* with *b* ≠0, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."* **Evidence Required:** 2. The student determines the unit rate associated with a real-world ratio. **Tools:** Calculator	**Prompt Features:** The student is prompted to identify the unit rate that corresponds to a ratio of real-world quantities. **Stimulus Guidelines:** • Context should be familiar to students 11 to 13 years old. • Item difficulty can be adjusted via these example methods: o Both numbers and unit rate are whole numbers. o Both numbers are whole numbers and unit rate is a fraction. **TM2** **Stimulus:** The student is presented with a real-world ratio problem. **Example Stem:** Carl can type 180 words in 2 minutes. How many words per minute can Carl type? **Rubric:** (1 point) Student enters correct value (e.g., 90). Units should be assumed from the problem. **Response Type:** Equation/Numeric

Task Model 3

Response Type: Fill-in Table

DOK Level 1

6.RP.A.3a
Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

Evidence Required:
3. The student finds missing values in tables of equivalent ratios.

Tools: Calculator

Prompt Features: The student is prompted to find missing values in tables of equivalent ratios.

Stimulus Guidelines:

- If used, context should be familiar to students 11 to 13 years old.
- The values for the table should be whole numbers.
- Tables should be labeled and have two columns and 3–5 rows of data.
- Either one *x*- or *y*-value should be missing from the table.
- All table formats in an item should be the same.
- Unit rate should be a whole number or non-complex fraction.
- Item difficulty can be adjusted via these example methods:
  - All numbers and unit rates are whole numbers. Unit rate is given in the table (i.e., 1:3).
  - All numbers and unit rates are whole numbers. Unit rate is not given in the table.
  - All numbers are whole numbers and unit rate is a non-complex fraction.

TM3a
Stimulus: The student is presented with a table that has an equivalent ratio and a single missing value.

Example Stem 1: The table shows the number of tennis balls that fit into a given number of cans. Each can holds the same number of balls.

Cans	Balls
2	6
	15
7	21
9	27

Fill in the missing value in the table.

Example Stem 2: This table contains equivalent ratios between *x* and *y*.

x	***y***
2	6
5	
7	21
9	27

Fill in the missing value in the table.

Rubric: (1 point) Student enters correct missing value (e.g., 5; 15).

Response Type: Fill-in Table

Task Model 3 **Response Type: Fill-in Table** **DOK Level 2** **6.RP.A.3a** Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. **Evidence Required:** 3. The student finds missing values in tables of equivalent ratios. **Tools:** Calculator	**TM3b** **Stimulus:** The student is presented with a table that has an equivalent ratio and two missing values. **Example Stem:** The table shows the number of tennis balls that fit into a given number of cans. Each can holds the same number of balls. **Cans** | **Balls** 1 | 4 | 12 13 | 15 | 45 Fill in the missing values to complete the table. **Rubric:** (1 point) Student enters the two correct values into the table (e.g., 3 and 39). **Response Type:** Fill-in Table

Task Model 4

Response Type: Graphing

DOK Level 1

6.RP.A.3a
Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

Evidence Required:
4. The student plots coordinate pairs to represent equivalent ratios.

Tools: Calculator

Accessibility Note: Graphing items are not currently able to be Brailled. Minimize the number of items developed to this TM.

Prompt Features: The student is prompted to plot ordered pairs in the coordinate plane that correspond to ratios in a table.

Stimulus Guidelines:

- Table should have two columns and 3–5 rows of data.
- All table formats in an item should be the same.
- The *x*- and *y*-values for the table should be whole numbers.
- Unit rates should be a whole number or non-complex fraction.
- If used, context should be familiar to students 11 to 13 years old.
- Graph should have a title and have both axes labeled.
- Item difficulty can be adjusted via these example methods:
  - All numbers are whole numbers. The independent values are all consecutive numbers.
  - All numbers are whole numbers. Some independent values are consecutive numbers.
  - All numbers are whole numbers. All independent values are non-consecutive numbers.

TM4
Stimulus: The student is presented with a completed table that has an equivalent ratio.

Example Stem: The table shows the number of tennis balls that fit into a given number of cans.

Cans	Balls
2	6
5	15
7	21
8	24

Use the Add Point tool to plot the ordered pairs in the coordinate plane.

Interaction: Students will be given a graph with axes numbered and labeled appropriately. Students will need the Add Point and Delete tools.

Rubric: (1 point) Student correctly plots all coordinate pairs on the graph.

Response Type: Graphing

Task Model 5

Response Type: Fill-in Table

DOK Level 2

6.RP.A.3a
Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

Evidence Required:
5. The student makes tables of equivalent ratios relating quantities with whole-number measurements.

Tools: Calculator

Version 3 Update:
Revised TM5 including prompt features, stimulus guidelines, and example stem.

Prompt Features: The student is prompted to create a table given a ratio.

Stimulus Guidelines:

- Ratios use whole numbers
- Tables should have 3 rows of values

TM5
Stimulus: The student is presented with a partially completed table and information about a specific ratio.

Example Stem: To make popcorn, a movie theater uses 9 tablespoons of oil for each cup of popcorn kernels.

Using this information, complete the table for the missing amounts of oil and popcorn kernels.

Tablespoons of Oil	Cups of Popcorn Kernels
18	
	4
	9

Rubric: (1 point) Student enters the correct missing values in the table (e.g., 2, 36, 81).

Response Type: Fill-in Table

Task Model 6 **Response Type: Equation / Numeric** **DOK Level 2** **6.RP.A.3b** Solve unit rate problems including those involving unit pricing and constant speed. **Evidence Required:** 6. The student solves real-world problems involving unit rate. **Tools:** Calculator	**Prompt Features:** The student is prompted to identify the solution to problems involving a unit rate. **Stimulus Guidelines:** • Context should be familiar to students 11 to 13 years old. • Unit rate should be a whole number or non-complex fraction. • Unit of measurement values should be whole numbers appropriate for the given situation. **TM6** **Stimulus:** The student is presented with a real-world problem involving unit rate. **Example Stem:** Carl types 180 words in 2 minutes. Enter the number of words Carl types in 5 minutes at this rate. **Rubric:** (1 point) Student enters correct numeric value (e.g., 450). **Response Type:** Equation/Numeric

Task Model 7 **Response Type: Equation / Numeric** **DOK Level 2** **6.RP.A.3c** Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. **Evidence Required:** 7. The student solves mathematical problems involving finding the whole, given a part and the percent. **Tools:** Calculator	**Prompt Features:** The student is prompted to solve a mathematical problem involving finding the whole, given a part and the percent. **Stimulus Guidelines:** • If used, context should be familiar to students 11 to 13 years old. • Percent and total quantities should be whole numbers. • Item difficulty can be adjusted via these example methods: o Benchmark percentages (such as 100% and 50%) are used. o Benchmark percentages (such as 75%, 25%, and 10%) are used. o Non-benchmark percentages are used. **TM7** **Stimulus:** The student is presented with a part and a percent. Enter the unknown value that makes this statement true: 30% of ☐ is 60. **Rubric:** (1 point) Student enters the correct numeric value representing the total amount (e.g., 200). **Response Type:** Equation/Numeric

Task Model 8 **Response Type: Equation / Numeric** **DOK Level 2** **6.RP.A.3c** Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. **Evidence Required:** 8. The student solves real-world and mathematical problems involving finding a percent of a quantity as a rate per 100. **Tools:** Calculator	**Prompt Features:** The student is prompted to solve a real-world or mathematical problem involving finding a percent of a quantity as a rate per 100. **Stimulus Guidelines:** • If used, context should be familiar to students 11 to 13 years old. • Generally percentages and quantities should be whole numbers unless appropriate for the situation. • Item difficulty can be adjusted via these example methods: o Benchmark percentages (such as 100% and 50%) are used. o Benchmark percentages (such as 75%, 25%, and 10%) are used. o Non-benchmark percentages are used. **TM8a** **Stimulus:** The student is presented with a part and a whole. **Example Stem 1:** Janet correctly answers 45 questions on her science test. There are 50 questions on the test. Enter the percent of the questions Janet did **not** answer correctly. **Example Stem 2:** Enter the unknown value that makes this statement true: 45 is ☐ % of 50. **Rubric:** (1 point) Student enters the correct numeric value representing the percent (e.g., 10; 90) and 0.90 is not an acceptable answer. Percent symbol (%) is not required for a correct response. **Response Type:** Equation/Numeric

Task Model 8 **Response Type: Multiple Choice, multiple correct response** **DOK Level 1** **6.RP.A.3c** Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. **Evidence Required:** 8. The student solves real-world and mathematical problems involving finding a percent of a quantity as a rate per 100. **Tools:** Calculator **Version 3 Update:** Evidence required statement 9 and TM9 have been retired.	**Prompt Features:** The student is prompted to identify solution methods for problems involving finding a percent of a quantity as a rate per 100. **Stimulus Guidelines:** • If used, context should be familiar to students 11 to 13 years old. • Percentages and quantities should be whole numbers. **TM8b** **Stimulus:** The student is presented with a real-world or mathematical percent problem. **Example Stem 1:** In a school with 200 students, 45% are males. Select **all** expressions that can be used to find the total number of male students**.** A. $\frac{45}{100} \bullet 200$ B. $\frac{0.45}{100} \bullet 200$ C. $0.45 \bullet 200$ D. $\frac{45}{10} \bullet 200$ **Example Stem 2:** Select **all** expressions that can be used to find 45% of 200. A. $\frac{45}{100} \bullet 200$ B. $\frac{0.45}{100} \bullet 200$ C. $0.45 \bullet 200$ D. $\frac{45}{10} \bullet 200$ **Answer Choices:** At least two expressions must be correct. **Rubric:** (1 point) Student selects all the correct mathematical expressions (e.g., A and C; A and C). **Response Type:** Multiple Choice, multiple correct response

Task Model 10 **Response Type: Equation / Numeric** **DOK Level 2** **6.RP.A.3d** Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. **Evidence Required:** 10. The student uses ratio reasoning to manipulate and transform units appropriately when multiplying or dividing quantities. **Tools:** Calculator **Version 3 Update:** Revised stimulus guidelines and example stem to eliminate multi-unit conversions.	**Prompt Features:** The student is prompted to use ratio reasoning to convert measurement units. **Stimulus Guidelines:** • If used, context should be familiar to students 11 to 13 years old. • Units of measurement should be rational numbers appropriate for the given situation. • Specify measurement relationship when needed (e.g., 1 inch = 2.54 cm). • Item difficulty can be adjusted via these example methods: o All numbers used in conversion are whole numbers. o Some numbers used in conversion are decimals. **TM10** **Stimulus:** The student is presented with a measurement and is asked to convert it to an equivalent measurement. **Example Stem:** Aaron needs 24 inches of copper wire for an experiment. The wire is sold by the centimeter. Given that 1 inch = 2.54 centimeters, how many **centimeters** of wire does Aaron need? **Rubric:** (1 point) Student enters the correct numeric value for the converted unit of measurement [e.g., 60.96 (accept 61 because of the real-word context)]. **Response Type:** Equation/Numeric

Claim 1: Concepts and Procedures
Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.

Content Domain: **The Number System**

Target B [m]: Apply and extend previous understandings of multiplication and division to divide fractions by fractions. (DOK Levels 1, 2)

Tasks for this target will ask students to divide fractions by fractions, including using this as a strategy to solve one-step contextual problems.

Standards: 6.NS.A, 6.NS.A.1	**6.NS.A Apply and extend previous understanding of multiplication and division to divide fractions by fractions.** **6.NS.A.1** Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ac/bd.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*
Related Below-Grade and Above-Grade Standards for Purposes of Planning for Vertical Scaling: 5.NF.B, 5.NF.B.7 7.NS.A, 7.NS.A.2	**Related Grade 5 Standards** **5.NF.B Apply and extend previous understandings of multiplication and division to multiply and divide fractions.** **5.NF.B.7** Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain the (1/3) ÷ 4 = 1/12 because 1/12 x 4 = 1/3.* b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ 1/5, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ 1/5= 20 because 20 x (1/5) = 4.* c. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?* **Related Grade 7 Standards** **7.NS.A Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.** **7.NS.A.2** Apply and extend previous understandings of multiplication and division of fractions to multiply and divide rational

	numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring the operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
DOK Levels:	1, 2
Achievement Level Descriptors:	
RANGE Achievement Level Descriptor (Range ALD) Target B: Apply and extend previous knowledge of multiplication and division to divide fractions by fractions.	**Level 1** Students should be able to apply and extend previous understandings of multiplication and division to multiply a fraction by a fraction, divide a fraction by a whole number, and be able to connect to a visual model. They should understand the effect that a fraction greater than or less than 1 has on a whole number when multiplied and use or create visual models when multiplying a whole number by a fraction between 0 and 1.
	Level 2 Students should be able to apply and extend previous understandings of multiplication and division to divide a whole number by a fraction between 0 and 1, divide a mixed number by a whole number, and be able to connect to a visual model.
	Level 3 Students should be able to apply and extend previous understandings of multiplication and division to divide a fraction by a fraction and be able to connect to a visual model.
	Level 4 Students should be able to use visual models in settings where smaller fractions are divided by larger fractions. They should also understand and apply the fact that a fraction multiplied or divided by 1 in the form of a/a is equivalent to the original fraction.
Evidence Required:	1. The student interprets quotients of fractions using visual fraction models, equations, and the relationship between multiplication and division. 2. The student solves real-world and mathematical one-step problems involving division of fractions by fractions.
Allowable Response Types:	Multiple Choice, single correct response; Drag and Drop; Equation/Numeric
Allowable Stimulus Materials:	visual fraction models
Construct-Relevant Vocabulary:	fraction, quotient, product
Allowable Tools:	none
Target-Specific Attributes:	The problems involving division of fractions by fractions should be able to be solved in one step.
Non-Targeted Constructs:	none

Accessibility Guidance:	Item writers should consider the following Language and Visual Element/Design guidelines[1] when developing items. Language Key Considerations: • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level • Avoid ambiguous or obscure words, idioms, jargon, unusual names and references Visual Elements/Design Key Considerations: • Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context • Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary • Avoid crowding of details and graphics Items are selected for a student's test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.[2]

[1] For more information, refer to the General Accessibility Guidelines at: http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf

[2] For more information about student accessibility resources and policies, refer to http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

Task Model 1 **Response Type: Equation / Numeric** **DOK Level 2** **6.NS.A.1** Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. **Evidence Required:** 1. The student interprets quotients of fractions using visual fraction models, equations, and the relationship between multiplication and division. **Tools:** None **Version 3 Update:** Retired TM1a, TM1b, and TM1c.	**Prompt Features:** The student is prompted to recognize and use the relationship between multiplication and division. **Stimulus Guidelines:** • All fractions should be positive. • Item difficulty can be adjusted via these example methods: o Students find an unknown number in a division problem. o Students find an unknown dividend in a given equation involving division of two fractions. o Students find an unknown divisor in a given equation involving division of two fractions. **TM1d** **Stimulus:** The student is presented with a quotient equation with an unknown fraction or number. **Example Stem 1:** The equation shown has an unknown number. $\square \div \frac{2}{3} = \frac{3}{4}$ Enter a number that makes the equation true. **Example Stem 2:** The equation shown has an unknown number. $\frac{2}{3} \div \square = \frac{6}{8}$ Enter a number that makes the equation true. **Rubric:** (1 point) Student enters the correct fraction (e.g., $\frac{1}{2}$; $\frac{8}{9}$ or equivalent value). **Response Type:** Equation/Numeric

Task Model 2 **Response Type: Equation / Numeric** **DOK Level 1** **6.NS.A.1** Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. **Evidence Required:** 2. The student solves real-world and mathematical one-step problems involving division of fractions by fractions. **Tools:** None **Version 3 Update:** Added more example stems to TM2b and added new TM2c.	**Prompt Features:** The student is prompted to solve a one-step mathematical or real-world problem involving division of fractions by fractions. **Stimulus Guidelines:** • Context should be familiar to students 11 to 13 years old. • Numbers used could be positive fractions and/or mixed numbers. • Answers should be appropriate for the context. • Item difficulty can be adjusted via these example methods: o Students solve a problem involving division of two fractions (no mixed numbers). o Students solve a problem involving division of two fractions (at least one mixed number). o divide two fractions (at least one mixed number). • Do not allow operation symbols in the response keypad. **TM2a** **Stimulus:** The student is is asked to compute the quotient of two fractions. **Example Stem 1:** What is the value of $\frac{2}{3} \div \frac{3}{4}$? **Example Stem 2:** What is the value of $2\frac{2}{3} \div \frac{3}{4}$? **Rubric:** (1 point) Student enters a whole number, mixed number, or fraction equivalent to the correct quotient (e.g., $\frac{8}{9}$; $3\frac{5}{9}$). **Response Type:** Equation/Numeric **TM2b** **Stimulus:** The student is presented with a real-world one-step problem involving division of fractions by fractions. **Example Stem 1:** A recipe requires $\frac{3}{4}$ cup of nuts for 1 batch of muffins. Enter the number of batches of muffins that can be made using $7\frac{1}{2}$ cups of nuts. **Example Stem 2:** Nina used $3\frac{3}{4}$ liters of water to completely fill 3 water bottles. If the water bottles are all the same size, how many liters of water does each bottle hold? Enter your answer in the response box.

	Example Stem 3: Joey made $\frac{1}{2}$ of a recipe and used $\frac{3}{4}$ cups of peas. How many cups of peas are required for a whole recipe? Enter your answer in the response box. **Rubric:** (1 point) Student enters the correct quotient (e.g., 10; $1\frac{1}{4}$ or $\frac{5}{4}$; $1\frac{1}{2}$ or $\frac{6}{4}$ or equivalents). **Response Type:** Equation/Numeric

Task Model 2 **Response Type: Multiple choice, multiple select response** **DOK Level 2** **6.NS.A.1** Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. **Evidence Required:** 1. The student interprets quotients of fractions using visual fraction models, equations, and the relationship between multiplication and division. **Tools:** None **Version 3 Update:** Added new TM2c	**Prompt Features:** The student is prompted to interpret fraction division in a context. **Stimulus Guidelines:** • Context should be familiar to students 11 to 13 years old. • Numbers used could be positive fractions and/or mixed numbers. • Item difficulty can be adjusted via these example methods: o by including different combinations of whole numbers, o fractions less than 1, o fractions greater than 1, o mixed numbers as dividend, divisor, and quotient. **TM2c** **Stimulus:** The student is asked to interpret fraction division in a context. **Example Stem 1:** Select **all** the questions that can be answered by determining the value of $1\frac{3}{4} \div \frac{1}{2}$? A. Chloe has $1\frac{3}{4}$ kilograms of rice she is using to fill $\frac{1}{2}$ kilogram packets. How many packets can she fill? B. Terry ran $1\frac{3}{4}$ miles. This is $\frac{1}{2}$ the distance that Kim ran. What is the distance, in miles, that Kim ran? C. Danielle has a cat who is $1\frac{3}{4}$ years old. Her dog is $\frac{1}{2}$ that age. How old is her dog? D. Jeri had $1\frac{3}{4}$ pounds of gummi worms, which she shared equally with her best friend. How many pounds of gummi worms did they each get? **Rubric:** (1 point) The student selects all of the contexts that can be represented by the given quotient (e.g., A, B). **Response Type:** Multiple choice, multiple select response

Claim 1: Concepts and Procedures
Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.

Content Domain: **The Number System**

Target C [a]: Compute fluently with multi-digit numbers and find common factors and multiples. (DOK Levels 1, 2)

Tasks for this target will ask students to divide multi-digit numbers and add, subtract, multiply, and divide multi-digit decimals. Other tasks will ask students to find the greatest common factor of two whole numbers less than or equal to 100; find the least common multiple of two whole numbers less than or equal to 12; and express the sum of two whole numbers 1–100 with a common factor as a multiple of the sum of two whole numbers with no common factor, or find the missing value in an equation representing such equivalence (see connections to 6.EE Targets E and F to generate items with greater range of difficulty).

Standards: 6.NS.B, 6.NS.B.2, 6.NS.B.3, 6.NS.B.4	**6.NS.B Compute fluently with multi-digit numbers and find common factors and multiples.** **6.NS.B.2** Fluently divide multi-digit numbers using the standard algorithm. **6.NS.B.3** Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. **6.NS.B.4** Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*
Related Below-Grade and Above-Grade Standards for Purposes of Planning for Vertical Scaling: 5.NBT.B, 5.NBT.B.6, 5.NBT.B.7 7.NS.A, 7.NS.A.2	**Related Grade 5 Standards** **5.NBT.B Perform operations with multi-digit whole numbers and with decimals to the hundredths.** **5.NBT.B.6** Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **5.NBT.B.7** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. **Related Grade 7 Standards** **7.NS.A Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.** **7.NS.A.2** Apply and extend previous understandings of multiplication and division of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring the operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (−1)(−1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

	b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
DOK Levels:	1, 2
Achievement Level Descriptors:	
RANGE Achievement Level Descriptor (Range ALD) Target C: Compute fluently with multi-digit numbers and find common factors and multiples.	**Level 1** Students should be able to add, subtract, and multiply multi-digit whole numbers and decimals to hundredths. They should be able to use the distributive property to express the sum of two whole numbers with a common factor.
	Level 2 Students should be able to divide multi-digit whole numbers and add and subtract multi-digit decimal numbers. They should be able to find common factors of two numbers less than or equal to 100 and multiples of two numbers less than or equal to 12.
	Level 3 Students should be able to fluently divide multi-digit numbers and add, subtract, multiply, and divide multi-digit decimal numbers. They should be able to find the greatest common factor of two numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12.
	Level 4 Students should be able to make generalizations regarding multiples and factors of sets of numbers (e.g., state that a particular set of numbers is relatively prime).
Evidence Required:	1. The student divides multi-digit numbers. 2. The student adds, subtracts, multiplies, and divides multi-digit decimals. 3. The student determines the greatest common factor of two whole numbers. 4. The student determines the least common multiple of two whole numbers. 5. The student uses the distributive property to express a sum of two whole numbers with a common factor as a multiple of a sum of two whole numbers with no common factor.
Allowable Response Type:	Equation/Numeric
Allowable Stimulus Materials:	None
Construct-Relevant Vocabulary:	sum, difference, product, quotient, common factor, greatest common factor, common multiple, least common multiple, distributive property
Allowable Tools:	None
Target-Specific Attributes:	A multi-digit dividend should have at least 4 digits. A multi-digit divisor should have at least 2 digits. A multi-digit decimal can be to the thousandths. The greatest common factor must be of two whole numbers less than or equal to 100.

	The least common multiple must be of two whole numbers less than or equal to 12. When using the distributive property to express a sum of two whole numbers, the whole numbers must be 1–100.
Non-Targeted Constructs:	
Accessibility Guidance:	Item writers should consider the following Language and Visual Element/Design guidelines[1] when developing items. Language Key Considerations: • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level • Avoid ambiguous or obscure words, idioms, jargon, unusual names and references Visual Elements/Design Key Considerations: • Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context • Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary • Avoid crowding of details and graphics Items are selected for a student's test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.[2]

[1] For more information, refer to the General Accessibility Guidelines at: http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf

[2] For more information about student accessibility resources and policies, refer to http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

Task Model 1 **Response Type: Equation / Numeric** **DOK Level 1** **6.NS.B.2** Fluently divide multi-digit numbers using the standard algorithm. **Evidence Required:** 1. The student divides multi-digit numbers. **Tools:** None	**Prompt Features:** The student is prompted to find the quotient of multi-digit numbers with or without a remainder. **Stimulus Guidelines:** • The expression should be in the form $x \div y$, where x is a 4–6-digit positive integer and y is a 2–5-digit positive integer. Exception: do not have x as a 4-digit number and y as a 2-digit number without a remainder. • Generally answers with remainders should terminate no greater than the hundredths place. • Item difficulty can be adjusted via these example methods: o Students find quotient with no remainder (4-digit divided by 3-digit). o Students find quotient with no remainder (5- or 6-digit divided by 2- or 3-digit). o Students find quotient with a remainder (4- or 5-digit divided by 2- or 3-digit). o Students find quotient with a remainder or students interpret a division algorithm (4-digit divided by 4-digit; 6-digit divided by 2- or 3-digit). o Students find quotient with a remainder (5-digit divided by 4- or 5-digit; 6-digit divided by 4-, 5-, or 6-digit). **TM1** **Stimulus:** The student is presented with a division expression. **Example Stem 1:** Divide. 16,536 ÷ 24 Enter the exact quotient. **Example Stem 2:** Divide. 35,702 ÷ 25 Enter the exact quotient. **Rubric:** (1 point) Student enters the correct quotient (e.g., 689; 1428.08). **Response Type:** Equation/Numeric

Task Model 2 **Response Type: Equation / Numeric** **DOK Level 1** **6.NS.B.3** Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. **Evidence Required:** 2. The student adds, subtracts, multiplies, and divides multi-digit decimals. **Tools:** None	**Prompt Features:** The student is prompted to find the sum, difference, product, or quotient of multi-digit numbers with or without a remainder using the standard algorithm**.** **Stimulus Guidelines:** • Given numbers and answers should be positive. • Item difficulty can be adjusted via these example methods: o Students add two multi-digit decimals; at least one decimal in thousandths. o Students add two multi-digit decimals, at least one decimal in ten-thousandths OR add three multi-digit decimals, at least one decimal in thousandths or ten-thousandths. o Students subtract two multi-digit decimals, at least one decimal in thousandths or ten thousandths. o Students multiply two multi-digit decimals, at least one decimal in thousandths. o Students find quotient of multi-digit decimals, at least one decimal to thousandths OR product of two multi-digit decimals, at least one decimal in ten-thousandths. **TM2a** **Stimulus:** The student is presented with an addition expression with two or three terms. **Example Stem:** Add. 34.381 + 8.2 Enter the exact sum. **Rubric:** (1 point) Student enters the correct sum (e.g., 42.581). **Response Type:** Equation/Numeric **TM2b** **Stimulus:** The student is presented with a subtraction expression with two terms. **Example Stem:** Subtract. 48.235 – 29.67 Enter the exact difference. **Rubric:** (1 point) Student enters the correct difference (e.g., 18.565). **Response Type:** Equation/Numeric

Task Model 2 **Response Type: Equation / Numeric** **DOK Level 1** **6.NS.B.3** Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. **Evidence Required:** 2. The student adds, subtracts, multiplies, and divides multi-digit decimals. **Tools:** None	**TM2c** **Stimulus:** The student is presented with a multiplication expression with two decimals. **Example Stem:** Multiply. 8.296 ● 0.8 Enter the exact product. **Rubric:** (1 point) Student enters the correct product (e.g., 6.6368). **Response Type:** Equation/Numeric **TM2d** **Stimulus:** The student is presented with a division expression with two decimals. • The divisor place value should be to the tenths or hundredths and the dividend place value should be at the thousandths or the ten-thousandths. • Answers should be a positive answer that terminates no greater than the thousandths place. **Example Stem:** Divide. 0.912 ÷ 0.24 Enter the exact quotient. **Rubric:** (1 point) Student enters the correct quotient (e.g., 3.8). **Response Type:** Equation/Numeric

Task Model 2 **Response Type: Equation / Numeric** **DOK Level 2** **6.NS.B.3** Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. **Evidence Required:** 2. The student adds, subtracts, multiplies, and divides multi-digit decimals. **Tools:** None	**Prompt Features:** The student is prompted to reason and interpret about addition, subtraction, multiplication, or division problems. **Stimulus Guidelines:** Given numbers and answers should be positive and item difficulty can be adjusted by changing whether the given equation is addition, subtraction, multiplication, or division. **TM2e** **Stimulus:** The student is presented with an addition/subtraction/multiplication/division equation. **Example Stem:** Use the fact that 12 ● 218 = 2616. Enter the exact product of 1.2 ● 2.18. **Rubric:** (1 point) Student enters the correct product (e.g., 2.616). **Note:** Students should be able to determine the product without calculating it, but instead by using the given computation and reasoning skills. **Response Type:** Equation/Numeric

Task Model 3 **Response Type: Equation / Numeric** **DOK Level 1** **6.NS.B.4** Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4(9 + 2).* **Evidence Required:** 3. The student determines the greatest common factor of two whole numbers. **Tools:** None	**Prompt Features:** The student is prompted to find the greatest common factor of two whole numbers**.** **Stimulus Guidelines:** • Greatest common factor should be greater than 1. • Whole numbers should be less than or equal to 100. • Item difficulty can be adjusted via these example methods: o Students find GCF of two whole numbers (both numbers are even; GCF less than 10). o Students find GCF of two whole numbers (GCF between 10 and 20). o Students find GCF of two whole numbers (one of the numbers is a multiple of 5, the other is a multiple of 10). o Students find GCF of two whole numbers (one of the numbers is a prime number greater than 20 and is a factor of the other number). **TM3** **Stimulus:** The student is presented with two whole numbers less than 100. **Example Stem:** Enter the greatest common factor of 24 and 36. **Rubric:** (1 point) Student enters the correct greatest common factor (e.g., 12). **Response Type:** Equation/Numeric

Task Model 4 **Response Type: Equation / Numeric** **DOK Level 1** **6.NS.B.4** Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4(9 + 2).* **Evidence Required:** 4. The student determines the least common multiple of two whole numbers. **Tools:** None	**Prompt Features:** The student is prompted to find the least common multiple of two whole numbers**.** **Stimulus Guidelines:** • Whole numbers should be less than or equal to 12. • Item difficulty can be adjusted via these example methods: o Students find LCM of two whole numbers (one of the numbers is 2). o Students find LCM of two whole numbers (one of the numbers is 5; both numbers lower than 6; LCM is less than 30). o Students find LCM of two whole numbers (one of the numbers is less than 6, the other number is greater than 6; LCM is less than 40). o Students find LCM of two whole numbers (LCM is greater than 40). **TM4** **Stimulus:** The student is presented with two whole numbers less than 12. **Example Stem:** Enter the least common multiple of 6 and 8. **Rubric:** (1 point) Student enters the correct least common multiple (e.g., 24). **Response Type:** Equation/Numeric

Task Model 5
 	
Response Type: Equation / Numeric
 	
DOK Level 2
 	
6.NS.B.4
Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4(9 + 2).*
 	
Evidence Required:
5. The student uses the distributive property to express a sum of two whole numbers with a common factor as a multiple of a sum of two whole numbers with no common factor.
 	
Tools: None
	Prompt Features: The student is prompted to identify equivalent expressions using the distributive property.
	
	Stimulus Guidelines:
	• The expression should be in the form $x + y = a\ (b + c)$ or $a\ (b + c) = x + y$ where x, y, a, b, and c are whole numbers between 1 and 100.
	• x and y should have a common factor greater than 1.
	• The missing number may be any of the variables x, y, a, b, and c.
	• Item difficulty can be adjusted via these example methods:
	  o Use only even numbers less than 20.
	  o Use only numbers less than 70.
	  o Use at least two numbers greater than 70.
	
	TM5
	Stimulus: The student is presented with an equation showing the distributive property with a missing number.
	
	Example Stem: Consider the equation.
	
	$24 + 30 = 6(4 + \square)$
	
	Enter the unknown number that makes the equation true.
	
	Rubric: (1 point) Student enters the correct value (e.g., 5).
	
	Response Type: Equation/Numeric

Claim 1: Concepts and Procedures
Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.

Content Domain: **The Number System**

Target D [m]: Apply and extend previous understandings of numbers to the system of rational numbers. (DOK Levels 1, 2)

Tasks for this claim will ask students to place numbers on a number line (positive and negative rational numbers, including those expressed using absolute value notation). Some tasks will ask students to interpret the meaning of zero in a context related to other given quantities in the problem.

Claim 3 tasks will integrate the work of this target by incorporating students' understanding of interpretations and explanations of common misconceptions related to inequalities for negative rational numbers (e.g., explaining that $-3^{o}C$ is warmer than $-7^{o}C$). Claims 2 and 4 will include items that ask students to solve problems in the four quadrants of the coordinate plane, including distances between points with the same first and second coordinate.

Standards: 6.NS.C, 6.NS.C.5, 6.NS.C.6, 6.NS.C.7, 6.NS.C.8	**6.NS.C Apply and extend previous understandings of numbers to the system of rational numbers.** **6.NS.C.5** Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. **6.NS.C.6** Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. **6.NS.C.7** Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.* b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3°C > –7°C to express the fact that –3°C is warmer than –7°C.* c. Understand the absolute value of a rational number as its

	distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.* d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.* **6.NS.C.8** Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.
Related Below-Grade and Above-Grade Standards for Purposes of Planning for Vertical Scaling: 5.G.A, 5.G.A.1, 5.G.A.2 7.NS.A, 7.NS.A.2, 7.NS.A.3	**Related Grade 5 Standards** **5.G.A Graph points on the coordinate plane to solve real-world and mathematical problems.** **5.G.A.1** Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate). **5.G.A.2** Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. **Related Grade 7 Standards** **7.NS.A Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.** **7.NS.A.2** Apply and extend previous understandings of multiplication and division of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring the operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If *p* and *q* are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates

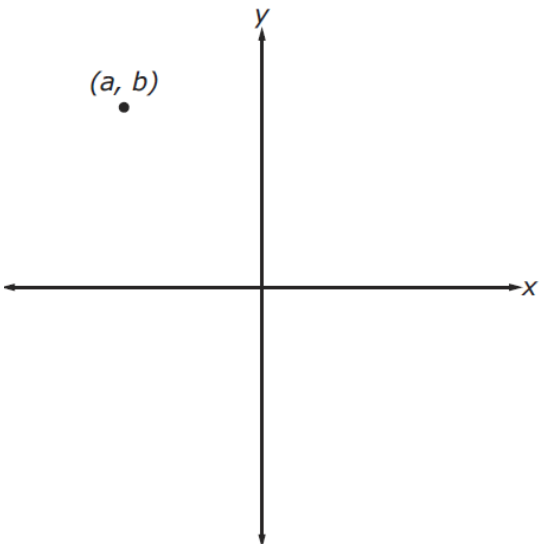
	in 0s or eventually repeats. **7.NS.A.3** Solve real-world and mathematical problems involving the four operations with rational numbers.
DOK Levels:	1, 2
Achievement Level Descriptors:	
RANGE Achievement Level Descriptor (Range ALD) Target D: Apply and extend previous understandings of numbers to the system of rational numbers.	**Level 1** Students should be able to place all integers on a number line and integer pairs on a coordinate plane with one-unit increments on both axes.
	Level 2 Students should be able to apply and extend previous understandings of whole numbers to order rational numbers and interpret statements of their order in the context of a situation. They should be able to place all rational numbers on a number line and integer pairs on a coordinate plane with various axis increments. They should be able to relate changes in sign to placements on opposite sides of the number line and understand the absolute value of a number as its distance from zero on a number line.
	Level 3 Students should be able to apply and extend previous understandings of numbers to relate statements of inequality to relative positions on a number line, place points with rational coordinates on a coordinate plane, and solve problems involving the distance between points when they share a coordinate. They should be able to understand absolute value and ordering by using number lines and models and relate reflection across axes to changes in sign.
	Level 4: No descriptor
Evidence Required:	1. The student uses positive and negative numbers to represent quantities in real-world contexts. 2. The student can identify the location of ordered pairs on the coordinate plane based on the signs of the numbers in an ordered pair. 3. The student locates and positions integers and other rational numbers on a number line. 4. The student positions ordered pairs of integers and other rational numbers on a coordinate plane. 5. [Evidence Required statement retired] 6. The student writes and interprets statements about the order of rational numbers in real-world contexts. 7. The student represents the absolute value of a rational number as the distance from zero on a number line. 8. The student can make comparisons of absolute value from statements about order. 9. The student solves real-world and mathematical problems by graphing ordered pairs on a coordinate plane and using coordinates and absolute value to find the distances between points with same first coordinate or same second coordinate.
Allowable Response Types:	Multiple Choice, single correct response; Multiple Choice, multiple correct response; Equation/Numeric; Matching Tables; Drag and

	Drop; Graphing; Hot Spot
Allowable Stimulus Materials:	horizontal and vertical number lines, coordinate planes
Construct-Relevant Vocabulary:	positive, negative, integer, absolute value, coordinate, ordered pair, coordinate grid/plane, quadrant, number line, relative position, magnitude
Allowable Tools:	None
Target-Specific Attributes:	
Non-Targeted Constructs:	
Accessibility Guidance:	Item writers should consider the following Language and Visual Element/Design guidelines[1] when developing items. Language Key Considerations: • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level • Avoid ambiguous or obscure words, idioms, jargon, unusual names and references Visual Elements/Design Key Considerations: • Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context • Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary • Avoid crowding of details and graphics Items are selected for a student's test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.[2]
Development Notes:	Claim 3 tasks will integrate the work of this target by incorporating students' understanding of interpretations and explanations of common misconceptions related to inequalities for negative rational numbers (e.g., explaining that $-3^{o}C$ is warmer than $-7^{o}C$). Claims 2 and 4 will include items that ask students to solve problems in the four quadrants of the coordinate plane, including distances between points with the same first and second coordinate.

[1] For more information, refer to the General Accessibility Guidelines at: http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf

[2] For more information about student accessibility resources and policies, refer to http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

Task Model 1 **Response Type: Equation / Numeric** **DOK Level 1** **6.NS.C.5** Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. **Evidence Required:** 1. The student uses positive and negative numbers to represent quantities in real-world contexts. **Tools:** None	**Prompt Features:** The student is prompted to interpret negative numbers in context. **Stimulus Guidelines:** • Context should be familiar to students 11 to 13 years old. • The context should involve quantities where negative values can be interpreted in an appropriate and unambiguous way such as references to temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge. • Students use a rational number to represent a given real-world scenario. **TM1** **Stimulus:** The student is presented with a context that can be unambiguously represented by a negative number or zero. **Example Stem:** A Fahrenheit thermometer shows that the temperature is 15 degrees below zero. Enter the integer that represents the temperature in degrees Fahrenheit. **Rubric:** (1 point) The student enters the correct number (e.g., –15). **Response Type:** Equation/Numeric

Task Model 2 **Response Types: Multiple Choice, multiple correct response; Hot Spot** **DOK Level 1** **6.NS.C.6b** Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. **Evidence Required:** 2. The student can identify the location of ordered pairs on the coordinate plane based on the signs of the numbers in an ordered pair. **Tools:** None **Version 3 Update:** Retired TM2a and TM2b. Added new TM2c. **Accessibility Note:** Graphing items are not currently able to be Brailled. Minimize the number of items developed to this TM.	**Prompt Features:** The student is prompted to locate a point in a different quadrant of the coordinate plane than a given point. **Stimulus Guidelines:** • Ordered pairs in the form ($\pm x$, $\pm y$), where x and y are rational numbers. • x and y cannot be equal to 0. **TM2c** **Stimulus:** The student is presented with coordinate axes and a point labeled (a, b) in one of the quadrants. **Example Stem:** The point that corresponds to (a, b) is shown in the coordinate plane. Use the Add Point tool to graph ($-a$, b).  **Rubric:** (1 point) The student places a point in the correct location with some tolerance. **Response Type:** Graphing

Task Model 3

Response Type: Multiple Choice, single correct response

DOK Level 1

6.NS.C.6c
Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

Evidence Required:
3. The student locates and positions integers and other rational numbers on a number line.

Tools: None

Version 3 Update:
Retired TM3a.

Prompt Features: The student is prompted to identify a number line containing correctly plotted rational numbers.

Stimulus Guidelines:

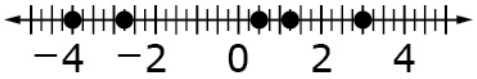
- Number lines should have tick marks and labels appropriate for the given numbers.
- Item difficulty can be adjusted via these example methods:
  - Values are integers.
  - Values are decimal numbers up to the hundredths.
  - Values are fractions/mixed numbers.
  - Values are fractions/mixed numbers and decimals.

TM3b
Stimulus: The student is presented with a list of rational numbers.

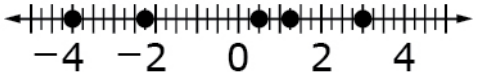
Example Stem: Which number line shows the correct positions of all the values shown?

$\frac{1}{2}$, -4, $-2\frac{3}{4}$, 3, $1\frac{1}{4}$

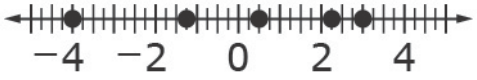
A.



B.



C.



D.

Answer Choices: Answer choices will be number lines with points plotted. Distractors will include incorrect placement of one or more numbers on a number line based on signs and/or positions of rational numbers.

Rubric: (1 point) Student selects the correct number line (e.g., A).

Response Type: Multiple Choice, single correct response

Task Model 3

Response Type: Matching Tables

DOK Level 1

6.NS.C.6c
Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

Evidence Required:
3. The student locates and positions integers and other rational numbers on a number line.

Tools: None

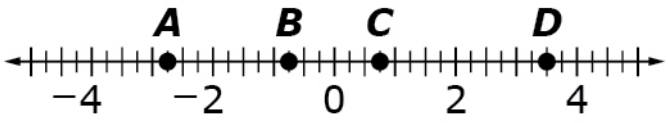
Prompt Features: The student is prompted to identify the rational numbers that occupy locations on a given number line.

Stimulus Guidelines:

- Number lines should have tick marks and labels appropriate for the given numbers.
- Item difficulty can be adjusted via these example methods:
  - Values are integers.
  - Values are decimal numbers up to the hundredths.
  - Values are fractions/mixed numbers.
  - Values are fractions/mixed numbers and decimals.

TM3c
Stimulus: The student is presented with a number line with labeled tick marks that contains 3–5 labeled points.

Example Stem: Consider the points plotted on the number line shown.



Select True or False for each statement about the number line.

Statement	True	False
The value of Point A is less than –3.		
The value of Point B is greater than the value of Point A.		
The value of Point D is $3\frac{1}{2}$.		

Rubric: (1 point) The student identifies all three statements correctly as true or false (e.g., F, T, T). Statements will include the opposite of the given number, failure to correctly plot fractions on a number line, etc.

Response Type: Matching Tables

Task Model 3

Response Type: Drag and Drop

DOK Level 1

6.NS.C.6c
Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

6.NS.7c
Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.*

Evidence Required:
3. The student locates and positions integers and other rational numbers on a number line.

Tools: None

Accessibility Note:
Drag and Drop items are not currently able to be Brailled. Minimize the number of items developed to this TM.

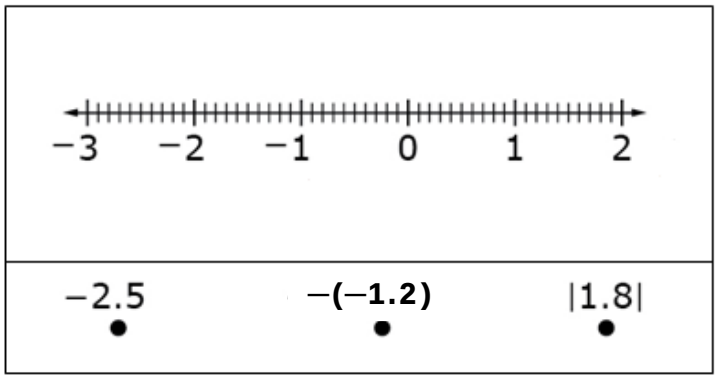
Prompt Features: The student is prompted to position rational numbers on a number line.

Stimulus Guidelines:

- At least one number should be in the form "–(–*x*)."
- The number line should be labeled appropriately.
- Numbers may be integers, fractions, or decimals. Appropriate tic marks should be identified on the number line with sufficient spacing.
- Item difficulty can be adjusted via these example methods:
  - Values are integers.
  - Values are decimal numbers up to the hundredths.
  - Values are fractions/mixed numbers.

TM3d
Stimulus: The student is presented with three rational numbers and an incomplete number line.

Example Stem: Drag each number to its correct location on the number line.



Interaction: The student uses a preplaced drag-and-drop tool. The points are labeled with a rational number value which students can drag to the number line. Use the snap-to feature for each tick mark.

Rubric: (1 point) Student plots all numbers correctly on the number line.

Response Type: Drag and Drop

Task Model 4

Response Type: Multiple Choice, single correct response

DOK Level 1

6.NS.C.6c
Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

Evidence Required:
4. The student positions ordered pairs of integers and other rational numbers on a coordinate plane.

Tools: None

Prompt Features: The student is prompted to identify the coordinate plane showing correctly graphed ordered pairs and vice versa.

Stimulus Guidelines:

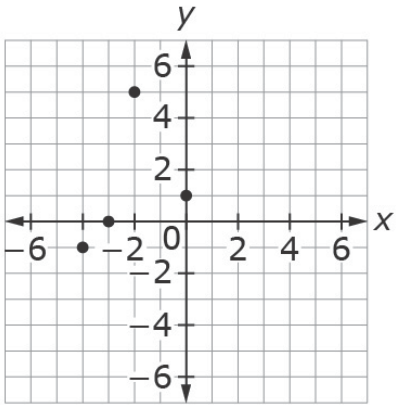
- There should be three to five total ordered pairs.
- At least two ordered pairs should contain negative coordinates.
- Ordered pairs are in the form $(\pm x, \pm y)$, where x and y may be integers and/or other rational numbers.
- For plotting rational numbers, coordinate plane scale should be such that students must use number line sense to place the points.
- Item difficulty can be adjusted via these example methods:
  - Students identify the ordered pairs for a given graph and vice versa (ordered pairs are integers).
  - Students identify the ordered pairs for a given graph and vice versa (ordered pairs include rational numbers).

TM4a
Stimulus: The student is presented with coordinates of ordered pairs and a coordinate plane with the ordered pairs plotted.

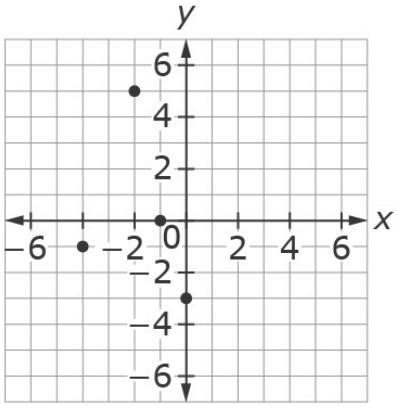
Example Stem 1: Which coordinate plane best represents the graph of these ordered pairs?

(–1, 0), (0, –3), (–4, –1), (–2, 5)

A.



B.



Task Model 4

Response Type: Multiple Choice, single correct response

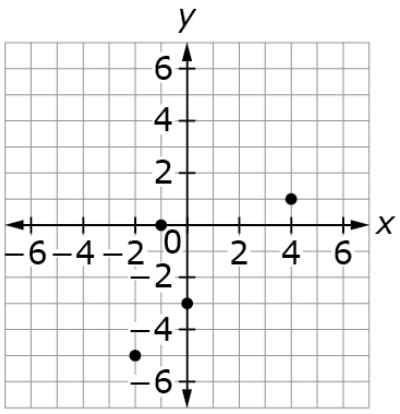
DOK Level 1

6.NS.C.6c
Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

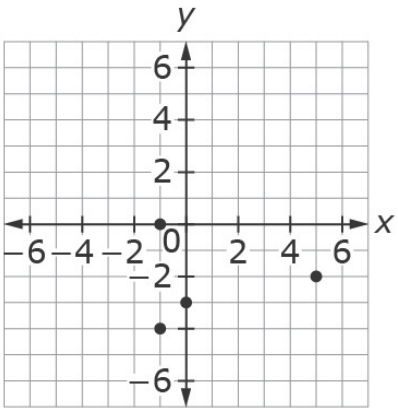
Evidence Required:
4. The student positions ordered pairs of integers and other rational numbers on a coordinate plane.

Tools: None

C.



D.



Rubric: (1 point) The student identifies the correct graph (e.g., B).

Answer Choices: Answer choices will be coordinate planes with three to five ordered pairs plotted. Distractors will include errors in signs of numbers and/or confusing *x*- and *y*-axis or coordinates.

Response Type: Multiple Choice, single correct response

Task Model 4

Response Type: Multiple Choice, single correct response

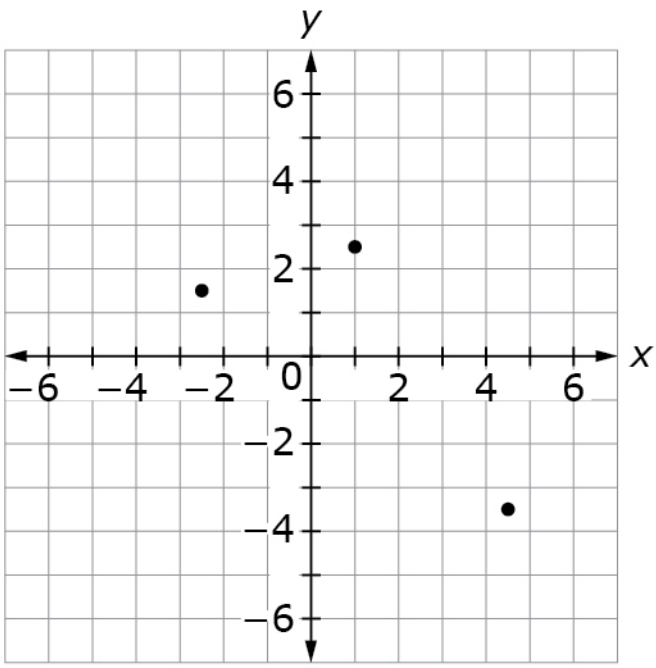
DOK Level 1

6.NS.C.6c
Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

Evidence Required:
4. The student positions ordered pairs of integers and other rational numbers on a coordinate plane.

Tools: None

Example Stem 2: Consider the coordinate plane.



Which list of ordered pairs corresponds to the points on the coordinate plane?

A. $(-4\frac{1}{2}, -3\frac{1}{2})$, $(-1, 2\frac{1}{4})$, $(-2\frac{1}{2}, -1\frac{1}{2})$

B. $(-2\frac{1}{2}, 1\frac{1}{2})$, $(4\frac{1}{2}, -3\frac{1}{2})$, $(1, 2\frac{1}{4})$

C. $(-3\frac{1}{2}, -4\frac{1}{2})$, $(1, -2\frac{1}{4})$, $(2\frac{1}{2}, 1\frac{1}{2})$

D. $(2\frac{1}{2}, -1\frac{1}{2})$, $(4\frac{1}{2}, 3\frac{1}{2})$, $(1, -2\frac{1}{4})$

Rubric: (1 point) Student selects the correct set of ordered pairs (e.g., B).

Answer Choices: Answer choices will be lists of ordered pairs. Distractors will include errors in signs of numbers and/or confusing *x*- and *y*-axis or coordinates.

Response Type: Multiple Choice, single correct response

Task Model 4

Response Type: Graphing

DOK Level 1

6.NS.C.6c
Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

Evidence Required:
4. The student positions ordered pairs of integers and other rational numbers on a coordinate plane.

Tools: None

Accessibility Note: Graphing items are not currently able to be Brailled. Minimize the number of items developed to this TM.

Version 3 Update: Retired Evidence Required statement 5 and TM5.

Prompt Features: The student is prompted to position ordered pairs on a coordinate plane.

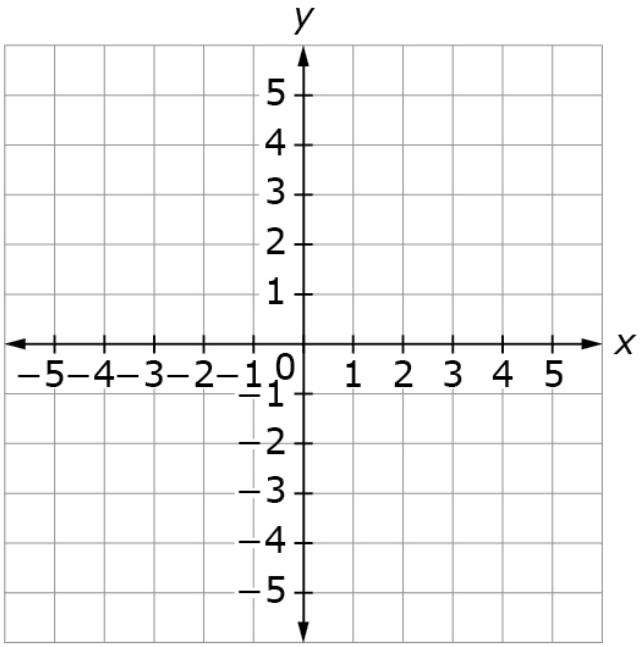
Stimulus Guidelines:

- The coordinate plane should have axes and values labeled.
- The ordered pairs may contain combinations of positive and negative integers and rational numbers that could be graphed in all four quadrants.
- For plotting rational numbers, coordinate plane scale should be such that student must use number line sense to place the points.
- Item difficulty can be adjusted via these example methods:
  - Both coordinates are positive integers.
  - At least one coordinate is a negative integer.
  - At least one coordinate is a rational number.
  - Both coordinates are rational numbers.

TM4b
Stimulus: The student is presented with three ordered pairs and a graphic of a coordinate plane.

Example Stem: Use the Add Point tool to plot these three ordered pairs on the coordinate grid:

(–2, 3)
(0, 3)
(–4, –2)



Interaction: The student uses the Add Point and Delete tools to graph the ordered pairs. Use the snap-to feature for each intersection of the grid.

Rubric: (1 point) Student plots all three points correctly on the coordinate plane.

Response Type: Graphing

Task Model 6

Response Type: Drag and Drop

DOK Level 2

6.NS.C.7b
Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3°C > –7°C to express the fact that –3°C is warmer than –7°C.*

Evidence Required:
6. The student writes and interprets statements about the order of rational numbers in real-world contexts.

Tools: None

Accessibility Note: Drag and Drop items are not currently able to be Brailled. Minimize the number of items developed to this TM.

Prompt Features: The student is prompted to order rational numbers in a real-world context.

Stimulus Guidelines:

- The context should involve opposite directions or values such as temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge.
- Context should be familiar to students 11 to 13 years old.
- Table should have three to five rows of data.
- Item difficulty can be adjusted via these example methods:
  - Numbers contain positive and negative integers.
  - Numbers contain positive and negative decimals.
  - Numbers contain positive and negative fractions/mixed numbers.
  - All numbers are fractions/mixed numbers and decimals.

TM6a
Stimulus: The student is presented with a real-world context involving rational numbers.

Example Stem: Sea level is defined as being at an elevation of 0 feet. The elevation of land is defined to be its height above or below sea level. The table shows the lowest elevations in some states.

Drag the numbers to each empty box to place the elevations in order from least to greatest.

State	Elevation
Arizona	72 ft
California	–282 ft
Louisiana	–68 ft
Tennessee	178 ft

☐ ☐ ☐ ☐

Least Greatest

72 ft -282 ft -68 ft 178 ft

Interaction: The student is given four empty boxes below the table and a palette at the bottom. The palette should contain the four numbers preplaced from the table (i.e., 72 ft, -282 ft, -68 ft, and 178 ft). Students use the drag-and-drop feature to place numbers in the boxes. Numbers may be used only once.

Rubric: (1 point) The student drags all four rational numbers in order from least to greatest.

Response Type: Drag and Drop

Task Model 6

Response Type: Matching Tables

DOK Level 2

6.NS.C.7b
Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3°C > –7°C to express the fact that –3°C is warmer than –7°C.*

Evidence Required: 6. The student writes and interprets statements about the order of rational numbers in real-world contexts.

Tools: None

Prompt Features: The student is prompted to determine whether statements relating to the order of rational numbers are true or false in a real-world context.

Stimulus Guidelines:

- The context should involve opposite directions or values such as temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge.
- Context should be familiar to students 11 to 13 years old.
- Tables should have three to five rows of data.
- Item difficulty can be adjusted via these example methods:
  - One number is negative.
  - Both integers are negative.
  - Numbers are negative decimals.
  - Numbers are negative fractions/mixed numbers.

TM6b
Stimulus: The student is presented with a real-world context involving rational numbers.

Example Stem: Sea level is defined as being at an elevation of 0 feet. The elevation of land is defined to be its height above or below sea level. This table shows the lowest elevations in some states.

State	Elevation
Arizona	72 ft
California	–282 ft
Louisiana	–68 ft
Tennessee	178 ft

Determine whether each statement about the elevations is correct. Select True or False for each statement.

Statement	True	False
California has a higher elevation than Louisiana because –282 is greater than –68.		
Tennessee's elevation is farther from 0 than Louisiana's elevation.		
Louisiana has a higher elevation than California because –68 is closer to zero than –282.		

Rubric: (1 point) The student identifies all three statements correctly as true or false (e.g., F, T, T). Statements should deal with the order of the numbers.

Response Type: Matching Tables

Task Model 6 **Response Type: Equation / Numeric** **DOK Level 2** **6.NS.C.7b** Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3°C > –7°C to express the fact that –3°C is warmer than –7°C.* **Evidence Required:** 6. The student writes and interprets statements about the order of rational numbers in real-world contexts. **Tools:** None	**Prompt Features:** The student is prompted to give an inequality based on a verbal description of a real-world context involving rational numbers. **Stimulus Guidelines:** • The context should involve opposite directions or values such as temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge. • Context should be familiar to students 11 to 13 years old. • Item difficulty can be adjusted via these example methods: o One number is negative. o Both integers are negative. o Numbers are negative decimals. o Numbers are negative fractions/mixed numbers. **TM6c** **Stimulus:** The student is presented with a real-world context involving rational numbers. **Example Stem:** Sea level is defined as being at an elevation of 0 feet. • The lowest elevation in Arizona is 72 feet. • The lowest elevation in Louisiana is –68 feet. Enter an inequality that compares these two elevations. **Rubric:** (1 point) The student enters a correct inequality statement. Students are allowed credit for putting either "–68 < 72" or "72 > –68." **Response Type:** Equation/Numeric

Task Model 7

Response Type: Drag and Drop

DOK Level 2

6.NS.C.7c
Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.

Evidence Required:
7. The student represents the absolute value of a rational number as the distance from zero on a number line.

Tools: None

Accessibility Note: Drag and Drop items are not currently able to be Brailled. Minimize the number of items developed to this TM.

Prompt Features: The student positions numbers on the number line, including numeric expressions that involve the absolute value of numbers.

Stimulus Guidelines:

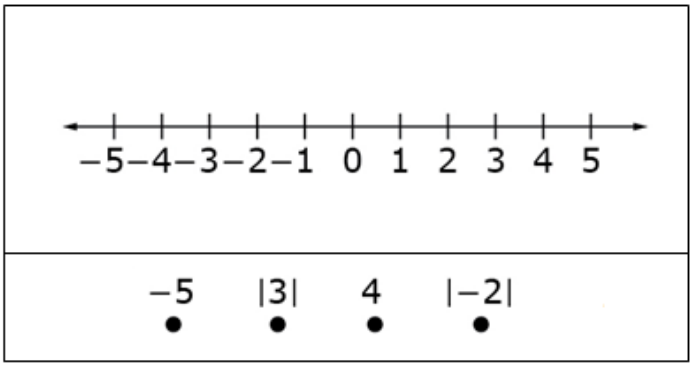
- Rational numbers should be a mixture of positive, negative, and absolute value.
- At least two of the numbers need to contain absolute values.
- Item difficulty can be adjusted via these example methods:
  - Values are integers and include absolute values.
  - Values are decimal numbers up to the hundredths and include absolute values.
  - Values are fractions/mixed numbers and include absolute values.

TM7a
Stimulus: The student is presented with a set of four or five rational numbers and a number line.

Example Stem: Consider this set of numbers.

$-5,\ |3|,\ 4,\ |-2|$

Drag the four values to their correct locations on the number line.



Interaction: The points are pre-labeled with the given rational number values and placed in a palette below the number line in which students can drag the points to the number line. Use the snap-to feature for each tick mark.

Rubric: (1 point) Student plots all four numbers correctly on the number line.

Response Type: Drag and Drop

Task Model 7

Response Type: Matching Tables

DOK Level 2

6.NS.C.7c
Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.

Evidence Required:
7. The student represents the absolute value of a rational number as the distance from zero on a number line.

Tools: none

Prompt Features: The student is prompted to determine whether statements relating to absolute value are true or false.

Stimulus Guidelines:

- Item difficulty can be adjusted via these example methods:
  - Numbers used are integers, decimals, and fractions/mixed numbers.
  - Using two negative numbers may be more difficult than a positive and a negative.

TM7b
Stimulus: The student is presented with statements about the absolute value of numbers in relation to a number line.

Example Stem: Consider the statements in the table shown. Select True or False for each statement.

Statement	True	False
The distance from –3 to 0 is the same as the distance from 3 to 0 on the number line.		
The distance between –21 and 0 on a number line is |–21| units.		
On a number line, |4| and –4 are the same point.		

Rubric: (1 point) The student identifies all three statements correctly as true or false (e.g., T, T, F). Statements will be about the location of numbers with absolute values. False statements include statements that ignore absolute value signs and suggest an absolute value sign means “the opposite of” the number.

Response Type: Matching Tables

Task Model 8

Response Type: Matching Tables

DOK Level 1

6.NS.C.7d
Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.*

Evidence Required:
8. The student can make comparisons of absolute value from statements about order.

Tools: None

Prompt Features: The student is prompted to determine whether statements comparing numbers containing absolute value in real-world contexts are true.

Stimulus Guidelines:

- The context should involve opposite directions or values such as temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge.
- Context should be familiar to students 11 to 13 years old.
- Item difficulty can be adjusted by varying the numbers to compare absolute value, fractions, and mixed numbers.

TM8
Stimulus: The student is presented with statements involving absolute value in a real-world context.

Example Stem: Sea level is defined as being at an elevation of 0 feet. Objects can be above or below sea level.

- Submarine J is 35.6 feet below sea level.
- Submarine Q is 21.5 feet below sea level.
- Submarine Z is 43.8 feet below sea level.

Determine whether each statement comparing the submarines is true. Select True or False for each statement.

Statement	True	False
Submarine J is deeper than Submarine Q because $\|-35.6\| > \|-21.5\|$.		
Submarine Q is deeper than Submarine Z because $\|-21.5\| > \|-43.8\|$.		
Submarine J is deeper than Submarine Z because $\|-35.6\| > \|-43.8\|$.		

Rubric: (1 point) The student correctly identifies all three statements as true or false (e.g., T, F, F).

Response Type: Matching Tables

Task Model 9

Response Type: Equation / Numeric

DOK Level 2

6.NS.C.8
Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Evidence Required:
9. The student solves real-world and mathematical problems by graphing ordered pairs on a coordinate plane and using coordinates and absolute value to find the distances between points with same first coordinate or same second coordinate.

Tools: None

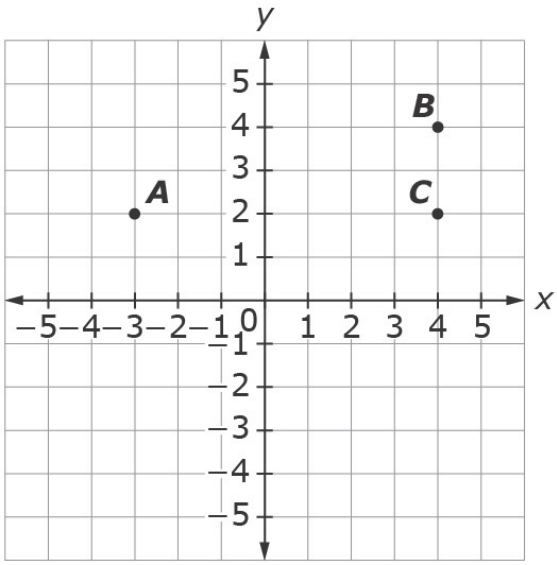
Prompt Features: The student is prompted to solve real-world or mathematical problems by using ordered pairs on a coordinate plane and absolute value to find distances between points with the same first coordinate or same second coordinate.

Stimulus Guidelines:

- Coordinates of the ordered pairs generally should be limited to integers unless appropriate for the situation.
- Multiple ordered pairs should have the same first coordinate or same second coordinate.
- If used, context should be familiar to students 11 to 13 years old.
- Item difficulty can be adjusted via these example methods:
  - Students find the distance between points in first quadrant only.
  - Students find the distance between points in adjacent quadrants.

TM9
Stimulus: The student is presented with a real-world or mathematical context and a graph of ordered pairs.

Example Stem 1: This grid shows the location of three points.



Enter the distance, in units, between point A and point C.

Rubric: (1 point) Student enters the correct numeric value for the distance (e.g., 7). Units of measure should be assumed from the stem.

Response Type: Equation/Numeric

Task Model 9

Response Type: Equation / Numeric

DOK Level 2

6.NS.C.8
Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

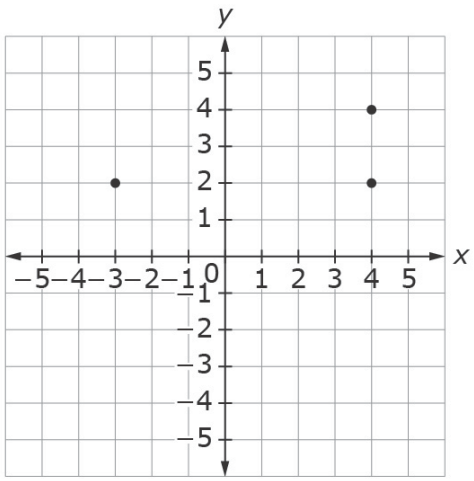
Evidence Required:
9. The student solves real-world and mathematical problems by graphing ordered pairs on a coordinate plane and using coordinates and absolute value to find the distances between points with same first coordinate or same second coordinate.

Tools: None

Version 3 Update: Added new example stem 3.

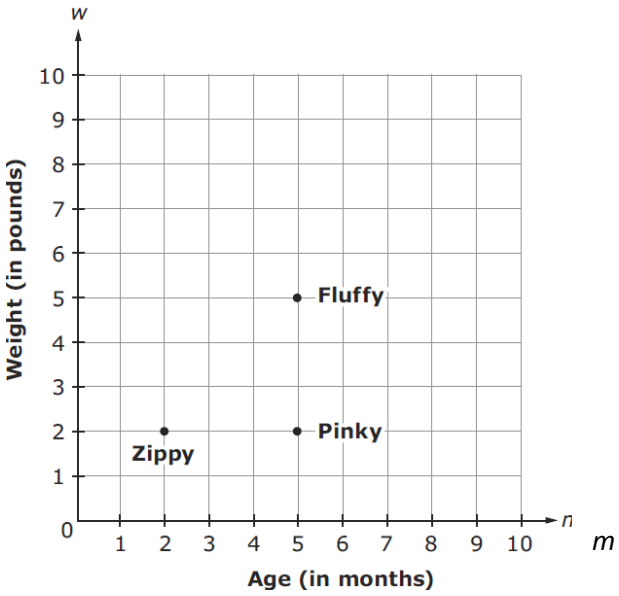
Example Stem 2: This grid represents the layout of Tom's neighborhood. Each unit on the grid represents 1 square mile.

- Tom's house is located at (4, 2)
- A store is located at (–3, 2)
- Tom's neighbors are located at (4, 4).



What is the distance, in miles, from Tom's house to the store?

Example Stem 3: Barry raises rabbits. The age, in months, and the weight, in pounds, of three of his rabbits are shown.



How many more pounds does Fluffy weigh than Pinky?

Rubric: (1 point) Student enters the correct numeric value for the distance (e.g., 7; 3). Units of measure should be assumed from the stem.

Response Type: Equation/Numeric

Claim 1: Concepts and Procedures
Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.

Content Domain: **Expressions and Equations**

Target E [m]: Apply and extend previous understandings of arithmetic to algebraic expressions. (DOK 1)

Tasks for this target will ask students to write and evaluate expressions (numerical expressions with whole-number exponents; algebraic expressions; and expressions arising from formulas in real-world problems). Other tasks will ask students to identify or generate equivalent expressions using understanding of properties or operations.

Standards: 6.EE.A, 6.EE.A.1, 6.EE.A.2, 6.EE.A.3, 6.EE.A.4	**6.EE.A Apply and extend previous understandings of arithmetic to algebraic expressions.** **6.EE.A.1** Write and evaluate numerical expressions involving whole-number exponents. **6.EE.A.2** Write, read, and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.* b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2(8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.* c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V = s^3$ and $A = 6 s^2$ to find the volume and surface area of a cube with sides of length s = 1/2.* **6.EE.A.3** Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.* **6.EE.A.4** Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*
Related Below-Grade and Above-Grade Standards for Purposes of Planning for Vertical Scaling: 5.OA.A, 5.OA.A.1, 5.OA.A.2	**Related Grade 5 Standards** **5.OA.A Write and interpret numerical expressions.** **5.OA.A.1** Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. **5.OA.A.2** Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 x (8 + 7). Recognize that 3 x (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*

7.EE.A, 7.EE.A.1, 7.EE.A.2	**Related Grade 7 Standards** **7.EE.A Use properties of operations to generate equivalent expressions.** **7.EE.A.1** Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. **7.EE.A.2** Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."*
DOK Levels:	1, 2
Achievement Level Descriptors:	
RANGE Achievement Level Descriptor (Range ALD) Target E: Apply and extend previous understandings of arithmetic to algebraic expressions.	**Level 1** Students should be able to evaluate numerical expressions without exponents; write one- or two-step numerical expressions; and identify parts of an expression, using terms (e.g., coefficient, term, sum, product, difference, quotient, factor).
	Level 2 Students should be able to evaluate numerical expressions with nonnegative integer exponents that do not need to be distributed across a set of parentheses. They should be able to apply and extend previous understandings of arithmetic to evaluate expressions with variables that do not contain exponents. They should also be able to write one- and two-step algebraic expressions that introduce a variable and identify equivalent expressions.
	Level 3 Students should be able to write and evaluate numerical expressions with nonnegative integer exponents and expressions from formulas in real-world problems, and they should be able to apply and extend previous understandings of arithmetic to evaluate expressions with variables that include nonnegative integer exponents. They should be able to apply properties of operations to generate equivalent expressions.
	Level 4 Students should be able to apply the understanding of the properties of operations and use the properties to show why two expressions are equivalent.
Evidence Required:	1. The student evaluates numerical expressions involving whole-number exponents. 2. The student writes numerical expressions involving whole-number exponents, algebraic expressions, and expressions from formulas in real-world problems. 3. The student uses mathematical terms to describe expressions. 4. The student evaluates algebraic expressions and expressions from formulas in real-world problems. 5. The student creates equivalent expressions by applying properties of operations. 6. The student identifies when expressions are equivalent by utilizing properties of operations.
Allowable Response Types:	Multiple Choice, multiple correct response; Equation/Numeric; Drag and Drop
Allowable Stimulus Materials:	

Construct-Relevant Vocabulary:	sum, product, quotient, difference, negative, term, factor, coefficient, expression, algebraic expression, numerical expression, order of operations, distributive property, associative property, commutative property
Allowable Tools:	Calculator
Target-Specific Attributes:	Only whole-number exponents can be used in items that involve the use of exponents.
Non-Targeted Constructs:	Parentheses when the student is applying the use of real-world problems or properties of operations. Distributive property of multiplication over addition. Associative property. Commutative property. Properties of addition and multiplication.
Accessibility Guidance:	Item writers should consider the following Language and Visual Element/Design guidelines[1] when developing items. Language Key Considerations: • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level • Avoid ambiguous or obscure words, idioms, jargon, unusual names and references Visual Elements/Design Key Considerations: • Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context • Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary • Avoid crowding of details and graphics Items are selected for a student's test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.[2]

[1] For more information, refer to the General Accessibility Guidelines at: http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf

[2] For more information about student accessibility resources and policies, refer to http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

Task Model 1 **Response Type: Equation / Numeric** **DOK Level 1** **6.EE.A.1** Write and evaluate numerical expressions involving whole-number exponents. **Evidence Required:** 1. The student evaluates numerical expressions involving whole-number exponents. **Tools:** Calculator	**Prompt Features:** The student is prompted to evaluate numerical expressions involving exponents. **Stimulus Guidelines:** • Expressions contain at least four numbers and one multiplication/division symbol. • Parentheses may be utilized to change the order of operations. • Expression should not be properly computed by simply going from left to right. • Numbers in expressions should be positive rational numbers. • Exponents should be whole numbers. • Answers should be positive numbers (up to hundredths, if a decimal). **TM1** **Stimulus:** The student is presented with a numerical expression with exponents. **Example Stem:** Enter the value of $3^3 \bullet 7^2 - 8 \div 4$. **Rubric:** (1 point) Student enters the correct value for the expression (e.g., 1321). **Response Type:** Equation/Numeric

Task Model 2 **Response Type: Equation / Numeric** **DOK Level 1** **6.EE.A.1** Write and evaluate numerical expressions involving whole-number exponents. **6.EE.A.2a** Write expressions that record operations with numbers and with letters standing for numbers. **Evidence Required:** 2. The student writes numerical expressions involving whole-number exponents, algebraic expressions, and expressions from formulas in real-world problems. **Tools:** Calculator	**Prompt Features:** The student is prompted to write an expression to represent a given verbal description of that expression. **Stimulus Guidelines:** • Expressions should be one- or two-step problems. • Exponents should be whole numbers. • Numbers in expressions should be positive rational numbers. • Item difficulty can be adjusted via these example methods: o Students write a numeric expression with exponents. o Students write an algebraic expression/formula without exponents. o Students write an algebraic expression/formula with exponents. **TM2** **Stimulus:** The student is presented with a verbal numerical expression with exponents or verbal algebraic expression with or without exponents. **Example Stem 1:** Enter a numerical expression that represents the sum of eight squared and thirty-two. **Example Stem 2:** Enter an algebraic expression that represents eight times the sum of *y* squared and twenty-eight. **Rubric:** (1 point) Student enters a correct numerical/algebraic expression for the given verbal expression (e.g., $8^2 + 32$; $8(y^2 + 28)$). **Response Type:** Equation/Numeric

Task Model 3 **Response Type: Multiple Choice, multiple correct response** **DOK Level 1** **6.EE.A.1** Write and evaluate numerical expressions involving whole-number exponents. **6.EE.A.2b** Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. **Evidence Required:** 3. The student uses mathematical terms to describe expressions. **Tools:** Calculator **Version 3 Update:** Revised the options for example stem for TM3a. Retired TM3b.	**Prompt Features:** The student is prompted to use mathematical terms to describe an expression. **Stimulus Guidelines:** • Mathematical terms include sum, term, product, factor, quotient, and coefficient. • Exponents used should be whole numbers. • Numbers in expressions should be rational numbers. • Item difficulty can be adjusted by presenting expressions that contain parentheses. **TM3a:** **Stimulus:** The student is presented with a numerical or algebraic expression. **Example Stem:** Select **all** the statements that correctly describe the expression $4^3 \bullet (8w - 7)$. A. 3 is a factor of the expression. B. The difference of 8w and 7 is a factor of the expression. C. The expression represents the product of 4^3 and $8w - 7$. D. The expression represents the difference of $4^3 \bullet 8w$ and 7. **Answer Choices:** Answer choices should be statements that include the following vocabulary: sum, term, product, factor, quotient, and coefficient. Distractors will include confusing the meaning of sum, term, product, factor, quotient, and coefficient. At least two statements must be correct. **Rubric:** (1 point) Student selects all the correct statements (e.g., B and C). **Response Type:** Multiple Choice, multiple correct response

Task Model 4 **Response Type: Equation / Numeric** **DOK Level 1** **6.EE.A.2c** Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). **Evidence Required:** 4. The student evaluates algebraic expressions and expressions from formulas in real-world problems. **Tools:** Calculator **Version 3 Update:** Added new example stem 4 to TM4.	**Prompt Features:** The student is prompted to find the value of a given expression. **Stimulus Guidelines:** • Expression cannot be properly computed by simply going from left to right. • Numbers in expressions should be rational numbers. • If used, exponents should be whole numbers. • Item difficulty can be adjusted via these example methods: o Students enter the value of an algebraic expression without fractions/decimals or exponents. o Students enter the value of an algebraic expression with exponents and no fractions/decimals. o Students enter the value of an algebraic expression that contains fractions/decimals. o Students enter the value of an algebraic expression that contains fractions/decimals and exponents. **TM4** **Stimulus:** The student is presented with an algebraic expression and specific values for variables in the expression. **Example Stem 1:** The formula $C = \frac{5}{9}(F - 32)$ is used to convert degrees Fahrenheit (F) to degrees Celsius (C). Enter the temperature, in degrees Celsius (C), equal to 113 degrees Fahrenheit (F). **Example Stem 2:** Enter the value of $2 \bullet y - 8 \div 4$ when $y = 7$. **Example Stem 3:** Enter the value of $3^3 \bullet y^2 - 8 \div 4$ when $y = 7$. **Example Stem 4:** A baker uses the expression $5.75c + 3.45p$ to calculate his profit when he sells c cakes and p pies. What is the baker's profit, in dollars, when he sells 33 cakes and 42 pies? **Rubric:** (1 point) Student enters the correct value for the expression or formula (e.g., 45; 12; 1321; 334.65). Units should be assumed from the problem. **Response Type:** Equation/Numeric

Task Model 5 **Response Type: Equation / Numeric** **DOK Level 2** **6.EE.A.3** Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.* **Evidence Required:** 5. The student creates equivalent expressions by applying properties of operations. **Tools:** Calculator	**Prompt Features:** The student is prompted to create equivalent expressions based on given parameters. **Stimulus Guidelines:** • Expressions could contain one or two variables. • For expressions in the form $a(bx + cy)$, b and c do not have a common factor. • The correct answer choice will use properties of operations to generate an equivalent expression. **TM5a** **Stimulus:** The student is presented with an algebraic expression or an incomplete algebraic expression. **Example Stem 1:** Consider this expression: $3(2x + 5y)$. Enter an expression that shows the **sum of exactly two terms** that is equivalent to $3(2x + 5y)$. **Example Stem 2:** An equivalent expression to $6x + 15y$ can be written as the product of two factors. One of the factors is 3. Enter the **second factor** that will result in $6x + 15y$ when the two factors are multiplied. **Rubric:** (1 point) Student enters the correct algebraic expression (e.g., $6x + 15y$; $2x + 5y$). **Response Type:** Equation/Numeric

Task Model 5

Response Type: Drag and Drop

DOK Level 2

6.EE.A.3
Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*

Evidence Required:
5. The student creates equivalent expressions by applying properties of operations.

Tools: Calculator

Accessibility Note: Drag and Drop items are not currently able to be Brailled. Minimize the number of items developed to this TM.

Prompt Features: The student is prompted to use given parameters to create an expression that is equivalent to a given expression.

Stimulus Guidelines:

- For expressions in the form $a(bx + cy)$, b and c do not have a common factor.
- Blanks represent terms; at least two blanks should be provided.
- Expressions could contain one or two variables.
- If expressions are in the form $ax + by$, then they must have a common factor greater than one.
- Item difficulty can be adjusted via these example methods:
  - Students enter an equivalent expression that represents a given expression.
  - Students enter missing parts of an equivalent expression that represents a given expression.

TM5b
Stimulus: The student is presented with an expression and the parameters to create an equivalent expression.

Example Stem 1: Consider this equation.

$$3(2x + 5y) = \square + \square$$

Drag an expression into each box to create an expression equivalent to $3(2x + 5y)$.

Example Stem 2: Consider this equation.

$$6x + \square = 3(\square + 5)$$

Drag an expression into each box to create a true equation.

Interaction: Students will use the drag-and-drop feature to place expressions in the boxes. A palette will be given on the left-hand side with 8–12 terms. Snap-to feature should be used and Delete tool needs to be provided.

Rubric: (1 point) Student correctly creates an equivalent expression (e.g., $6x$ and $15y$; 15 and $2x$).

Response Type: Drag and Drop

Task Model 6 **Response Type: Multiple Choice, multiple correct response** **DOK Level 2** **6.EE.A.4** Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions* $y + y + y$ *and* $3y$ *are equivalent because they name the same number regardless of which number* y *stands for.* **Evidence Required:** 6. The student identifies when expressions are equivalent by utilizing properties of operations. **Tools:** Calculator	**Prompt Features:** The student is prompted to identify equivalent expressions. **Stimulus Guidelines:** • If used, exponents should be whole numbers. • Item difficulty can be adjusted via these example methods: o Having multiple correct answers increases the difficulty. o Expressions can involve the distributive property or just combining or expanding terms. **TM6** **Stimulus:** The student is presented with an algebraic expression. **Example Stem 1:** Select **all** expressions that are equivalent to $4(3x + 6y)$. A. $12x + 6y$ B. $12x + 24y$ C. $2(6x + 12y)$ D. $4(12x+24y)$ **Example Stem 2:** Select **all** expressions that are equivalent to $3 + w + w + w$. A. $3(1 + w)$ B. $3 + 3w$ C. $3+w^3$ D. $3w^3$ **Answer Choices:** Answer choices will be algebraic expressions. Distractors will include confusing the meaning of sum, term, product, factor, quotient, and coefficient and/or the properties of operations. At least two expressions must be correct. **Rubric:** (1 point) Student selects all of the correct expressions (e.g., B and C; A and B). **Response Type:** Multiple Choice, multiple correct response

Claim 1: Concepts and Procedures
Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.

Content Domain: **Expressions and Equations**

Target F [m]: Reason about and solve one-variable equations and inequalities. (DOK Levels 1, 2)

Tasks for this target will ask students to solve and write one-variable equations and inequalities, some of which provide substitution of given numbers as an entry point to a solution.

Claim 3 tasks will tap into students' abilities to explain that there are infinitely many solutions to an inequality (some connecting the content of this target to 6.NS Target C).

Standards: 6.EE.B, 6.EE.B.5, 6.EE.B.6, 6.EE.B.7, 6.EE.B.8	**6.EE.B Reason about and solve one-variable equations and inequalities.** **6.EE.B.5** Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. **6.EE.B.6** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. **6.EE.B.7** Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q, and x are all nonnegative rational numbers. **6.EE.B.8** Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.
Related Below-Grade and Above-Grade Standards for Purposes of Planning for Vertical Scaling: 5.OA.A, 5.OA.A.2 7.EE.B, 7.EE.B.3, 7.EE.B.4	**Related Grade 5 Standards** **5.OA.A Write and interpret numerical expressions.** **5.OA.A.2** Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 x (8 + 7). Recognize that 3 x (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.* **Related Grade 7 Standards** **7.EE.B Solve real-life and mathematical problems using numerical and algebraic expressions and equations.** **7.EE.B.3** Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar*

	9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation. **7.EE.B.4** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve real-world problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* b. Solve real-word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*
DOK Levels:	1, 2
Achievement Level Descriptors:	
RANGE Achievement Level Descriptor (Range ALD) Target F: Reason about and solve one-variable equations and inequalities.	**Level 1** Students should be able to use substitution to determine when a given number makes an equation or inequality true.
	Level 2 Students should be able to solve one-variable equations and inequalities of the form $x + p = \leq/\geq/</> q$ or $px = \leq/\geq/</> q$, where p and q are nonnegative rational numbers. They should be able to identify and use variables when writing equations.
	Level 3 Students should be able to write one-variable equations and inequalities of the form $x + p = \leq/\geq/</> q$ or $px = \leq/\geq/</> q$, where p and q are nonnegative rational numbers. They should be able to reason about and solve equations and inequalities by writing and graphing their solutions on a number line.
	Level 4 Students should be able to solve equations and inequalities of the form $x + p = \leq/\geq/</> q$ or $px = \leq/\geq/</> q$, where p and q are rational numbers. They should be able to write and graph solutions on the number line.
Evidence Required:	1. The student uses substitution in one-variable equations and inequalities. 2. The student writes one-variable equations and inequalities and solves one-variable equations in real-world and mathematical problems. 3. The student represents solutions of inequalities in real-world and mathematical problems on a number line.
Allowable Response Types:	Multiple Choice, single correct response; Multiple Choice, multiple correct response; Drag and Drop; Equation/Numeric; Matching Tables
Allowable Stimulus Materials:	
Construct-Relevant Vocabulary:	variable, equation, inequality, solution, solution set

Allowable Tools:	None
Target-Specific Attributes:	p, q, and x must all represent nonnegative rational numbers when solving equations of the form $x + p = q$ and $px = q$
Non-Targeted Constructs:	Parentheses when the student is applying the use of real-world problems. Properties of addition and multiplication.
Accessibility Guidance:	Item writers should consider the following Language and Visual Element/Design guidelines[1] when developing items. Language Key Considerations: • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level • Avoid ambiguous or obscure words, idioms, jargon, unusual names and references Visual Elements/Design Key Considerations: • Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context • Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary • Avoid crowding of details and graphics Items are selected for a student's test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.[2]
Development Notes:	Claim 3 tasks will tap into a student's ability to explain inequalities as a set of infinitely many solutions (some connecting the content of this target to 6.NS Target C).

[1] For more information, refer to the General Accessibility Guidelines at: http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf

[2] For more information about student accessibility resources and policies, refer to http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

Task Model 1 **Response Type: Multiple Choice, multiple correct response** **DOK Level 1** **6.EE.B.5** Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. **Evidence Statement:** 1. The student uses substitution in one-variable equations and inequalities. **Tools:** None	**Prompt Features:** The student is prompted to identify equations that have a given solution. **Stimulus Guidelines:** The student is presented with a solution and one equation per answer choice. • Equations are one-step equations in the form $x + p = q$ or $px = q$ in which p, q, and x must all represent nonnegative rational numbers. • Item difficulty can be adjusted via these example methods: o p and q are whole numbers. o At least one number is a decimal to the hundredths. o At least one number is a fraction or mixed number. **TM1a** **Example Stem:** Select **all** equations that have $x = 3$ as a solution. A. $x + 7 = 10$ B. $3 + x = 3$ C. $x \bullet 3 = 1$ D. $4 \bullet x = 12$ **Answer Choices:** Answer choices will be equations in the form $x + p = q$ or $px = q$, in which p and q must represent nonnegative rational numbers. Distractors will include confusing addition, subtraction, multiplication, or division, computation errors, and/or incorrect substitution. At least two equations must be correct. **Rubric:** (1 point) Student selects all the correct equations (e.g., A and D). **Response Type:** Multiple Choice, multiple correct response

Task Model 1 **Response Type: Multiple Choice, multiple correct response** **DOK Level 2** **6.EE.B.5** Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. **Evidence Statement:** 1. The student uses substitution in one-variable equations and inequalities. **Tools:** None **Version 3 Update:** Revised TM1a example stem 1 and added new example stem 2. Revised TM1c.	**Prompt Features:** The student is prompted to use substitution to identify a solution set for an inequality. **Stimulus Guidelines:** • Inequalities should be in the form $x > c$ or $x < c$ where c must represent a rational number. • Item difficulty can be adjusted via these example methods: o c is an integer. o c is a decimal to the hundredths. o c is a fraction or mixed number. **TM1b** **Stimulus:** The student is presented with a solution set and one inequality per answer choice. **Example Stem 1:** Select **all** inequalities that include 0 in the solution set. A. $x > -4.24$ B. $x < -5.5$ C. $x > -5.13$ D. $x < 4.5$ **Example Stem 2:** Select **all** inequalities that include all numbers less than -6 in the solution set. A. $x > -4.24$ B. $x < -5.5$ C. $x > -5.13$ D. $x < 4.5$ **Answer Choices:** Answer choices will be inequalities in the form $x > c$ or $x < c$. Distractors will include misinterpreting the inequality symbols and/or not all the values in a given set satisfy the inequality. At least two inequalities must be correct. **Rubric:** (1 point) Student selects all the correct inequalities (e.g., A,C,D; B,D). **TM1c** **Stimulus:** The student is presented with a one-variable inequality. **Example Stem:** Select **all** the values that satisfy the inequality $x < 7\frac{1}{2}$. A. $x = -8$ B. $x = -7$ C. $x = 7$ D. $x = 8$ **Rubric:** (1 point) Student selects all the correct sets of numbers (e.g., A, B and C). **Response Type:** Multiple Choice, multiple correct response

Task Model 1

Response Type: Matching Tables

DOK Level 2

6.EE.B.5
Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

Evidence Statement:
1. The student uses substitution in one-variable equations and inequalities.

Tools: None

Prompt Features: The student is prompted to use substitution to identify multiple solutions to one-variable inequalities.

Stimulus Guidelines: The student is presented with a one-variable inequality.

- Inequalities should be in the form $x > c$ or $x < c$ in which c must represent a rational number.
- The table may include four to five values.
- Item difficulty can be adjusted by varying the types of numbers used as values in the table (e.g., positive and negative integers, fractions, decimals).

TM1d
Example Stem: Consider the inequality $x > 7$.

Determine whether each value of x makes this inequality true. Select Yes or No for each value.

x	Yes	No
22		
-7		
13		
5		
-39		

Rubric: (1 point) Student correctly determines whether all five values make the inequality true (e.g., Y, N, Y, N, N).

Response Type: Matching Tables

Task Model 2 **Response Type: Multiple Choice, single correct response** **DOK Level 1** **6.EE.B.8** Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams. **Evidence Statement:** 2. The student writes one-variable equations and inequalities and solves one-variable equations in real-world and mathematical problems. **Tools:** None	**Prompt Features:** The student is prompted to identify one-variable inequalities in real-world and mathematical problems. **Stimulus Guidelines:** The student is presented with verbal constraints in a real-world or mathematical problem involving one-variable inequalities. • Inequalities should be in the form $x > c$, $x < c$, $c > x$, or $c < x$ in which c must represent a rational number. • Context should be familiar to students 11 to 13 years old. • Item difficulty can be adjusted by varying the types of numbers used as values (e.g., positive and negative integers, fractions, decimals). **TM2a** **Example Stem:** John is planning to put a rectangular pool in his backyard. The length (l) of the pool must be greater than 24 feet and the width (w) must be less than 14 feet. Select the pair of inequalities that models the possible measurements for each dimension. A. $l > 14$ and $w < 24$ B. $l > 24$ and $w < 14$ C. $24 > l$ and $14 > w$ D. $24 < l$ and $14 < w$ **Answer Choices:** Each answer choice will be two inequalities in the form $x > c$, $x < c$, $c > x$, or $c < x$. Distractors will include misinterpreting the inequality symbols and/or incorrect placement of variable and numerical terms. **Rubric:** (1 point) Student selects the correct inequality pair (e.g., B). **Response Type:** Multiple Choice, single correct response

Task Model 2 **Response Type: Equation / Numeric** **DOK Level 2** **6.EE.B.7** Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. **Evidence Statement:** 2. The student writes one-variable equations and inequalities and solves one-variable equations in real-world and mathematical problems. **Tools:** None	**Prompt Features:** The student is prompted to solve one-variable equations in mathematical and real-world contexts. **Stimulus Guidelines:** The student is presented with a one-variable equation of the form $x + p = q$ or $px = q$ in context. • p and q must represent nonnegative rational numbers • If used, context should be familiar to students 11 to 13 years old. • Item difficulty can be adjusted by varying the types of numbers used as values (e.g., positive and negative integers, fractions, decimals). **TM2b** **Example Stem:** Julia has some peaches**.** She gathers 6 more peaches. She now has 58 peaches. **Part A**: In the first box, enter an **equation** to represent the number of peaches, p, that Julia has before she gathers 6 more peaches. **Part B:** In the second box, enter the **number** of peaches represented by p in this situation. **Rubric:** (2 points) Student enters the correct equation (e.g., $p + 6 = 58$) and the correct solution (e.g., 52). (1 point) Student enters the correct equation or the correct solution. **Response Type:** Equation/Numeric (2 response boxes)

Task Model 2 **Response Type: Equation / Numeric** **DOK Level 1** **6.EE.B.7** Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. **Evidence Statement:** 2. The student writes one-variable equations and inequalities and solves one-variable equations in real-world and mathematical problems. **Tools:** None	**Prompt Features:** The student is prompted to write or solve one-variable equations in mathematical and real-world contexts. **Stimulus Guidelines:** • Equations should be in the form $x + p = q$ or $px = q$, where p and q must represent nonnegative rational numbers. • If used, context should be familiar to students 11 to 13 years old. • Item difficulty can be adjusted by varying the types of numbers used as values (e.g., positive and negative integers, fractions, decimals). **TM2c** **Stimulus:** The student is presented with an equation in a mathematical context. **Example Stem:** The sum of 32 and n is equal to 59.13. Enter the equation described in the sentence. **Rubric:** (1 point) Student enters a correct equation (e.g., $32 + n = 59.13$ or equivalent). **Response Type:** Equation/Numeric **TM2d** **Stimulus:** The student is presented with an equation containing an unknown variable. **Example Stem:** Enter the value of y that makes the given equation true. $y + 3\frac{2}{9} = 5\frac{5}{6}$. **Rubric:** (1 point) Student enters the correct value (e.g., $2\frac{11}{18}$). **Response Type:** Equation/Numeric

Task Model 3

Response Type: Multiple Choice, single correct response

DOK Level 1

6.EE.B.8
Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Evidence Statement:
3. The student represents solutions of inequalities in real-world and mathematical problems on a number line.

Tools: None

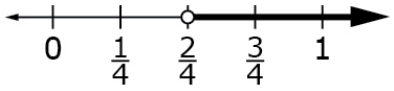
Prompt Features: The student is prompted to identify a number line that represents the solution to a one-variable inequality presented in a mathematical or real-world context.

Stimulus Guidelines: The student is presented with a one-variable inequality in a mathematical or real-world context.

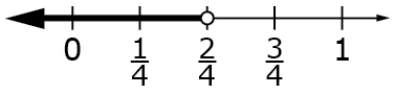
- Inequalities should be in the form $x > c$ *or* $x < c$ in which c must represent a rational number.
- Number lines should have evenly spaced tick marks.
- If used, context should be familiar to students 11 to 13 years old.
- Item difficulty can be adjusted via these example methods, but are not limited to these methods:
  - c is a whole number; number line has integers labeled.
  - c is an integer; number line has integers labeled.
  - c is a fraction.
  - c is a decimal.

TM3a
Example Stem: Select the number line that represents all solutions of $x < -\frac{2}{4}$.

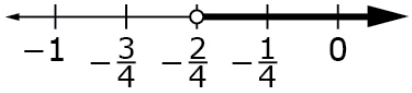
A.



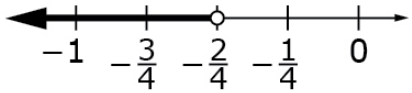
B.



C.



D.



Rubric: (1 point) Student selects the correct number line (e.g., D).

Response Type: Multiple Choice, single correct response

Task Model 3

Response Type: Drag and Drop

DOK Level 2

6.EE.B.8
Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Evidence Statement:
3. The student represents solutions of inequalities in real-world and mathematical problems on a number line.

Tools: None

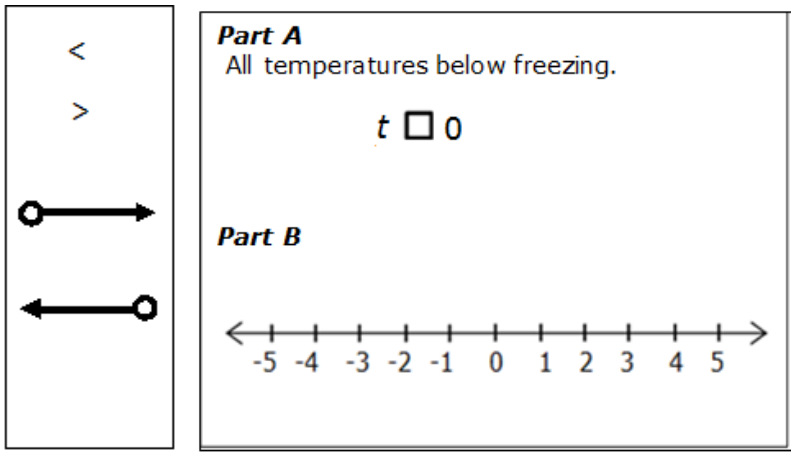
Accessibility Note:
Drag and Drop items are not currently able to be Brailled. Minimize the number of items developed to this TM

Prompt Features: The student is prompted to create and represent, on a number line, a one-variable inequality that corresponds to a verbal constraint in a mathematical or real-world problem.

Stimulus Guidelines: The student is presented with a verbal constraint in a mathematical or real-world problem.

- Inequalities should be in the form $x > c$ or $x < c$ in which c must represent a rational number.
- Drag elements should include: an arrow going to the left with an open circle, an arrow going to the right with an open circle, <, and >.
- Number lines should have evenly spaced tick marks. Each tick mark should have snap-to regions that can fit the circles and arrows.
- Context should be familiar to students 11 to 13 years old.
- Item difficulty can be adjusted via these example methods, but are not limited to these methods:
  - c is a whole number; number line has whole numbers labeled.
  - c is an integer; number line has integers labeled.
  - c is a decimal; number line is appropriately labeled.
  - c is a fraction; number line is appropriately labeled.

TM3b
Example Stem: The freezing point of water is 0 degrees Celsius.



Part A: Drag the correct symbol into the box to create an inequality that describes all temperatures (t) below freezing.

Part B: Drag the correct ray to the number line to represent all temperatures, t, that are below freezing, in degrees Celsius.

Interaction:
Students given Delete tool as well as the following:
Part A

- Students use the drag-and-drop tool to place an inequality symbol in the open box.

Part B

- Students use the drag-and-drop tool to place a ray on the

	number line. • Snap-to feature used at each tick mark on the number line. **Rubric:** (1 point) Student places the correct inequality symbol in the box and places the correct ray at the proper location on the number line. **Response Type:** Drag and Drop

Claim 1: Concepts and Procedures
Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.

Content Domain: **Expressions and Equations**

Target G [m]: Represent and analyze quantitative relationships between dependent and independent variables. (DOK 2)

Tasks for this target will ask students to select or write an equation that expresses one quantity in terms of another. Some tasks will target the relationship between the variables in an equation and their representation in a table or graph.

Some tasks may connect the content of this target with 6.EE Target F.

Standards: 6.EE.C, 6.EE.C.9	**6.EE.C Represent and analyze quantitative relationships between dependent and independent variables.** **6.EE.C.9** Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*
Related Below-Grade and Above-Grade Standards for Purposes of Planning for Vertical Scaling: 5.OA.B, 5.OA.B.3, 5.G.A, 5.G.A.2 7.EE.B, 7.EE.B.4	**Related Grade 5 Standards** **5.OA.B Analyze patterns and relationships.** **5.OA.B.3**. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number is 0, and the given rule "Add 6" and the starting number is 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.* **5.G.A Graph points on the coordinate plane to solve real-world and mathematical problems** **5.G.A.2** Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. **Related Grade 7 Standards** **7.EE.B Solve real-life and mathematical problems using numerical and algebraic expressions and equations.** **7.EE.B.4** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve world problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently.

	Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*
DOK Level:	2
Achievement Level Descriptors:	
RANGE Achievement Level Descriptor (Range ALD) Target G: Represent and analyze quantitative relationships between dependent and independent variables.	**Level 1** Students should be able to identify a table that represents a relationship between two variables of the forms $y = kx$ and $y = x \pm c$ with rational numbers and plot points corresponding to equations on coordinate planes.
	Level 2 Students should be able to use variables to represent and analyze two quantities that change in relationship to each other of the form $y = kx$ or $y = x \pm c$ with rational numbers; identify and create an equation that expresses one quantity in terms of another; and use graphs and tables to represent the relationship.
	Level 3 Students should be able to use graphs, tables, or context to analyze the relationship between dependent and independent variables and relate them to a linear equation.
	Level 4 Students should be able to use graphs, tables, or context to analyze nonlinear polynomial relationships between dependent and independent variables and relate them to nonlinear polynomial equations.
Evidence Required:	1. The student writes an equation to express one quantity versus another quantity using dependent and independent variables. 2. The student identifies the relationship between dependent and independent variables from graphs and tables and relates them to equations.
Allowable Response Types:	Multiple Choice, single correct response; Multiple Choice, multiple correct response; Equation/Numeric; Matching Tables; Fill-in Table
Allowable Stimulus Materials:	graphs, tables
Construct-Relevant Vocabulary:	variable, equation, inequality, dependent variable, independent variable, relation
Allowable Tools:	Calculator
Target-Specific Attributes:	
Non-Targeted Constructs:	

Accessibility Guidance:	Item writers should consider the following Language and Visual Element/Design guidelines[1] when developing items. Language Key Considerations: • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level • Avoid ambiguous or obscure words, idioms, jargon, unusual names and references Visual Elements/Design Key Considerations: • Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context • Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary • Avoid crowding of details and graphics Items are selected for a student's test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.[2]

[1] For more information, refer to the General Accessibility Guidelines at: http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf

[2] For more information about student accessibility resources and policies, refer to http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

Task Model 1 **Response Type: Equation / Numeric** **DOK Level 2** **6.EE.C.9** Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. **Evidence Required:** 1. The student writes an equation to express one quantity versus another quantity using dependent and independent variables. **Tools:** Calculator	**Prompt Features:** The student is prompted to give an equation that uses dependent and independent variables to relate two quantities. **Stimulus Guidelines:** • Equations should be in the form of $y = kx$ or $y = x \pm c$ where k and c are positive rational numbers. • Context should be familiar to students 11 to 13 years old. • Item difficulty can be adjusted via these example methods: o Equations are in the form $y = kx$ or $y = x \pm c$ where k and c are positive integers. o Equations are in the form $y = kx$ or $y = x \pm c$ where k and c are fractions, mixed numbers, or decimals. **TM1** **Stimulus:** The student is presented with independent and dependent quantities in a real-world context. **Example Stem:** Emily studies 40 minutes after lunch for a science exam. She studies x more minutes that evening. Enter an **equation** that represents the total number of minutes, y, Emily studies for the science exam. **Rubric:** (1 point) Student gives a correct equation (e.g., $40 + x = y$). **Response Type:** Equation/Numeric

Task Model 2

Response Type: Multiple Choice, single correct response

DOK Level 2

6.EE.C.9
Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.

Evidence Required:
2. The student identifies the relationship between dependent and independent variables from graphs and tables and relates them to equations.

Tools: Calculator

Prompt Features: The student is prompted to identify the correct graph that represents a relationship between quantities that are related in a real-world context.

Stimulus Guidelines:

- Graph values should be linear in the form of $y = kx$ or $y = x \pm c$ where k and c are positive rational numbers.
- The coordinate plane should be limited to Quadrant I.
- Context should be familiar to students 11 to 13 years old.

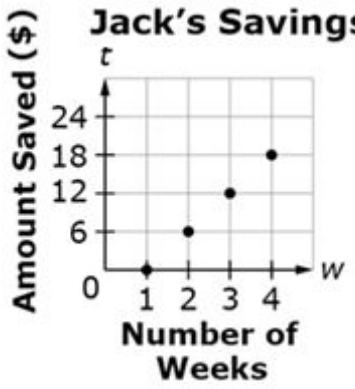
TM2a
Stimulus: The student is presented with two related quantities in a real-world context.

Example Stem: Jack saves $6.00 each week.

- Let w represent the number of weeks that Jack saves $6.00.
- Let t represent the total amount saved, in dollars.

Which graph shows the relationship between t, the amount of money Jack saves, and w, the number of weeks he has been saving?

A.



B.

Task Model 2 **Response Type: Multiple Choice, single correct response** **DOK Level 2** **6.EE.C.9** Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. **Evidence Required:** 2. The student identifies the relationship between dependent and independent variables from graphs and tables and relates them to equations. **Tools:** Calculator	C.  D.  **Answer Choices:** Answer choices will be a graph with three to five ordered pairs plotted. Distractors will include switching the two variables and/or incorrectly plotting the points. **Rubric:** (1 point) Student selects the correct graph (e.g., D). **Response Type:** Multiple choice, single correct response

Task Model 2

Response Type: Matching Tables

DOK Level 2

6.EE.C.9
Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.

Evidence Required:
2. The student identifies the relationship between dependent and independent variables from graphs and tables and relates them to equations.

Tools: Calculator

Prompt Features: The student is prompted to use a table or a graph to identify correct statements about a relationship between two quantities.

Stimulus Guidelines:

- Table and graph values should be linear in the form of $y = kx$ or $y = x \pm c$ where k and c are positive rational numbers.
- Tables should have three to five rows of data.
- The coordinate plane should be limited to Quadrant I.
- Context should be familiar to students 11 to 13 years old.

TM2b
Stimulus: The student is presented with a relationship between two quantities represented by a table or a graph.

Example Stem 1: Jack saves the same amount of money each week as shown in the table.

- Let w represent the number of weeks that Jack saves.
- Let t represent the total amount saved, in dollars.

Number of Weeks w	Total Amount Saved t
1	$ 6
2	$12
3	$18
4	$24

Determine whether each statement is true. Select True or False for each statement.

Statement	True	False
The equation $t = 6 + w$ represents the relationship between the number of weeks and the total amount saved.		
The total amount saved is 6 times the number of weeks.		
The number of weeks that Jack saves depends on the total amount of money Jack saves.		

Rubric: (1 point) Student correctly identifies each statement as being either true or false (e.g., F, T, F).

Response Type: Matching Tables

Task Model 2

Response Type: Matching Tables

DOK Level 2

6.EE.C.9
Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.

Evidence Required:
2. The student identifies the relationship between dependent and independent variables from graphs and tables and relates them to equations.

Tools: Calculator

Example Stem 2: Jack saves the same amount of money each week as shown in the graph.

- *w* represents the number of weeks that Jack saves.
- *t* represents the total amount saved, in dollars.

Determine whether each statement is true. Select True or False for each statement.

Statement	True	False
Jack saved a total of $12 at the end of week 2.		
The equation $t = 6w$ represents the relationship between the number of weeks and the total amount saved.		
The total amount of money Jack saves depends on the number of weeks that Jack saves.		

Rubric: (1 point) Student correctly identifies each statement as being either true or false (e.g., T, T, T).

Response Type: Matching Tables

Task Model 2

Response Type: Equation / Numeric

DOK Level 2

6.EE.C.9
Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.

Evidence Required:
2. The student identifies the relationship between dependent and independent variables from graphs and tables and relates them to equations.

Tools: Calculator

Prompt Features: The student is prompted to use a table or a graph to analyze a relationship between two quantities.

Stimulus Guidelines:

- Table and graph values should be linear in the form of $y = kx$ or $y = x \pm c$ where k and c are positive rational numbers.
- Tables should have three to five rows of data.
- The coordinate plane should be limited to Quadrant I.
- Context should be familiar to students 11 to 13 years old.

TM2c
Stimulus: The student is presented with a relationship between two quantities represented by a table or a graph.

Example Stem 1: Jack saves the same amount of money each week as shown in the table.

- Let w represent the number of weeks that Jack saves.
- Let t represent the total amount saved, in dollars.

Number of Weeks w	Total Amount Saved t
1	$ 6
2	$12
3	$18
4	$24

Enter the total amount of money, in dollars, that Jack saves after 6 weeks.

Rubric: (1 point) Student enters the correct value (e.g., 36).

Response Type: Equation/Numeric

Task Model 2 **Response Type: Equation / Numeric** **DOK Level 2** **6.EE.C.9** Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. **Evidence Required:** 2. The student identifies the relationship between dependent and independent variables from graphs and tables and relates them to equations. **Tools:** Calculator	**Example Stem 2:** Jack saves the same amount of money each week as shown in the graph. • Let *w* represent the number of weeks that Jack saves. • Let *s* represents the total amount saved, in dollars.  Enter the total amount of money, in dollars, that Jack saves after 6 weeks. **Rubric:** (1 point) Student enters the correct value (e.g.,36). **Response Type:** Equation/Numeric

Task Model 2

Response Type: Fill-in Table

DOK Level 2

6.EE.C.9
Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.

Evidence Required:
2. The student identifies the relationship between dependent and independent variables from graphs and tables and relates them to equations.

Tools: Calculator

Prompt Features: The student is prompted to complete a table to represent the relationship between two quantities.

Stimulus Guidelines:

- Table values should be linear in the form of $y = kx$ or $y = x \pm c$ where k and c are positive rational numbers.
- Tables should have three to five rows of data.
- Context should be familiar to students 11 to 13 years old.

TM2d
Stimulus: The student is presented with independent or dependent variables in the form of a table.

Example Stem: The band members are selling chocolate bars for a fundraiser. The amount of money collected for each box of bars sold is the same.

- Let n represent the number of boxes sold.
- Let d represent the amount of money collected, in dollars.

Number of Boxes Sold *n*	**Amount of Money Collected, in Dollars *d***
	30
2	
3	90
4	120
6	

Fill in the table for all missing values of n and d.

Rubric: (1 point) Student correctly enters all missing values in the table (e.g., 1, 60, and 180).

Response Type: Fill-in Table

Adapted from http://www.illustrativemathematics.org/standards/k8.

Task Model 2

Response Type: Equation / Numeric

DOK Level 2

6.EE.C.9
Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.

Evidence Required:
2. The student identifies the relationship between dependent and independent variables from graphs and tables and relates them to equations.

Tools: Calculator

Prompt Features: The student is prompted to give an equation from a graph or table that relates two quantities.

Stimulus Guidelines:

- Table and graph values should be linear in the form of $y = kx$ or $y = x \pm c$ where k and c are positive rational numbers.
- Tables should have three to five rows of data.
- The coordinate plane should be limited to Quadrant I.
- Context should be familiar to students 11 to 13 years old.
- Item difficulty can be adjusted via these example methods:
  - Students enter an equation for a table/graph of values for a linear relationship in the form $y = kx$ or $y = x \pm c$ where k and c are positive integers.
  - Students enter an equation for a table/graph of values for a linear relationship in the form $y = kx$ or $y = x \pm c$ where k and c are positive fractions, mixed numbers, or decimals.

TM2e
Stimulus: The student is presented with a relationship between two quantities represented by a table or a graph.

Example Stem 1: Jack saves the same amount of money each week as shown in the table.

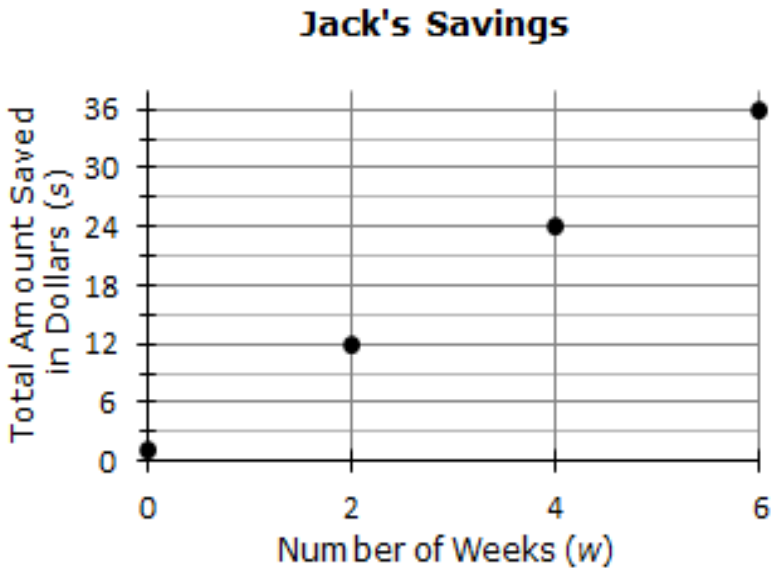
- Let w represent the number of weeks that Jack saves.
- Let t represent the total amount saved, in dollars.

Number of Weeks w	Total Amount Saved t
1	$ 6
2	$12
3	$18
4	$24

Enter an equation that represents the relationship between the number of weeks Jack saves and the total amount of money saved.

Rubric: (1 point) Student enters the correct equation (e.g., $t = 6w$).

Response Type: Equation/Numeric

Task Model 2 **Response Type: Equation / Numeric** **DOK Level 2** **6.EE.C.9** Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. **Evidence Required:** 2. The student identifies the relationship between dependent and independent variables from graphs and tables and relates them to equations. **Tools:** Calculator	**Example Stem 2:** Jack saves the same amount of money each week as shown in the graph. • Let *w* represent the number of weeks that Jack saves. • Let *s* represent the total amount saved, in dollars.  Enter an equation that represents the relationship between the number of weeks Jack saves and the total amount of money saved. **Rubric:** (1 point) Student enters the correct equation (e.g., $s = 6w$). **Response Type:** Equation/Numeric

Claim 1: Concepts and Procedures
Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.

Content Domain: **Geometry**

Target H [s]: Solve real-world and mathematical problems involving area, surface area, and volume. (DOK Levels 1, 2)

Tasks for this target will ask students to find area (triangles, special quadrilaterals, and polygons) using composition and decomposition; to find volume of right rectangular prisms with fractional edge lengths (see connections to 6.NS Target B); identify and use nets of three-dimensional figures to find surface area; and draw polygons in the coordinate plane with given coordinates or determine the length of a side of a polygon given the coordinates for the vertices.

Many tasks for this target will provide context for Claims 2–4 and connect the content of this target to several other targets across Claim 1 (see, for example, 6.NS Targets B and C, 6.EE Targets E, F, and G).

Standards: 6.G.A, 6.G.A.1, 6.G.A.2, 6.G.A.3, 6.G.A.4	**6.G.A Solve real-world and mathematical problems involving area, surface area, and volume.** **6.G.A.1** Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. **6.G.A.2** Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. **6.G.A.3** Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. **6.G.A.4** Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.
Related Below-Grade and Above-Grade Standards for Purposes of Planning for Vertical Scaling: 5.MD.C, 5.MD.C.4, 5.MD.C.5 7.G.A, 7.G.A.1, 7.G.A.2, 7G.B, 7.G.B.6	**Related Grade 5 Standards** **5.MD.C Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.** **5.MD.C.4** Measure volumes by counting unit cubes, using cubic cm, cubic in, cubit ft, and improvised units. **5.MD.C.5** Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

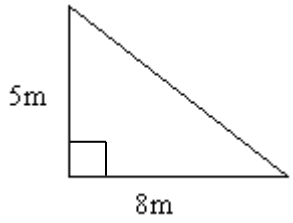
	b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems. c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems. **Related Grade 7 Standards** **7.G.A Draw, construct, and describe geometrical figures and describe the relationships between them.** **7.G.A.1** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing at a different scale. **7.G.A.2** Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures or angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. **7.G.B Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.** **7.G.B.6** Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.
DOK Levels:	1, 2
Achievement Level Descriptors:	
RANGE Achievement Level Descriptor (Range ALD) Target H: Solve real-world and mathematical problems involving area, surface area, and volume.	**Level 1** Students should be able to find areas of right triangles; draw polygons with positive coordinates on a grid with a scale in one-unit increments, given nonnegative integer-valued coordinates for the vertices; and find the volume of right rectangular prisms with one side expressed as a fraction or a mixed number in halves or fourths.
	Level 2 Students should be able to find areas of special quadrilaterals and triangles; draw polygons in the four-quadrant coordinate plane with scales in one-unit increments, given integer-valued coordinates for the vertices; and find the volume of right rectangular prisms with one side expressed as a fraction or a mixed number.
	Level 3 Students should be able to solve problems that involve finding areas of polygons and special quadrilaterals and triangles and find the volume of right rectangular prisms with all sides expressed as a fraction or a mixed number. They should be able to solve problems by drawing polygons in the four-quadrant coordinate plane with scales in various integer increments, given integer-valued coordinates for the vertices or coordinates containing a mix of integers and half, quarter, or tenth units.
	Level 4 Students should be able to solve problems by finding surface areas of three-dimensional shapes composed of rectangles and triangles. They should be able to find the volume of a compound figure composed of right rectangular prisms to solve problems.

Evidence Required:	1. The student determines the area of triangles, special quadrilaterals, and polygons using composition and decomposition in solving real-world and mathematical problems. 2. The student determines the volume of right rectangular prisms with fractional edge lengths in solving real-world and mathematical problems. 3. The student draws polygons in the coordinate plane, given coordinates for the vertices in the context of solving real-world and mathematical problems. 4. The student determines the length of a side of a polygon in the coordinate plane, given coordinates for the vertices in the context of solving real-world and mathematical problems. 5. The student determines the surface area of three-dimensional figures formed by nets of polygons in the context of solving real-world and mathematical problems.
Allowable Response Types:	Equation/Numeric; Graphing
Allowable Stimulus Materials:	coordinate planes, diagrams representing two– and three–dimensional figures
Construct-Relevant Vocabulary:	coordinate, ordered pair, coordinate plane, compose/decompose, vertices, right triangle, unit fraction, edge length, area, surface area, volume, nets, faces, edges, vertices
Allowable Tools:	Calculator
Target-Specific Attributes:	Given dimensions should be positive integers, decimals, or fractions; radicals should not be used as given dimensions. Nets must only be composed of rectangles, triangles, or a combination of both.
Non-Targeted Constructs:	
Accessibility Guidance:	Item writers should consider the following Language and Visual Element/Design guidelines[1] when developing items. Language Key Considerations: • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level • Avoid ambiguous or obscure words, idioms, jargon, unusual names and references Visual Elements/Design Key Considerations: • Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context • Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary • Avoid crowding of details and graphics

[1] For more information, refer to the General Accessibility Guidelines at: http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf

	Items are selected for a student's test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.[2]
Development Notes:	Many tasks for this target will provide context for Claims 2–4 and connect the content of this target to several other targets across Claim 1 (see, for example, 6.NS Targets B and C, 6.EE Targets E, F, and G).

[2] For more information about student accessibility resources and policies, refer to http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

Task Model 1 **Response Type: Equation / Numeric** **DOK Level 1** **6.G.A.1** Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. **Evidence Required:** 1. The student determines the area of triangles, special quadrilaterals, and polygons using composition and decomposition in solving real-world and mathematical problems. **Tools:** Calculator	**Prompt Features:** The student is prompted to determine the areas of triangles in solving mathematical and real-world problems. **Stimulus Guidelines:** • If used, context should be familiar to students 11 to 13 years old. • Rational numbers used should be appropriate for the situation. • Item difficulty can be adjusted via these example methods: o Measurements of shapes can be whole numbers, fractions, or decimals. o Students find the area of right triangles. o Students find the area of non-right triangles such as isosceles triangle, equilateral triangle, or scalene triangle. **TM1a** **Stimulus:** The student is presented with a mathematical problem involving triangles. **Example Stem:** Consider this figure.  Enter the area of the right triangle in square meters. **Rubric:** (1 point) Student enters the correct area of the figure (e.g., 20). Correct answer should be a single numerical value and units should be assumed from the stem. **Response Type:** Equation/Numeric

Task Model 1

Response Type: Equation / Numeric

DOK Level 2

6.G.A.1
Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Evidence Required:
1. The student determines the area of triangles, special quadrilaterals, and polygons using composition and decomposition in solving real-world and mathematical problems.

Tools: Calculator

Accessibility Note: When including diagrams, clearly indicate dimensions. Where reasonable, include the dimensions in the stem.

Prompt Features: The student is prompted to determine the areas of triangles, special quadrilaterals, and other polygons in solving mathematical and real-world problems.

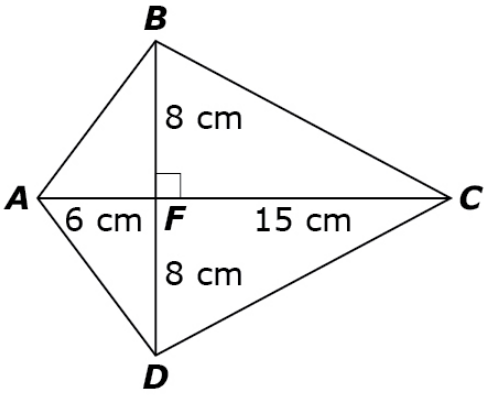
Stimulus Guidelines:

- If used, context should be familiar to students 11 to 13 years old.
- Rational numbers used should be appropriate for the situation.
- Item difficulty can be adjusted via these example methods:
  - Students find area of non-right triangles/special quadrilaterals with whole-number measures.
  - Students find area of polygon that can be decomposed into quadrilaterals and triangles with whole number measures.
  - Students find area of triangles/special quadrilaterals with fraction/decimal measures.
  - Students find area of polygon that can be decomposed into quadrilaterals and triangles with fraction/decimal measures.

TM1b

Stimulus: The student is presented with a mathematical or real-world problem involving composition or decomposition of a triangle, special quadrilateral, or other polygon.

Example Stem 1: Consider this figure.



Enter the total area, in square centimeters, of kite *ABCD*.

Example Stem 2: Figure A is composed of two shapes.

- A rectangle with length 9 inches and width 2 inches
- A square with side length 3 inches

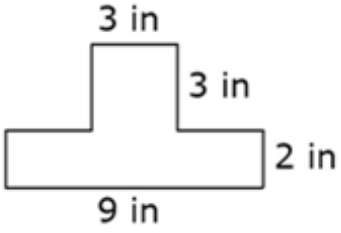
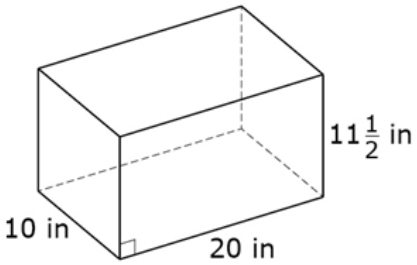
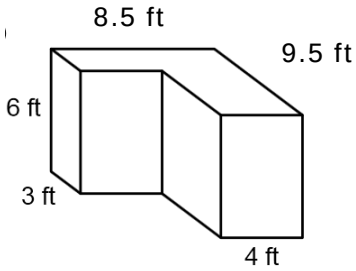


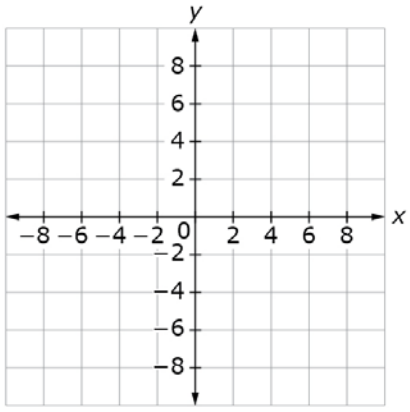
Figure A

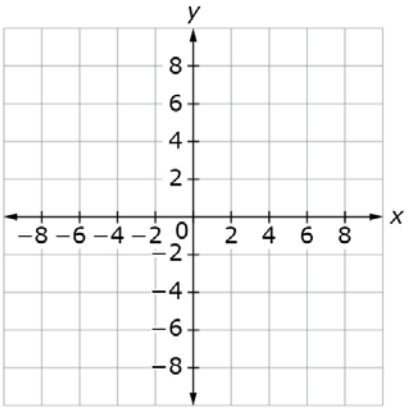
Enter the total area, in square inches, of the Figure A.

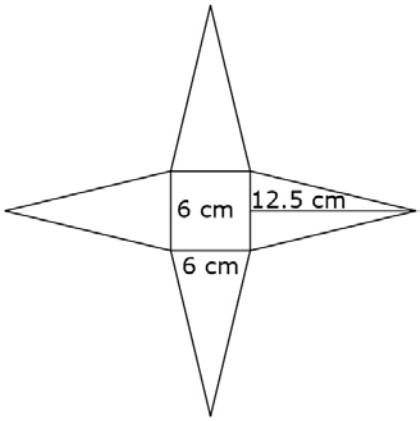
	Rubric: (1 point) Student enters the correct area of the figure (e.g., 168; 27). Correct answer should be a single numerical value and units should be assumed from the stem. **Response Type:** Equation/Numeric

Task Model 2 **Response Type: Equation / Numeric** **DOK Level 1** **6.G.A.2** Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = *lwh* and *V* = *bh* to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. **Evidence Required:** 2. The student determines the volume of right rectangular prisms with fractional edge lengths in solving real-world and mathematical problems. **Tools:** Calculator	**Prompt Features:** The student is prompted to determine the volume of a right rectangular prism by applying the formulas *V* = *lwh* and *V* = *bh*. **Stimulus Guidelines:** • If used, context should be familiar to students 11 to 13 years old. • Item difficulty can be adjusted via these example methods: o Students find volume of rectangular prism with one side measure expressed as fraction/mixed number in halves or fourths. o Students find volume of rectangular prism with one side measure expressed as fraction/mixed number. o Students find volume of rectangular prism with all side measures expressed as fractions/mixed numbers. **TM2a** **Stimulus:** The student is presented with a right rectangular prism with fractional edge lengths in the context of a mathematical or real-world problem. **Example Stem:** Consider this figure.  Enter the volume, in cubic inches, of the right rectangular prism. **Rubric:** (1 point) Student enters the correct volume (e.g., 2300). Correct answer should be a single numerical value and units should be assumed from the stem. **Response Type:** Equation/Numeric

Task Model 2 **Response Type: Equation / Numeric** **DOK Level 2** **6.G.A.2** Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = *lwh* and *V* = *bh* to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. **Evidence Required:** 2. The student determines the volume of right rectangular prisms with fractional edge lengths in solving real-world and mathematical problems. **Tools:** Calculator	**Prompt Features:** The student is prompted to determine the volume of a compound figure composed of right rectangular prisms by applying the formulas *V* = *lwh* and *V* = *bh*. **Stimulus Guidelines:** • If used, context should be familiar to students 11 to 13 years old. • Item difficulty can be adjusted via these example methods: o Use whole-numbers, fractions, or decimals for the side measurements. **TM2b** **Stimulus:** The student is presented with a compound figure composed of right rectangular prisms in the context of a mathematical or real-world problem. **Example Stem:** This figure was created by joining two right rectangular prisms.  Enter the volume, in cubic feet, of the figure. **Rubric:** (1 point) Student enters the correct volume (e.g., 309). Correct answer should be a single numerical value and units should be assumed from the stem. **Response Type:** Equation/Numeric

Task Model 3 **Response Type: Graphing** **DOK Level 1** **6.G.A.3** Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. **Evidence Required:** 3. The student draws polygons in the coordinate plane, given coordinates for the vertices in the context of solving real-world and mathematical problems. **Tools:** Calculator **Accessibility Note:** Graphing items are not currently able to be Brailled. Minimize the number of items developed to this TM.	**Prompt Features:** The student is prompted to draw polygons in the coordinate plane given coordinates for the vertices. **Stimulus Guidelines:** • If used, context should be familiar to students 11 to 13 years old. • Polygons should be limited to triangles, squares, rectangles, parallelograms, kites, rhombi, and trapezoids. • Coordinates of the ordered pairs should be integers. • Item difficulty can be adjusted via these example methods: o Students graph polygon in Quadrant I with one-unit increment axes. o Students graph polygon in all four quadrants with one-unit increment axes. o Students graph polygon in all four quadrants with varying integer increment axes. **TM3** **Stimulus:** The student is presented with the vertices of a polygon in the context of a real-world or mathematical problem. **Example Stem:** Consider these ordered pairs. Point A: (3, 2) Point B: (–3, 2) Point C: (3, –2)  Use the Connect Line tool to form triangle ABC. **Interaction:** The student is given the Connect Line, Add Point, and Delete tools to draw the polygon in the coordinate plane. **Rubric:** (1 point) Student plots all given points and connects the lines correctly. **Response Type:** Graphing

Task Model 4 **Response Type: Equation / Numeric** **DOK Level 2** **6.G.A.3** Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. **Evidence Required:** 4. The student determines the length of a side of a polygon in the coordinate plane, given coordinates for the vertices in the context of solving real-world and mathematical problems. **Tools:** Calculator	**Prompt Features:** The student is prompted to determine the length of a side of a polygon in the coordinate plane given coordinates for the vertices that have the same first coordinate or the same second coordinate. **Stimulus Guidelines:** • If used, context should be familiar to students 11 to 13 years old. • Polygons should be limited to triangles, squares, rectangles, parallelograms, kites, rhombi, and trapezoids. • Coordinates of the ordered pairs should be integers. • Item difficulty can be adjusted via these example methods: o Coordinates of the side used are in the same quadrant. o Coordinates of the side used are in different quadrants. **TM4** **Stimulus:** The student is presented with coordinates for the side of a polygon in the coordinate plane with either the same first coordinate or the same second coordinate in the context of a mathematical or real-world problem. **Example Stem 1:** A triangle has these coordinates: Point A: (–5, 2) Point B: (–5, 6) Point C: (7, 2) Enter the length of side AC. **Example Stem 2:** Refer to the map as a coordinate grid. On the map, the library is located at (–5, 2), the bus station is located at (–5, 6), and the courthouse is located at (7, 2). Each square unit in the grid represents 1 square kilometer.  Enter the distance, in kilometers, from the courthouse to the library. **Rubric:** (1 point) Student enters the correct length (e.g., 12; 12). Correct answer should be a single numerical value and units should be assumed from the stem. **Response Type:** Equation/Numeric

Task Model 5 **Response Type: Equation / Numeric** **DOK Level 2** **6.G.A.4** Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems. **Evidence Required:** 5. The student determines the surface area of three-dimensional figures formed by nets of polygons in the context of solving real-world and mathematical problems. **Tools:** Calculator	**Prompt Features:** The student is prompted to determine the surface area of a three-dimensional figure formed from a net. **Stimulus Guidelines:** • If used, context should be familiar to students 11 to 13 years old. • Rational numbers used should be appropriate for the situation. • Item difficulty can be adjusted via these example methods: o Students find surface area of polygon with all side measures expressed as whole numbers. o Students find surface area of polygon with some side measures expressed as decimals. o Students find surface area of polygon with some side measures expressed as fractions/mixed numbers. **TM5** **Stimulus:** The student is presented with a net composed of rectangles, triangles, or a combination of the two in the context of a real-world or mathematical problem. **Example Stem:** Susan is painting the outside of a square pyramid. The net for the pyramid is shown.  Enter the total surface area, in square centimeters, of the pyramid that Susan will paint. **Rubric:** (1 point) Student enters the correct surface area (e.g., 186). Correct answer should be a single numerical value and units should be assumed from the stem. **Response Type:** Equation/Numeric

Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.	
Content Domain: **Statistics and Probability**	
Target I [a]: Develop an understanding of statistics variability. (DOK 2) Tasks for this target will ask students to identify questions that lead to variable responses; identify a reasonable center and/or spread for a given context.	
Standards: 6.SP.A, 6.SP.A.1, 6.SP.A.2, 6.SP.A.3	**6.SP.A Develop understanding of statistical variability.** **6.SP.A.1** Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.* **6.SP.A.2** Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. **6.SP.A.3** Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
Related Below-Grade and Above-Grade Standards for Purposes of Planning for Vertical Scaling: 5.MD.B, 5.MD.B.2 7.SP.B, 7.SP.B.3	**Related Grade 5 Standards** **5.MD.B Represent and interpret data.** **5.MD.B.2** Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.* **Related Grade 7 Standards** **7.SP.B Draw informal comparative inferences about two populations.** **7.SP.B.3** Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*
DOK Level:	2
Achievement Level Descriptors:	
RANGE Achievement Level Descriptor (Range ALD) Target I: Develop understanding of statistical variability.	**Level 1** Students should be able to identify questions that lead to variable responses posed in familiar contexts and recognize that such questions are statistical questions.
	Level 2 Students should be able to recognize that questions that lead to variable responses are statistical questions and vice versa, and they should relate the concept of varying responses to the notion of a range of possible responses. They should develop an

	understanding that the responses to a statistical question will have a representative center and a given set of numerical data. They should be able to identify a reasonable measure of central tendency with respect to a familiar context.
	Level 3 Students should be able to pose statistical questions and understand that the responses to a statistical question have a distribution described by its center, spread, and overall shape. They should also understand that a measure of center summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. They should be able to identify a reasonable center and spread with respect to a context.
	Level 4 Students should be able to justify the reasonableness of their identified center and spread with respect to an unfamiliar context. They should be able to create or complete a data set with given measures (e.g., mean, median, mode, interquartile range).
Evidence Required:	1. The student recognizes a statistical question as one that anticipates variability. 2. The student identifies statements that describe the center and/or spread, and/or overall shape of a set of data. 3. The student recognizes that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
Allowable Response Types:	Multiple Choice, single correct response; Matching Tables
Allowable Stimulus Materials:	Dot/line plots, lists of numbers, tables, graphs, or other visual graphics to display a set of numbers
Construct-Relevant Vocabulary:	variation (variability), interquartile range, range, mean absolute deviation, center, spread, mean, median, outliers, shape (pertaining to statistics such as gap, cluster, peak, skew, bell curve, and uniform distribution)
Allowable Tools:	Calculator
Target-Specific Attributes:	
Non-Targeted Constructs:	
Accessibility Guidance:	Item writers should consider the following Language and Visual Element/Design guidelines[1] when developing items. Language Key Considerations: • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level • Avoid ambiguous or obscure words, idioms, jargon, unusual names and references

[1] For more information, refer to the General Accessibility Guidelines at: http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf

	Visual Elements/Design Key Considerations: • Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context • Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary • Avoid crowding of details and graphics Items are selected for a student's test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.[2]
Development Notes:	Tasks for this target will ask students to identify and pose questions that lead to variable responses; identify a reasonable center and/or spread for a given context. In Grade 6, the focus on assessment for the SP standards should be in Claim 4. The most important concept is distribution, which is a foundational idea for all future statistical work. Other concepts include shape, center, and spread of a distribution (not the more technical details often associated with those).

[2] For more information about student accessibility resources and policies, refer to http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

Task Model 1

Response Types: Multiple Choice, single correct response; Matching Tables

DOK Level 2

6.SP.A.1
Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*

Evidence Required:
1. The student recognizes a statistical question as one that anticipates variability.

Tools: Calculator

Version 3 Update: Retired TM2 and TM3 as they are more appropriately assessed in Claim 4.

Prompt Features: The student is prompted to identify whether questions are statistical in nature based on whether they anticipate variability in the answer data.

Stimulus Guidelines: Context should be familiar to students 11 to 13 years old.

TM1a
Stimulus: The student is presented with questions based on a statistical scenario.

Example Stem: Julie is writing a report about rainbows and needs to gather data from her classmates.

Which is a statistical question Julie could ask her classmates?

A. What are the colors of the rainbow?
B. When was the first rainbow seen?
C. Is there really a pot of gold at the end of a rainbow?
D. How many rainbows have you seen this month?

Rubric: (1 point) Student selects the statistical question (e.g., D)

Response Type: Multiple Choice, single correct response

TM1b
Stimulus: The student is presented with three statistical and non-statistical questions.

Example Stem: A statistical question anticipates variability in the data related to it. Determine whether each question can be classified as a statistical question. Select Yes or No for each question.

Question	Yes	No
How many hours a week do people exercise?		
How many hours are there in a day?		
How many rainbows have students seen this month?		

Rubric: (1 point) Student identifies all three questions correctly (e.g., Y, N, Y). At least one question should be statistical.

Response Type: Matching Tables

Claim 1: Concepts and Procedures
Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.

Content Domain: **Statistics and Probability**

Target J [a]: Summarize and describe distributions. (DOK Levels 1, 2)

Tasks for this target will ask students to create number lines, dot plots, histograms, and box plots. The reporting of quantitative measures (median and/or mean, interquartile range and/or mean absolute deviation) may be included in these tasks or delivered as separate tasks.

Other tasks for this target will ask students to match the shape of a data distribution to its quantitative measures.

Standards: 6.SP.B, 6.SP.B.4, 6.SP.B.5	**6.SP.B Summarize and describe distributions.** **6.SP.B.4** Display numerical data in plots on a number line, including dot plots, histograms, and box plots. **6.SP.B.5** Summarize numerical data sets in relation to their context, such as by: **a**. Reporting the number of observations. **b**. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. **c**. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. **d**. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.
Related Below-Grade and Above-Grade Standards for Purposes of Planning for Vertical Scaling: 5.MD.B, 5.MD.B.2 7.SP.B, 7.SP.B.3, 7.SP.B.4	**Related Grade 5 Standards** **5.MD.B Represent and interpret data.** **5.MD.B.2.** Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.* **Related Grade 7 Standards** **7.SP.B Draw informal comparative inferences about two populations.** **7.SP.B.3** Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.* **7.SP.B.4** Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the*

	words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade book.
DOK Levels:	1, 2
Achievement Level Descriptors:	
RANGE Achievement Level Descriptor (Range ALD) Target J: Summarize and describe distributions.	**Level 1** Students should be able to summarize or display numerical data on a number line, in dot plots, and in histograms; find the median of an odd number of data points; and find the mean when data points are nonnegative integers.
	Level 2 Students should be able to calculate mean and median, understand that mean and median can be different or the same, and use the measure of center to summarize data with respect to the context.
	Level 3 Students should be able to summarize or display data in box plots and find the interquartile range. They should be able to use the interquartile range along with the angle and measures of center to describe overall patterns in a data distribution, such as symmetry and clusters, and any striking deviations. They should also be able to examine a data set in context and explain the choice of the mean or median, as it relates to the data.
	Level 4 Students should be able to relate choice of measures of center and variability to the shape of the data distribution in context of the data; find mean absolute deviation and identify outliers with reference to the context of the situation; and predict effects on the mean and median, given a change in data points.
Evidence Required:	1. The student displays numerical data on line plots, dot plots, histograms, and box plots. 2. The student summarizes numerical data sets by describing the nature of the attribute under investigation, including how it was measured, its units of measurement, and number of observations. 3. The student summarizes numerical data sets by determining quantitative measures of center (median and/or mean) and variability (interquartile range, range, and/or mean absolute deviation). 4. [Retired Evidence Required statement] 5. [Retired Evidence Required statement]
Allowable Response Types:	Multiple Choice, single correct response; Multiple Choice, multiple correct response; Equation/Numeric; Drag and Drop; Hot Spot; Matching Tables
Allowable Stimulus Materials:	number line diagrams, dot plots, histograms, box plots
Construct-Relevant Vocabulary:	variability, interquartile range, range, mean absolute deviation, outliers, center, spread, mean, median, shape (pertaining to statistics such as gap, cluster, peak, skew, bell curve, and uniform distribution)
Allowable Tools:	Calculator
Target-Specific Attributes:	

Non-Targeted Constructs:	
Accessibility Guidance:	Item writers should consider the following Language and Visual Element/Design guidelines[1] when developing items. Language Key Considerations: • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level • Avoid ambiguous or obscure words, idioms, jargon, unusual names and references Visual Elements/Design Key Considerations: • Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context • Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary • Avoid crowding of details and graphics Items are selected for a student's test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.[2]
Development Notes:	In Grade 6, the focus on assessment for the SP standards should be in Claim 4. The most important concept is distribution, which is a foundational idea for all future statistical work. Other concepts include shape, center, and spread of a distribution (not the more technical details often associated with those).

[1] For more information, refer to the General Accessibility Guidelines at: http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf

[2] For more information about student accessibility resources and policies, refer to http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

Task Model 1

Response Types: Drag and Drop, Hot Spot, Multiple Choice, single correct response

DOK Level 2

6.SP.B.4
Display numerical data in plots on a number line, including dot plots, histograms, and box plots.

Evidence Required:
1. The student displays numerical data on line plots, dot plots, histograms, and box plots.

Tools: Calculator

Accessibility Note: Hot Spot items are not currently able to be Brailled. Minimize the number of items developed to this TM.

Prompt Features: The student is prompted to generate line plots, dot plots, histograms, or box plots that represent a set of numerical data.

Stimulus Guidelines:

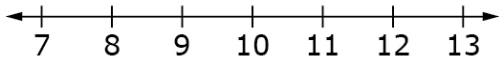
- If used, context should be familiar to students 11 to 13 years old.
- Numbers in the data set should be whole numbers.
- Vertical axis for histograms should be in one-unit increments.
- Item difficulty can be adjusted via these example methods:
  - Students create line plot/dot plot/histogram that corresponds to a given data set.
  - Students select/create box plot that corresponds to given data set.

TM1a
Stimulus: Students create a dot plot given a data set.

Example Stem: The ages of 9 students in a summer camp are shown.

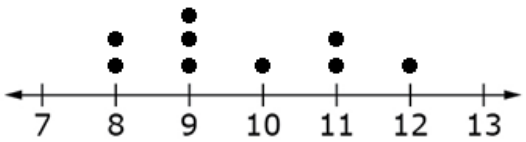
10	11	12
9	8	9
11	9	8

Click above the number line to create a dot plot for the data set.



Interaction: The student is given a labeled number line. Student uses the Hot Spot tool to click spaces above the number line to create a dot plot.

Rubric: (1 point) Student correctly creates a dot plot to represent the data (see below).



Response Type: Hot Spot

Task Model 1

Response Types: Drag and Drop, Hot Spot, Multiple Choice, single correct response

DOK Level 2

6.SP.B.4
Display numerical data in plots on a number line, including dot plots, histograms, and box plots.

Evidence Required:
1. The student displays numerical data on line plots, dot plots, histograms, and box plots.

Tools: Calculator

Accessibility Note: Hot Spot items are not currently able to be Brailled. Minimize the number of items developed to this TM.

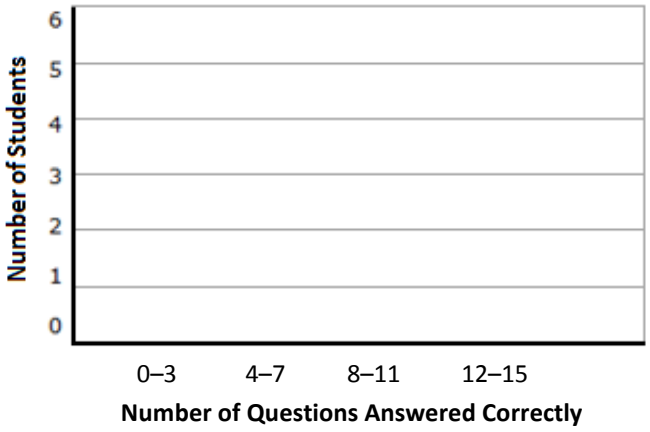
TM1b
Stimulus: Students create a histogram given a data set.

Example Stem: The numbers of test questions answered correctly by 9 students are shown.

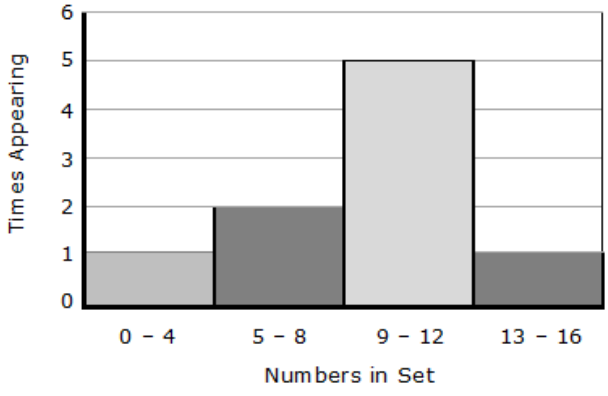
10	11	2
9	15	9
7	4	8

Click within the graph area to create a histogram for the data set.

Interaction: The student is given a graph with both axes labeled. Hot Spot tool is used to click unit squares on the graph to shade in and create a histogram.



Rubric: (1 point) Student correctly creates a histogram to represent the data (see below)



Response Type: Hot Spot

Task Model 1

Response Types: Drag and Drop, Hot Spot, Multiple Choice, single correct response

DOK Level 2

6.SP.B.4
Display numerical data in plots on a number line, including dot plots, histograms, and box plots.

Evidence Required:
1. The student displays numerical data on line plots, dot plots, histograms, and box plots.

Tools: Calculator

Accessibility Note: Drag and Drop items are not currently able to be Brailled. Minimize the number of items developed to this TM.

TM1c

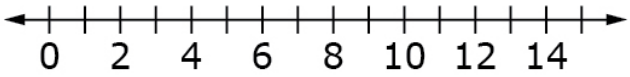
Stimulus: Students create a box plot given a data set.

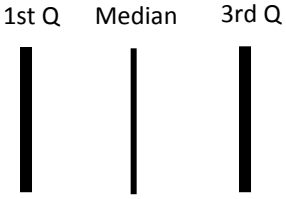
Example Stem: The numbers of test questions answered correctly by 9 students are shown.

10	11	12
9	15	9
7	4	8

The vertical line segments represent the 1st quartile (1st Q), median, and the 3rd quartile (3rd Q) of the data set.

Drag each line segment to the correct location on the number line.





Interaction: The student is given a number line and a palette at the bottom of the screen. The palette contains three images of line segments labeled “1st Q,” “Median,” and “3rd Q.” Students use the drag-and-drop tool to place the line segents in the appropriate place on the number line. Snap-to feature should be used at each tick mark on the number line.

Rubric: (1 point) Student places the three line segments in the correct locations on the number line.

Response Type: Drag and Drop

Task Model 1

Response Types: Drag and Drop, Hot Spot, Multiple Choice, single correct response

DOK Level 2

6.SP.B.4
Display numerical data in plots on a number line, including dot plots, histograms, and box plots.

Evidence Required:
1. The student displays numerical data on line plots, dot plots, histograms, and box plots.

Tools: Calculator

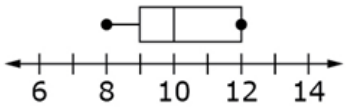
TM1d
Stimulus: Students identify the box plot that represents a given data set.

Example Stem: The ages of 9 students in a summer camp are shown in this frequency table.

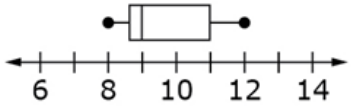
Age	Frequency
8	2
9	3
10	1
11	2
12	1

Which box plot correctly displays the data shown in the table?

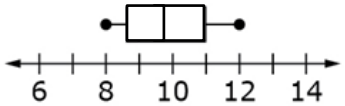
A.



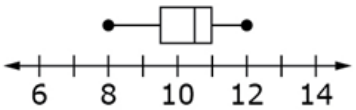
B.



C.



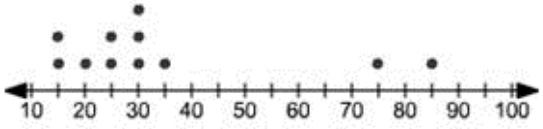
D.



Answer Choices: Answer choices will be box plots. Distractors will include incorrectly calculating the median, upper and lower quartile, and/or misrepresenting the data on a box plot.

Rubric: (1 point) The student selects the correct box plot (e.g., B).

Response Type: Multiple Choice, single correct response

Task Model 2 **Response Type: Equation / Numeric** **DOK Level 1** **6.SP.B.5a, 6.SP.B.5b** Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. **Evidence Required:** 2. The student summarizes numerical data sets by describing the nature of the attribute under investigation including how it was measured, its units of measurement, and number of observations. **Tools:** Calculator	**Prompt Features:** The student is prompted to summarize numerical data sets by writing how it was measured, its units of measurement, or number of observations. **Stimulus Guidelines:** • Context should be familiar to students 11 to 13 years old. • Data set may be presented as a: o table o line/dot plot o histogram • Item difficulty can be adjusted via these example methods: o Students give the number of observations that corresponds to a given data set. o Students describe how the attribute of a given data set is measured and the unit of measurement used. **TM2** **Stimulus:** The student is presented with a set of numerical data. **Example Stem:** Ted surveyed his neighbors to see how much money they spent on gasoline each week. The results are in the dot plot shown.  Amount of money (dollars) Enter the total number of people Ted surveyed. **Rubric:** (1 point) Student enters correct value (e.g., 11). **Response Type:** Equation/Numeric

Task Model 3

Response Type: Equation / Numeric

DOK Level 2

6.SP.B.5c
Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.

Evidence Required:
3. The student summarizes numerical data sets by determining quantitative measures of center (median and/or mean) and variability (interquartile range, range, and/or mean absolute deviation).

Tools: Calculator

Prompt Features: The student is prompted to write quantitative values for the measures of center (median or mean) or variability (interquartile range) for a given numerical data set.

Stimulus Guidelines:

- Context should be familiar to students 11 to 13 years old.
- Data set may be presented as a:
  - list
  - table
  - line/dot plot
  - box plot
- Item difficulty can be adjusted via these example methods:
  - Students find the range/median for a data set (odd number data set for median).
  - Students find the mean/median for a data set (even number data set for median).

TM3a
Stimulus: The student is presented with a set of numerical data.

Example Stem 1: Sophia surveyed her friends to see how many minutes they studied for their math test last evening. The results are in this list.

10, 15, 20, 15, 35, 25, 20, 30, 25

Enter the **mean** of the data.

Rubric: (1 point) Student gives the correct mean of the data. Students' answers should be within an acceptable range (e.g., 21.6–22).

Response Type: Equation/Numeric

Example Stem 2: Avery surveyed her friends to see how many minutes they studied for their math test last evening. The results are shown in the frequency table.

Minutes	Frequency
10	||
15	|||
20	||
25	||
30	||
35	|

Enter the **median** of the data.

Rubric: (1 point) Student gives the correct median of the data (e.g., 20).

Response Type: Equation/Numeric

Task Model 3

Response Type: Equation / Numeric

DOK Level 2

6.SP.B.5c
Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.

Evidence Required:
3. The student summarizes numerical data sets by determining quantitative measures of center (median and/or mean) and variability (interquartile range, range, and/or mean absolute deviation).

Tools: Calculator

Version 3 Update:
Removed example stem 2 from TM3b and retired TM3c, TM4, and TM5.

Prompt Features: The student is prompted to write quantitative values for the measures of variability (interquartile range) for a given numerical data set.

Stimulus Guidelines:

- Context should be familiar to students 11 to 13 years old.
- Data set may be presented as a:
  - list
  - table
  - line/dot plot
  - box plot
- Item difficulty can be adjusted via these example methods:
  - The data set has an odd amount of numbers.
  - The data set has an even amount of numbers.
  - Student finds the interquartile range.

TM3b
Stimulus: The student is presented with a set of numerical data.

Example Stem: Avery surveyed her friends to see how many minutes they studied for their math test last evening. The results are shown in the frequency table.

Minutes	Frequency
10	|
15	||
20	||
25	||
30	||
35	|

Enter the **interquartile range** of the data set.

Rubric: (1 point) Student enters the correct interquartile range of the data (e.g., 15).

Response Type: Equation/Numeric

Grades 6-8 Mathematics Item Specification Claim 2	
Problem solving, which of course builds on a foundation of knowledge and procedural proficiency, sits at the core of *doing* mathematics. Proficiency at problem solving requires students to choose to use concepts and procedures from across the content domains and check their work using alternative methods. As problem solving skills develop, student understanding of and access to mathematical concepts becomes more deeply established. (*Mathematics Content Specifications, p.56*)	
Primary Claim 2: Problem Solving Students can solve a range of well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.	
Secondary Claim(s): Items/tasks written primarily to assess Claim 2 will necessarily involve some Claim 1 content targets. Related Claim 1 targets should be listed below the Claim 2 targets in the item form. If Claim 3 or 4 targets are also directly related to the item/task, list those following the Claim 1 targets in order of prominence.	
Primary Content Domain: Each item/task should be classified as having a primary, or dominant, content focus. The content should draw upon the knowledge and skills articulated in the progression of standards leading up to and including the targeted grade within and across domains.	
Secondary Content Domain(s): While tasks developed to assess Claim 2 will have a primary content focus, components of these tasks will likely produce enough evidence for other content domains that a separate listing of these content domains needs to be included where appropriate. The standards in the NS domain in grades 6-8 can be used to construct higher difficulty items for the adaptive pool. The integration of the RP, EE, and G domains with NS allows for higher content limits within the grade level than might be allowed when staying within the primary content domain.	
DOK Levels	1, 2, 3
Allowable Response Types	**Response Types:** Multiple Choice, single correct response (MC); Multiple Choice, multiple correct response (MS); Equation/Numeric (EQ); Drag and Drop, Hot Spot, and Graphing (GI); Matching Tables (MA); Fill-in Table (TI) No more than six choices in MS and MA items. Short Text – Performance tasks only **Scoring:** Scoring rules and answer choices will focus on students' ability to solve problems and/or to apply appropriate strategies to solve problems. For some problems, multiple correct responses and/or strategies are possible. • MC will be scored as correct/incorrect (1 point) • If MS and MA items require two skills, they will be scored as:

	○ All correct choices (2 points); at least ½ but less than all correct choices (1 point) ○ Justification[1] for more than 1 point **must be** clear in the scoring rules ○ Where possible, include a "disqualifier" option that if selected would result in a score of 0 points, whether or not the student answered ½ correctly. ○ Numeric items scored as correct/incorrect (1 point) • GI,TI, and EQ items will be scored as: ○ Single requirement items: will be scored as correct/incorrect (1 point) ○ Multiple requirement items: All components correct (2 points); at least ½ but less than all correct (1 point) ○ Justification for more than 1 point **must be** clear in the scoring rules
Allowable Stimulus Materials	Effort must be made to minimize the reading load in problem situations. Use tables, diagrams with labels, and other strategies to lessen reading load. Use simple subject-verb-object (SVO) sentences; use contexts that are familiar and relevant to students at the targeted grade level. Target-specific stimuli will be derived from the Claim 1 targets used in the problem situation. All real-world problem contexts will be relevant to the age of the students. Stimulus guidelines specific to task models are given below.
Construct-Relevant Vocabulary	Refer to the Claim 1 specifications to determine construct-relevant vocabulary associated with specific content standards.
Allowable Tools	Any mathematical tools appropriate to the problem situation and the Claim 1 target(s). Some tools are identified in Standard for Mathematical Practice 5 and others can be found in the language of specific standards.
Target-Specific Attributes:	CAT items should take from 2 to 5 minutes to solve; Claim 2 items that are part of a performance task may take 5 to 10 minutes.

[1] For a CAT item to score multiple points, either distinct skills must be demonstrated that earn separate points or distinct levels of understanding of a complex skill must be tied directly to earning one or more points.

Accessibility Guidance	Item writers should consider the following Language and Visual Element/Design guidelines[2] when developing items. Language Key Considerations: • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level • Avoid ambiguous or obscure words, idioms, jargon, unusual names and references Visual Elements/Design Key Considerations: • Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context • Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary • Avoid crowding of details and graphics Items are selected for a student's test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.[3]
Development Notes	Tasks generating evidence for Claim 2 in a given grade will draw upon knowledge and skills articulated in the progression of standards up through that grade, though more complex problem-solving tasks may draw upon knowledge and skills from lower grade levels. Claim 1 *Specifications* that cover the following standards should be used to help inform an item writer's understanding of the difference between how these standards are measured in Claim 1 versus Claim 2. Development notes have been added to many of the Claim 1 specifications that call out specific topics that should be assessed under Claim 2. There are some other useful distinctions between Claim 1 and Claim 2 in grades 6-8 that have supported the approach to alignment. The following points describe some attributes of items in Claim 2: • Multiple approaches are feasible or a range of responses is expected (e.g., if a student can solve a word problem by identifying a key word or words and selecting

[2] For more information, refer to the General Accessibility Guidelines at: http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf

[3] For more information about student accessibility resources and policies, refer to http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

	operations, then it is Claim 1.) • The use of tools in Claim 2 is intended to support the problem solving process. In some cases, students may be asked to display their answer on the tool (e.g., by clicking the appropriate point or interval on a number line or ruler). • Assessing the reasonableness of answers to problems is a Claim 2 skill with items that align to Target C. In grades 6-7, Claim 2 tasks should be written to support three key themes: • Solving problems with ratios, rates, and proportions • Solving problems involving understanding of number systems • Solving problems with expressions and equations In grade 8, Claim 2 tasks should be written to support three key themes: • Solving problems with expressions and equations • Solving problems with functions • Solving problems involving geometry At least 80% of the items written to Claim 2 should primarily assess the standards and clusters listed in the table that follows.

Grade 6	**Grade 7**	**Grade 8**
6.RP.A	7.RP.A	8.EE.B
6.NS.A	7.NS.A	8.EE.C
6.NS.C	7.EE.A	8.F.A
6.EE.A	7.EE.B	8.F.B*
6.EE.B	7.G.A*	8.G.A
6.EE.C	7.G.B*	8.G.B
6.G.A*		8.G.C*

* Denotes additional and supporting clusters

Assessment Targets: Any given item/task should provide evidence for two or more Claim 2 assessment targets. Each of the following targets should not lead to a separate task: it is in *using* content from different areas, including work studied in earlier grades, that students demonstrate their problem-solving proficiency. Multiple targets should be listed in order of prominence as related to the item/task.

Target A: Apply mathematics to solve well-posed problems in pure mathematics and arising in everyday life, society, and the workplace. (DOK 1, 2, 3)
Under Claim 2, the problems should be completely formulated, and students should be asked to find a solution path from among their readily available tools.

Target B: Select and use appropriate tools strategically. (DOK 1, 2)
Tasks used to assess this target should allow students to find and choose tools; for example, using a "Search" feature to call up a formula (as opposed to including the formula in the item stem) or using a protractor in physical space.

Target C: Interpret results in the context of a situation. (DOK 2)
Tasks used to assess this target should ask students to link their answer(s) back to the problem's context. In early grades, this might include a judgment by the student of whether to express an answer to a division problem using a remainder or not based on the problem's context. In later grades, this might include a rationalization for the domain of a function being limited to positive integers based on a problem's context (e.g., understanding that the number of buses required for a given situation cannot be 32½, or that the negative values for the independent variable in a quadratic function modeling a basketball shot have no meaning in this context).

Target D: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas). (DOK 1, 2, 3)
For Claim 2 tasks, this may be a separate target of assessment explicitly asking students to use one or more potential mappings to understand the relationship between quantities. In some cases, item stems might suggest ways of mapping relationships to scaffold a problem for Claim 2 evidence.

What sufficient evidence looks like for Claim 2 (Problem Solving)[4]:

"Although items and tasks designed to provide evidence for this claim must primarily assess the student's ability to identify the problem and to arrive at an acceptable solution, mathematical problems nevertheless require students to apply mathematical concepts and procedures."

Properties of items/tasks that assess Claim 2: The assessment of many relatively discrete and/or single-step problems can be accomplished using short constructed-response items, or even computer-enhanced or selected-response items.

More extensive constructed-response items can effectively assess multi-stage problem solving and can also indicate unique and elegant strategies used by some students to solve a given problem, and can illuminate flaws in a student's approach to solving a problem. These tasks could:

- Present non-routine[5] problems where a substantial part of the challenge is in deciding what to do, and which mathematical tools to use; and
- Involve chains of autonomous[6] reasoning, in which some tasks may take a successful student 5 to 10 minutes, depending on the age of the student and the complexity of the task.

"A distinctive feature of both single-step and multi-step items and tasks for Claim 2 is that they are "well-posed." That is, whether the problem deals with pure or applied contexts, the problem itself is completely formulated; the challenge is in identifying or using an appropriate solution path."

[4] Text excerpted from the Smarter Balanced Mathematics Content Specifications (p. 56-57).
[5] As noted earlier, by "non-routine" we mean that the student will not have been taught a closely similar problem, so will not be expected to *remember* a solution path but will have to *adapt* or *extend* their earlier knowledge to find one.
[6] By "autonomous" we mean that the student responds to a single prompt, without further guidance within the task.

Grade 6 Content Combinations:	**The following standards can be effectively used in various combinations in Grade 6 Claim 2 items:** **Primary emphases for Claim 2 Items: Ratios and Proportional Relationships, The Number System, Expressions and Equations** **Ratios and Proportional Relationships (RP)** **6.RP.A: Understand ratio concepts and use ratio reasoning to solve problems.** **6.RP.A.1** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* **6.RP.A.2** Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."* **6.RP.A.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. **a.** Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. **b.** Solve unit rate problems including those involving unit pricing and constant speed. *For example, "If it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?"* **c.** Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. **d.** Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. **The Number System (NS)** **6.NS.A: Apply and extend previous understanding of multiplication and division to divide fractions by fractions.** **6.NS.A.1** Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ac/bd.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of*

yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?

6.NS.C: Apply and extend previous understandings of numbers to the system of rational numbers.

6.NS.C.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

6.NS.C.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

- **a.** Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite.
- **b.** Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
- **c.** Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

6.NS.C.7 Understand ordering and absolute value of rational numbers.

- **a.** Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.*
- **b.** Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3°C > –7°C to express the fact that –3°C is warmer than –7°C.*
- **c.** Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.*
- **d.** Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.*

6.NS.C.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Expressions and Equations (EE)

6.EE.A: Apply and extend previous understandings of arithmetic to algebraic expressions.

6.EE.A.1 Write and evaluate numerical expressions involving whole-number exponents.
6.EE.A.2 Write, read, and evaluate expressions in which letters stand for numbers.

- **a.** Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.*
- **b.** Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2(8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.*
- **c.** Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $V = s^3$ *and* $A = 6\ s^2$ *to find the volume and surface area of a cube with sides of length s = 1/2.*

6.EE.A.3 Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*
6.EE.A.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*

6.EE.B: Reason about and solve one-variable equations and inequalities.
6.EE.B.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
6.EE.B.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
6.EE.B.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q, and x are all nonnegative rational numbers.
6.EE.B.8 Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

6.EE.C: Represent and analyze quantitative relationships between dependent and independent variables.
6.EE.C.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between

	the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.* **Standards to integrate with the emphases:** **Geometry (G)** **6.G.A: Solve real-world and mathematical problems involving area, surface area, and volume.** **6.G.A.1** Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. **6.G.A.2** Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas *V = lwh* and *V = bh* to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. **6.G.A.3** Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. **6.G.A.4** Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.
Grade 7 Content Combinations:	**The following standards can be effectively used in various combinations in Grade 7 Claim 2 items:** **Primary emphases for Claim 2 Items at Grade 7: Ratios and Proportional Relationships, The Number System, Expressions and Equations** **Ratios and Proportional Relationships (RP)** **7.RP.A: Analyze proportional relationships and use them to solve real-world and mathematical problems.** **7.RP.A.1** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction ½/¼ miles per hour, equivalently 2 miles per hour.* **7.RP.A.2** Recognize and represent proportional relationships between quantities. **a.** Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent

ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn*.

d. Explain what a point *(x, y)* on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r)* where *r* is the unit rate.

7.RP.A.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

The Number System (NS)

7.NS.A: Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

7.NS.A.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*

b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

c. Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

d. Apply properties of operations as strategies to add and subtract rational numbers.

7.NS.A.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(–1)(–1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with a non-zero divisor) is a rational number. If p and q are integers, then $–(p/q) = (–p)/q = p/(–q)$. Interpret quotients of rational numbers by describing real-world contexts.

c. Apply properties of operations as strategies to multiply and divide rational numbers.

d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

7.NS.A.3 Solve real-world and mathematical problems involving the four operations with rational numbers.

Expressions and Equations (EE)

7.EE.A: Use properties of operations to generate equivalent expressions.

7.EE.A.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

7.EE.A.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."*

7.EE.B: Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.B.3 Solve multi-step, real-life, and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

7.EE.B.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Give an inequality for the number of sales you need to make, and describe the solutions.*

	Standards to integrate with the emphases: **Geometry (G)** **7.G.A: Draw, construct, and describe geometrical figures and describe the relationships between them.** **7.G.A.1** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. **7.G.A.2** Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. **7.G.A.3** Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right-rectangular prisms and right-rectangular pyramids. **7.G.B Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.** **7.G.B.4** Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. **7.G.B.5** Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. **7.G.B.6** Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
Grade 8 Content Combinations:	**The following standards can be effectively used in various combinations in Grade 8 Claim 2 items:** **Primary emphases for Grade 8 Claim 2 Items: Expressions and Equations, Functions, and Geometry** **Expressions and Equations (EE)** **8.EE.B: Understand the connections between proportional relationships, lines, and linear equations.** **8.EE.B.5** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* **8.EE.B.6** Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

8.EE.C: Analyze and solve linear equations and pairs of simultaneous linear equations.

8.EE.C.7 Solve linear equations in one variable.

- **a.** Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).
- **b.** Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

8.EE.C.8 Analyze and solve pairs of simultaneous linear equations.

- **a.** Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
- **b.** Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*
- **c.** Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line though the first pair of points intersects the line through and second pair.*

Functions (F)

8.F.A: Define, evaluate, and compare functions.

8.F.A.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

8.F.A.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

8.F.A.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1, 1), (2, 4) and (3, 9), which are not on a straight line.*

Geometry (G)

8.G.A: Understand congruence and similarity using physical models, transparencies, or

	geometry software. **8.G.A.1** Verify experimentally the properties of rotations, reflections, and translations: **a.** Lines are taken to lines, and line segments to line segments of the same length. **b.** Angles are taken to angles of the same measure. **c.** Parallel lines are taken to parallel lines. **8.G.A.2** Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. **8.G.A.3** Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. **8.G.A.4** Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two dimensional figures, describe a sequence that exhibits the similarity between them. **8.G.A.5** Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and explain, in terms of transversals why this is so.* **8.G.B: Understand and apply the Pythagorean Theorem.** **8.G.B.6** Explain a proof of the Pythagorean Theorem and its converse. **8.G.B.7** Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real–world and mathematical problems in two and three dimensions. **8.G.B.8** Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. **Standards to integrate with the primary emphases** **Functions (F)** **8.F.B: Use functions to model relationships between quantities.** **8.F.B.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. **8.F.B.5** Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

	Geometry (G) **8.G.C: Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.** **8.G.C.9** Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Range ALDs – Claim 2 Grades 6 - 8	**Level 1** Students should be able to identify important quantities in the context of a familiar situation and translate words to equations or other mathematical formulation. When given the correct math tool(s), students should be able to apply the tool(s) to problems with a high degree of scaffolding.
	Level 2 Students should be able to identify important quantities in the context of an unfamiliar situation and to select tools to solve a familiar and moderately scaffolded problem or to solve a less familiar or a nonscaffolded problem with partial accuracy. Students should be able to provide solutions to familiar problems using an appropriate format (e.g., correct units, etc.). They should be able to interpret information and results in the context of a familiar situation.
	Level 3 Students should be able to map, display, and identify relationships, use appropriate tools strategically, and apply mathematics accurately in everyday life, society, and the workplace. They should be able to interpret information and results in the context of an unfamiliar situation.
	Level 4 Students should be able to analyze and interpret the context of an unfamiliar situation for problems of increasing complexity and solve problems with optimal solutions.

Target 2A: Apply mathematics to solve well-posed problems in pure mathematics and arising in everyday life, society, and the workplace.

General Task Model Expectations for Target 2A:

- The student is asked to solve a well-posed problem arising in a mathematical context or a context from everyday life, society, or the workplace.
- Mathematical information from the context is presented in a table, graph, or diagram, or is extracted from a verbal description or pictorial representation of the context.
- Solving the problem requires, in Grades 6–7, understanding of and proficiency with ratios, rates and proportional relationships, the number system, or expressions and equations; in Grade 8, understanding of and proficiency with expressions and equations, functions, and geometry and geometric measurement.
- Understandings from statistics, probability, and geometry may be needed to set up the problem, but are not the primary focus of the problem (except that geometry is a legitimate primary focus in Grade 8). Claim 4 is the proper place for problems whose primary focus is statistics or probability.
- The task does not indicate by key words or other scaffolding which arithmetic and algebraic operations, and which geometry constructions or transformations, are to be performed or in what order.
- Difficulty of the task may be varied by varying (a) the difficulty of extracting information from the context (b) the number of steps or (c) the complexity of the expressions, equations, functions, or geometric figures or measurements used.
- Tasks have DOK Level 1, 2, or 3.

Task Model 2A.1

Expectations:

- Students use ratios, rates or proportional relationships to solve a problem arising in a real-world context.
- Dimensions along which to vary the task include
  a) Using ratios of whole numbers (Grade 6, Example Item a) versus fractions (Grade 7, Example Item c). The associated unit rate can be a fraction in Grade 6 (Example Item b).
  b) Working with single ratios or expecting students to find equivalent ratios, including making tables of equivalent ratios (Grade 6) versus expecting an understanding of proportional relationships (Grade 7, Example Item d) versus comparing proportional relationships (Grade 8, Example Item e).
  c) Complexity of percent problems, e.g., calculating the whole from a part or the part from a whole (Grade 6, Example Item f), versus calculating the total amount given a part and the change between the part and the whole (Grade 7, Example Item g).

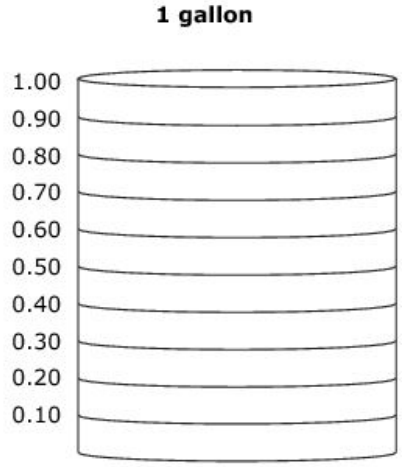
Example Item 2A.1a (Grade 6):
Primary Target 2A (Content Domain RP), Secondary Target 1A (CCSS 6.RP.A), Tertiary Target 2D

Tim made 80 gallons of paint by mixing 48 gallons of green paint with 32 gallons of blue paint.

What part of every gallon is from green paint?

The picture represents 1 gallon of mixed paint.

Click on the picture to show how much of the gallon is from green paint.



Rubric: (1 point) The student clicks on the picture so that 0.6 gallon is shaded.

Response Type: Hot Spot

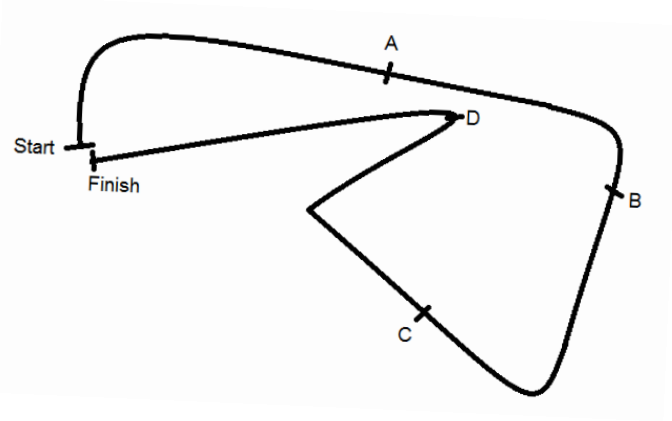
Example Item 2A.1b (Grade 6):
Primary Target 2A (Content Domain RP), Secondary Target 1A (CCSS 6.RP.A), Tertiary Target 2D

It takes Shaun 90 minutes to complete a 15 mile race. The route, with four checkpoints (labeled A, B, C, and D), is shown.

Assume Shaun runs at a constant rate during the race.

Complete the table to show Shaun's time, in minutes, and distance, in miles, at each checkpoint.

Checkpoint	A	B	C	D	Finish
Number of minutes		30		75	90
Number of miles	3		8.5		15



Rubric: (2 points) The student correctly enters all four missing values in the table.
(1 point) The student correctly determines both minutes (e.g., 18, 51) or both miles (e.g., 5, 12.5) or three out of four values correct.

Response Type: Fill-in Table

Commentary: Filling out the different cells in the table requires increasingly sophisticated skills moving from left to right. For students using a unit rate, they must first multiply one-digit whole numbers, then divide a two-digit by a one-digit number resulting in a whole number, then multiply a decimal and a whole number, then divide a two-digit whole number by a one-digit whole number resulting in a decimal. The item could be made easier by changing all entries to require whole-number arithmetic or harder by changing all entries to require decimal number arithmetic. Alternatively, students might notice that the entries in columns A and B are obvious factors of the entries of the columns labeled "Finish" and could easily find their corresponding entries; changing those numbers to less obvious factors would increase the difficulty for students as well.

Example Item 2A.1c (Grade 6):
Primary Target 2A (Content Domain RP), Secondary Target 1A (6.RP.A), Tertiary Standard 2D

Katie and Becca each bought a new book for $50.

- Katie sold her book to the used bookstore for 25% less than the original price.
- Becca sold her book to the used bookstore for 40% less than the original price.

Enter how much more money, in dollars, Katie received for her book than Becca received for her book.

Rubric: (1 point) The student enters the correct difference in the response box (e.g., 7.50 or 7 ½).

Response Type: Equation/Numeric

Example Item 2A.1d (Grade 7):
Primary Target 2A (Content Domain RP), Secondary Target 1A (CCSS 7.RP.A), Tertiary Target 2D

Luke buys a television that is on sale for 25% off the original price. The original price is $120 more than the sale price.

What is the original price of the television?

Rubric: (1 point) The student enters the correct original price in the response box (e.g., 480).

Response Type: Equation/Numeric

Example Item 2A.1e (Grade 7):
Primary Target 2A (Content Domain RP), Secondary Target 1A (CCSS 7.RP.A), Tertiary Target 2D

Elly poured $\frac{1}{10}$ gallon of water into an empty bottle. Now it is $\frac{1}{2}$ full. How many **cups** of water does a full bottle hold?

- There are 16 cups in one gallon.

Enter the total number of **cups** that are in the bottle when it is full.

Rubric: (1 point) The student enters the correct number of cups in the response box (e.g., $3\frac{1}{5}$ or 3.2).

Response Type: Equation/Numeric

Example Item 2A.1.f (Grade 7):
Primary Target 2A (Content Domain EE), Secondary Target 1C (CCSS 7.RP.A), Tertiary Target 2D

Justin's car can travel 77.5 miles using 3.1 gallons of gas.

At this rate, how far, in miles, can Justin travel using 8.2 gallons of gas?

Enter the distance in the response box.

Rubric: (1 point) The student enters the correct distances in the response boxes (e.g., 205).

Response Type: Equation/Numeric

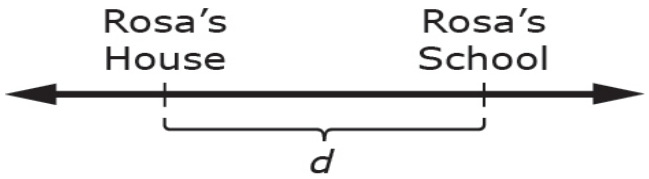
Task Model 2A.2

Expectations:

- Students solve real-world and mathematical problems involving understanding rational numbers and their operations.
- Items in this task model have a fairly straightforward connection between the context and the computation to be performed to solve the problem. They can be single step or multi-step. However, the item should not directly indicate the calculation to be performed.
- Items involving division of fractions can involve (a) division of fractions with like denominators (Example Item a) (b) division of a fraction by a whole number or a whole number by a fraction (Example Item b) (c) division of a fraction by a fraction (harder, Example Item c).
- Items involving operations with rational numbers can involve (a) operations with of integers (easier Grade 7) (b) operations with rational numbers that are not integers (harder Grade 7).

Example Item 2A.2a (Grade 6):
Primary Target 2A (Content Domain NS), Secondary Target 1B (CCSS 6.NS.A), Tertiary Target 2C
(Adapted from Illustrative Mathematics, Running to School, Variation 1)

The distance between Rosa's house and her school is $\frac{3}{4}$ mile. She ran $\frac{1}{2}$ mile.



What fraction of the distance, *d*, between her house and her school, did Rosa run?

Enter your answer in the response box.

Rubric: (1 point). The student enters the correct fraction in the response box (e.g., $\frac{2}{3}$).

Response Type: Equation/Numeric

Example Item 2A.2b (Grade 6):
Primary Target 2A (Content Domain NS), Secondary Target 1B (CCSS 6.NS.A), Tertiary Target 2C
(Adapted from Illustrative Mathematics, Making Hot Cocoa, Variation 1)

A serving of hot chocolate requires $\frac{3}{4}$ cup of milk.

How many servings can Nina make with $7\frac{1}{2}$ cups of milk?

Enter your answer in the response box.

Rubric: (1 point). The student enters the correct number of servings in the response box (e.g., 10).

Response Type: (Equation/Numeric)

Example Item 2A.2c (Grade 6):
Primary Target 2A (Content Domain NS), Secondary Target 1B (CCSS 6.NS.A), Tertiary Target 2C
(Adapted from Illustrative Mathematics, 6.NS How Many Containers in One Cup/Cups in One Container?)

It takes $\frac{1}{2}$ cup of water to fill $\frac{2}{3}$ of a plastic container.

How much water, in cups, will the full container hold?

Enter your answer in the response box.

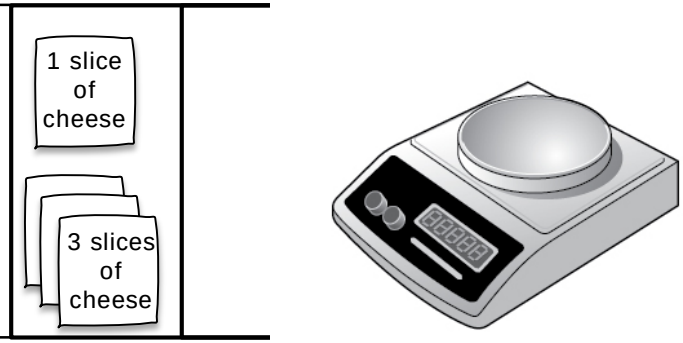
Rubric: (1 point). The student enters the correct number of cups in the response box (e.g., $\frac{3}{4}$).

Response Type: (Equation/Numeric)

Example Item 2A.2d (Grade 6)
Primary Target 2A (Content Domain NS), Secondary Target 1B (CCSS 6.NS.A), Tertiary Target 2C

Ellie ordered $\frac{3}{4}$ of a pound of cheese from the deli.

Drag the slices of cheese onto the scale so that together they weigh at least ¾ of a pound.

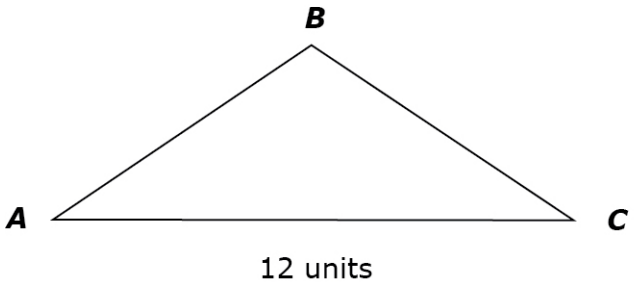


Interaction: The student drags pieces of cheese singly or in groups of three onto the scale. The weight of the cheese, to the nearest hundredth of a pound, is shown on the scale as the slices are added. Each slice is approximately 0.05 pounds, although they are not all equal.

Rubric: (1 point) The student drags the correct number of slices onto the scale (e.g., 8).

Response Type: Drag and drop

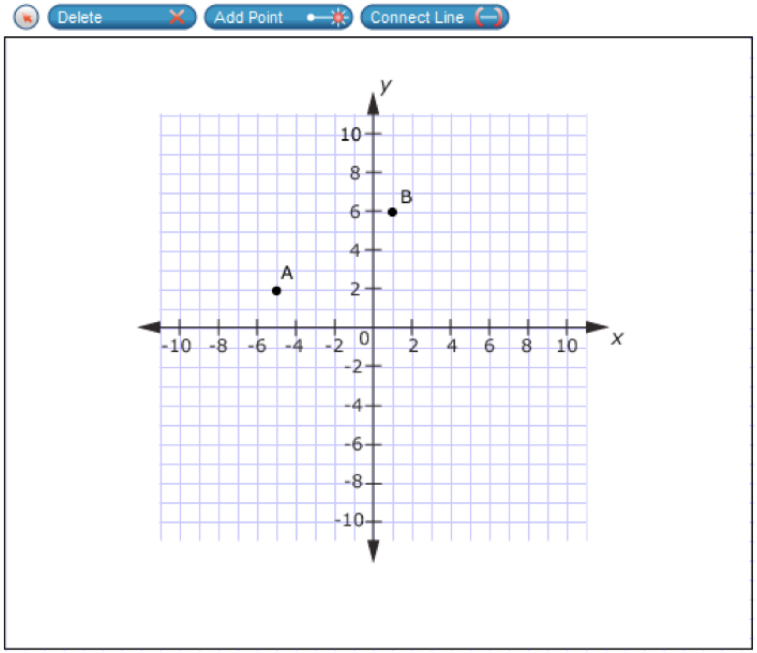
Example Item 2A.2e (Grade 7)
Primary Target 2A (Content Domain NS), Secondary Target 1D (CCSS 6.NS.C)



Complete the sketch of triangle ABC in the coordinate plane.

- Point A is plotted at (-5, 2)
- Point B is plotted at (1, 6)
- Side AC is parallel to the *x*-axis and is 12 units long

Use the Add Point and Connect Line Tool to plot C in the coordinate plane and connect the three points.



Rubric: (1 point). The student plots point C in the coordinate plane and draws the three line segments. (C is plotted at (7, 2); segments AB, AC, and BC are created)

Response Type: Graphing

Example Item 2A.2f (Grade 7):
Primary Target 2A (Content Domain NS), Secondary Target 1B (CCSS 7.NS.A), Tertiary Target 2C

The weather report predicted that the low temperature would be -8 degrees Fahrenheit. The radio announcer said,

"The low temperature was 5 degrees colder than predicted!"

What was the low temperature, in degrees Fahrenheit?

Enter your answer in the response box.

Rubric: (1 point). The student enters the correct temperature in the response box (e.g., -13).

Response Type: Equation/Numeric

Grades 6-8, Claim 2

Task Model 2A.3

Expectations:

- The student solves a real world and mathematical problems using expressions, equations, and functions (functions limited to Grade 8 problems).
- For problems involving equations in one variable, grade level may be varied by choosing equations of the form $px = q$ or $x+p = q$ (Grade 6) or equations of the form $px + q = r$ or $p(x+q) = r$ (Grade 7). (Note that there is no restriction on equation structure in Grade 8.)
- The equation should not be extractable by key words or other scaffolding.
- Items can simply ask for the equation and not its solution (Example Item 2A.3c), or they can ask for the solution as well.

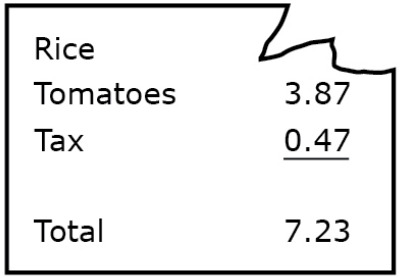
Example Item 2A.3a (Grade 6)

Primary Target 2A (Content Domain EE), Secondary Target 1F (CCSS 6.EE.B), Tertiary Target 2D

Sierra's bought a bag of rice and some tomatoes. The corner of her of her receipt got torn. The torn receipt is shown.

Write an equation that can be solved to determine the cost, x, of the bag of rice.

Enter your equation in the response box.



Rubric: (1 point) The student enters a correct equation in the response box (e.g., $x+3.87+0.47=7.23$).

Response Type: Equation/Numeric

Example Item 2A.3b (Grade 7):
Primary Target 2A (Content Domain EE), Secondary Target 1D (CCSS 7.EE.B), Tertiary Target 2D

The marching band has 85 members. There are 15 more girls than boys in the band.
How many boys are in the marching band?

Enter your answer in the response box.

Rubric: (1 point) The student enters the correct number of boys in the response box (e.g., 35).

Response Type: Equation/Numeric

Item Commentary: Notice that although the equation is simple, the item is a disguised 2-step problem, which prevents extracting the equation through simple keyword analysis. Indeed, keyword analysis might lead to the wrong equation.

Example Item 2A.3c (Grade 7):
Primary Target 2A (Content Domain RP), Secondary Target 1A (CCSS 7.RP.A), Tertiary Target 2D

The school bus driver follows the same route to pick students up in the morning and to drop them off in the afternoon. Because of traffic, the afternoon drive takes 1.5 times as long as the morning drive.

Enter an equation that represents the relationship between the number of minutes x, of the morning drive, to the **total** number of minutes, y, that the bus driver spends picking up and dropping off students each day.

Rubric: (1 point) The student enters a correct equation in the response box (e.g., $y=2.5x$).

Response Type: Equation/Numeric

Item Commentary: Notice that although the equation is simple, finding the constant of proportionality is not as straightforward as it would appear to be, which prevents extracting the equation through simple keyword analysis. Indeed, keyword analysis might lead to the wrong equation ($y=1.5x$).

Example Item 2A.3d (Grade 8):
Primary Target 2A (Content Domain F), Secondary Target 1E (CCSS 8.F.A), Tertiary Target 2D

Helga wants to have a lot of helium-filled balloons at her party.

- The helium tank costs $58 to rent.
- Balloons cost $0.29 each.
- She wants to have 5 helium-filled balloons for each party guest.

Enter an equation that represents the total cost, C, in dollars of the helium-filled balloons for n party guests.

Rubric: (1 point) The student enters a correct equation in the response box (e.g., $C=58+1.45n$).

Response Type: Equation/Numeric

Task Model 2A.4

Expectations:

- The student solves a problem related to the Pythagorean Theorem or volumes of cylinders, cones, and spheres.
- The task should require more than a routine application of the Pythagorean Theorem or a volume formula.

Example Item 2A.4a (Grade 8):
Primary Target 2A (Content Domain G), Secondary Target 1H (CCSS 8.G.B), Tertiary Target 2D

Two sides of a right triangle have lengths $\sqrt{10}$ centimeters and $\sqrt{6}$ centimeters. There are two possible lengths for the third side.

Enter the **longest** possible side length, in centimeters, for the third side of this triangle.

Rubric: (1 point) The student enters the correct length in the response box (e.g., 4).

Response Type: Equation/Numeric

Example Item 2A.4b (Grade 8):
Primary Target 2A (Content Domain G), Secondary Target 1I (CCSS 8.G.C), Tertiary Target 2D

A sphere and the base of a cone have a radius of 3 inches. The volume of the sphere equals the volume of the cone. What is the height of the cone, in inches?

Enter the height, in inches.

Rubric: (1 point) The student enters the correct radius in the response box (e.g., 12).

Response Type: Equation/Numeric

Example Item 2A.4c (Grade 8):
Primary Target 2B (Content Domain G), Secondary Target 1F (CCSS 8.G.C), Tertiary Target 2D

A right cylindrical tank has a height of 10 feet and a radius of 4 feet. Jane fills this tank with water at a rate of 8 cubic feet per minute. Using this rate, determine the number of minutes it will take Jane to completely fill the tank.

Enter your answer, rounded to the nearest minute, in the response box.

Rubric: (1 point) The student enters the correct number of minutes in the response box (e.g., 63).

Response Type: Equation/Numeric

Target 2B: Select and use appropriate tools strategically.

General Task Model Expectations for Target 2B:

- Mathematical information from the context is presented in a table, graph, or diagram, or is extracted from a verbal description or pictorial representation of the context.
- Tasks aligned to this task model focus on using tools to solve problems or making strategic choices about which tool to use or whether to use a tool to solve a problem.
- Difficulty of the task may be varied by varying (a) the difficulty of extracting information from the context, (b) the number of steps, (c) the complexity of the numbers used, or (d) the complexity of the interpretation required.
- Tasks have DOK Level 2 or 3.

Task Model 2B.1

Expectations:

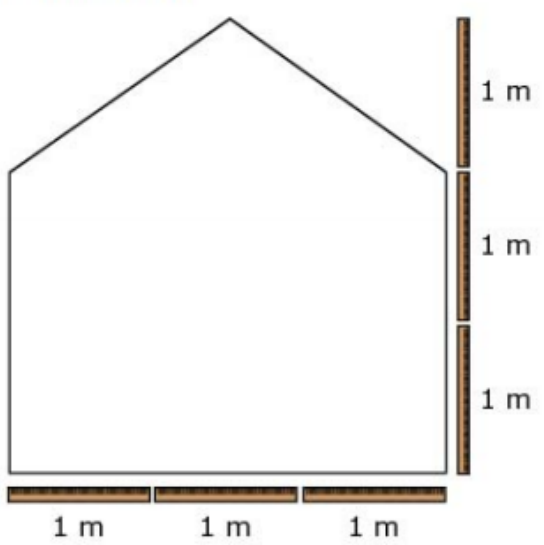
- The student uses a tool to solve a problem.
- The tool should have a mathematical purpose relevant to the solution of the problem. For example, in Example Item 2B.1a, the tool is needed to make measurements, and in Example Item 2B.1b, the tool helps the student think through the conditions.

Example Item 2B.1a (Grade 7):
Primary Target 2B (Content Domain RP), Secondary Target 1A (CCSS 7.RP.A), Tertiary Target 2D

John needs to paint one wall in his school. He knows that one can of paint covers an area of 24 square feet. John uses a meter stick to measure the dimensions of the wall, as shown.

- 1 meter is approximately 39 inches

What is the **fewest** number of cans of paint John can use to paint the wall?



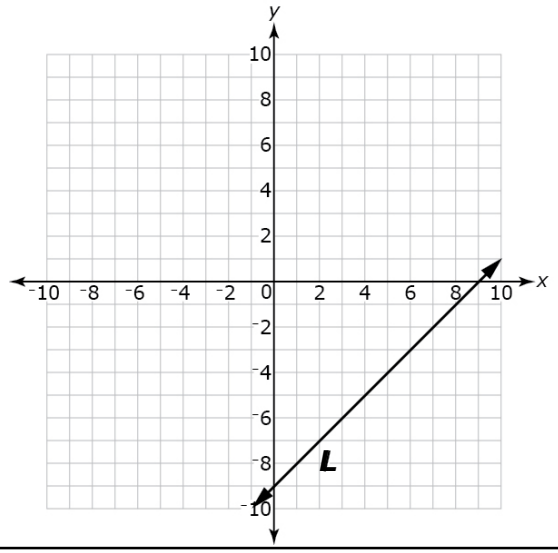
Rubric: (1 point) The student enters the correct number of cans of paint in the response box (e.g., 4).

Response Type: Equation/Numeric

Example Item 2B.1b (Grade 8):
Primary Target 2B (Content Domain EE), Secondary Target 1D (CCSS 8.EE.C)

Line ***L*** is shown on the coordinate plane. Use the Add Arrow tool to draw line ***M*** so that:

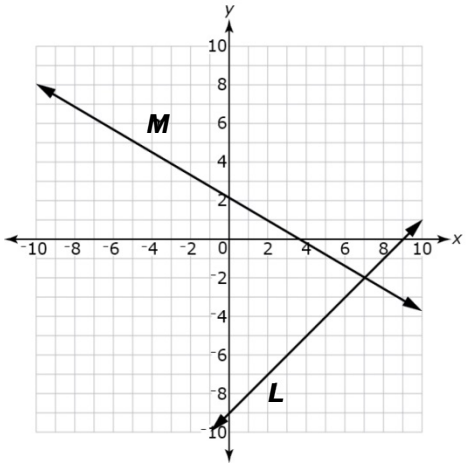
- Lines ***L*** and line ***M*** are graphs of a system of linear equations with a solution of (7, –2).
- The slope of line ***M*** is greater than –1 and less than 0.
- The *y*-intercept of line ***M*** is positive.



Interaction: The double arrow Add Arrow tool is available, as well as the Add Point tool.

Rubric: (1 point) The student draws a line that meets the requirements (e.g., see below).

Response Type: Graphing



Grades 6-8, Claim 2

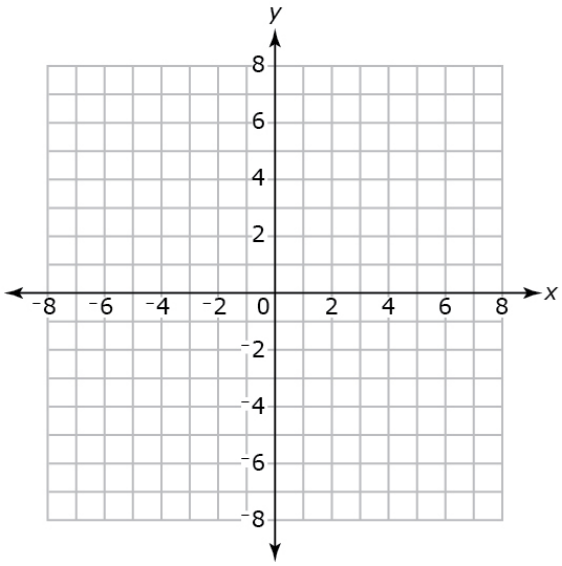
Example Item 2B.1c (Grade 8):

Primary Target 2B (Content Domain F), Secondary Target 1E (CCSS 8.F.A)

This table shows some values of a linear function.

x	*y*
–1	–5
1	–1
3	3

Use the Add Arrow tool to draw the graph of a **different** function that has the **same** rate of change as the one shown in the table of values.



Rubric: (1 point) The student draws a line with the correct slope and does not pass through the points shown in the function table (e.g., slope of 2, passes through any *y*-intercept **except** (0, –3))

Response Type: Graphing

Grades 6-8, Claim 2

Task Model 2B.2

Expectations:

- The student makes strategic choices about using tools.
- The student has access to a tool that is more appropriate for some problems than others. Students may choose to use the tool or not.
- Mathematical contexts involving computations that benefit from seeing structure or understanding numbers may be used in addition to real world contexts.
- Computations with numbers may draw on operations learned in earlier grades if the computations are particularly complex and lend themselves to making strategic choices whether or not to use a calculator.
- Dimensions along which to vary the item include (a) varying the context (b) varying the tool to be used (c) varying the complexity of the numbers to be used.

Example Item 2B.2a (Grade 6):

Primary Target 2B (Content Domain NS), Secondary Target 1C (CCSS 6.NS.B)

Perform the following calculations. You may use a calculator, but in some cases mental calculations might be faster and more reliable.

Part A:

$(1 - 1) + (2 - 2) + (3 - 3) + (4 - 4) + (5 - 5) + (6 - 6) + (7 - 7) + (8 - 8) + (9 - 9) + 10 = ?$

Enter your answer in the first response box.

Part B: $987 \times 654 = ?$

Enter your answer in the second response box.

Rubric: (1 point) The student correctly enters the correct values for both parts in the response boxes (e.g., 10; 645,498).

Response Type: Equation/Numeric (2 response boxes)

Commentary: It is more strategic to do the first problem without a calculator. Other examples of calculations that would be better done without a calculator include $(100 + 200 + 300 + 400 + 500) \div (500 + 400 + 300 + 200 + 100)$ and $(941{,}704{,}813 - 237{,}498) \times (1{,}234 - 1{,}000 - 200 - 30 - 4)$.

Example Item 2B.2b (Grade 7)

Primary Target 2B (Content Domain NS), Secondary Target 1B (CCSS 7.NS.A)

Determine whether each expression has a value that is positive, negative, or zero.

Select the correct comparison for each expression.

Expression	Positive	Zero	Negative
$\left(1\frac{2}{3}\right)+\left(-\frac{4}{3}\right)$			
$\frac{23}{56}-0.42$			
$(-0.025)\cdot\left(\frac{9}{16}\right)$			
$\left(-\frac{21}{5}\right)\div\left(-\frac{21}{5}\right)$			

Rubric: (1 point) The student selects the correct sign for each expression, as shown below.

Expression	Positive	Zero	Negative
$\left(1\frac{2}{3}\right)+\left(-\frac{4}{3}\right)$	✓		
$\frac{23}{56}-0.42$			✓
$(-0.025)\cdot\left(\frac{9}{16}\right)$			✓
$\left(-\frac{21}{5}\right)\div\left(-\frac{21}{5}\right)$	✓		

Response Type: Matching Tables

Commentary: It is more strategic to do all but the second problem without a calculator.

Grades 6-8, Claim 2

Example Item 2B.2.c

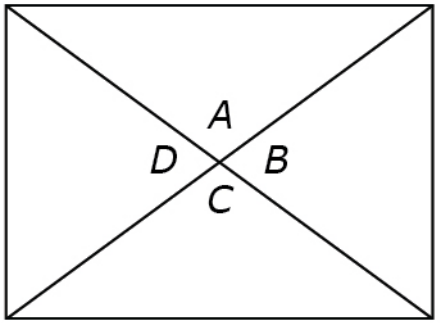
Primary Target 2B (Content Domain G), Secondary Target 1F (CCSS 7.G.B), Tertiary Target 2D

The figure shows a scale drawing of a window.

Find the measures of angles A, B, C, and D to the nearest degree.

Enter the measures in the table shown.

Angle	Measure, in degrees
A	
B	
C	
D	



Rubric: (1 point) The student enters correct angle measures in the response box within a tolerance of +/-3 degrees (e.g., 72, 108, 72, 108). Note that vertical angles should be equal and supplementary angles should sum to 180 degrees.

Response Type: Fill in Table

Commentary: The student has the choice of using a protractor and a ruler. Students will need to measure at least one angle with the protractor, but do not need the ruler at all. They could just measure one of the angles using the protractor and deduce the rest, which is more strategic, or they could measure all four angles, which is less strategic.

Target 2C: Interpret results in the context of a situation.

General Task Model Expectations for Target 2C

- The student is asked to interpret the solution of a well-posed problem arising in a context from everyday life, society, or the workplace, and then to interpret the solution in terms of the context.
- Possible interpretations include: giving the units of an answer and explaining their meaning, interpreting parts of an expression, and interpreting the solution to an equation. Problems involving interpreting data are more likely to fit into Claim 4C than Claim 2C.
- Because the focus is on interpreting the solution, items in this task model will generally have lower cognitive demand in the problem solving aspects than items in task models for 2A and 2B.
- Mathematical information from the context is presented in a table, graph, or diagram, or is extracted from a verbal description or pictorial representation of the context.
- Solving the problem requires either using units, writing an expression in an equivalent form, setting up and solving an equation or system of equations, or calculating geometric measures.
- Difficulty of the task may be varied by varying (a) the difficulty of extracting information from the context (b) the number of steps (c) the complexity of the numbers used or (d) the complexity of the interpretation required.
- Tasks have DOK Level 1 or 2.

Task Model 2C.1

Expectations:

- The student performs a calculation arising from a context and reports a number other than the direct result of the calculation because the context provides additional constraints on the allowable answers, for example.
  - choosing a value that falls into a range of acceptable values limited by information given in the context,
  - rounding up or down based on the constraints of the context.
- The student may be asked to interpret the meaning of points on the number line or in the coordinate plane in a real-world context.

Example Item 2C.1a (Grade 6):
Primary Target 2C (Content Domain RP), Secondary Target 1A (CCSS 6.RP.A), Tertiary Target 2D

A factory makes 12 bottles every 2 minutes. The factory makes bottles for 8 hours each work day. Enter a whole number to represent the **fewest** number of work days the factory will need to make 28,000 bottles.

Rubric: (1 point) The student enters the correct least number of days in the response box (e.g., 10).

Response Type: Equation/Numeric

Example Item 2C.1b (Grade 7)
Primary Target 2C (Content Domain NS), Secondary Target 1B (CCSS 7.NS.A)

This table shows the monthly change in Sara's bank account balance for each month listed. For example, the account balance change of -30 means that Sara's balance decreased by $30 from the beginning to the end of the month of February.

Month	Account Balance Change (Dollars)
January	+38
February	–30
March	–19
April	+49

Determine whether each statement about Sara's bank account balance is true or false, based on the information in the table. Select True or False for each statement.

Statement	True	False
Sara has less money in her account at the end of February than at the end of any other month.		
Sara's account balance is the same at the end of April as it is at the end of January.		
Sara has more money in her account at the end of April than she had at the **beginning** of January.		

Rubric: (1 point) The student correctly selects true or false for all three statements (e.g., FTT).

Response Type: Matching Tables

Task Model 2C.2

- The student interprets expression, equations, or graphs that represent a real-world context.
- Tasks involving expressions can involve interpreting the expression as representing a meaningful calculation arising from the context, or comparing two expressions, either equivalent or not, in terms of the calculation they represent. They can also involve interpreting constants, terms, or factors in terms of the context.
- Tasks involving solving equations in one variable can involve interpreting the solution in terms of the context.
- Tasks involving functions (Grade 8), either defined by an expression in one variable or an equation in two variables, can involve interpreting a parameter in the expression or equation; they can also involve interpreting graphical or tabular representations of the function, or making a connection between different representations.
- The wording of the problem should not reveal the answer to the interpretation step.
- Dimensions along which to vary the item include (a) varying the context (b) varying the type of expression or the type of equation to be solved (one- or two-step) (c) varying the complexity of the interpretation asked.

Example Item 2C.2a (Grade 7):
Primary Target 2C (Content Domain EE), Secondary Target 1D (CCSS 7.EE.B), Tertiary Target 2D
(Source: Adapted from *Illustrative Mathematics*, Grade 7.EE)

The students in Mr. Sanchez's class are converting distances measured in miles (m) to kilometers (km).

Abby and Renato use the following methods to convert miles to kilometers.

- Abby takes the number of miles, doubles it, and then subtracts 20% of the result.
- Renato first divides the number of miles by 5, then multiplies the result by 8.

Which equation correctly shows why both their methods produce the same result?

A. $2m - 0.20 = \frac{m}{5} \cdot 8$
B. $2m - 0.20(2m) = \frac{m}{5} \cdot 8$
C. $2m - 2.20m = \frac{m}{5} + 8\left(\frac{m}{5}\right)$
D. $0.20(2m) - 2m = \frac{m}{5} + 8\left(\frac{m}{5}\right)$

Rubric: (1 point) The student selects the correct equation (e.g., B).

Response Type: Multiple Choice, single correct response

Grades 6-8, Claim 2

Example Item 2C.2b (Grade 7):

Primary Target 2C (Content Domain EE), Secondary Target 1C (CCSS 7.EE.B), Tertiary Target 2D

A mail-order company sells jars of spices.

- An empty jar has a mass of 200 grams.
- A full jar contains 110 grams of a spice.
- The company sells n jars filled with spices.

Select the best interpretation of the expression $(200 + 110)n$.

A. The cost to ship 1 full jar
B. The cost to ship n full jars
C. The mass of 1 full jar
D. The mass of n full jars

Rubric: (1 point) The student selects the correct interpretation (e.g., D).

Response Type: Multiple Choice, single correct response

Grades 6-8, Claim 2

Example Item 2C.2c (Grade 8):

Primary Target 2C (Content Domain EE), Secondary Target 1C (CCSS 8.EE.B), Tertiary Target 2D

A comet is orbiting the sun.

The equation $d = 130{,}000t$ represents the relationship between d, the distance traveled by the comet in kilometers and t, the time, in hours, since astronomers first spotted the comet

What does the 130,000 in the equation tell us about the comet?

A. The comet will travel 130,000 kilometers in a year.
B. The comet is traveling at 130,000 kilometers per hour.
C. The comet has traveled 130,000 kilometers since astronomers spotted it.
D. The comet has been traveling for 130,000 hours since astronomers spotted it.

Rubric: (1 point) The student selects the correct interpretation (e.g., B).

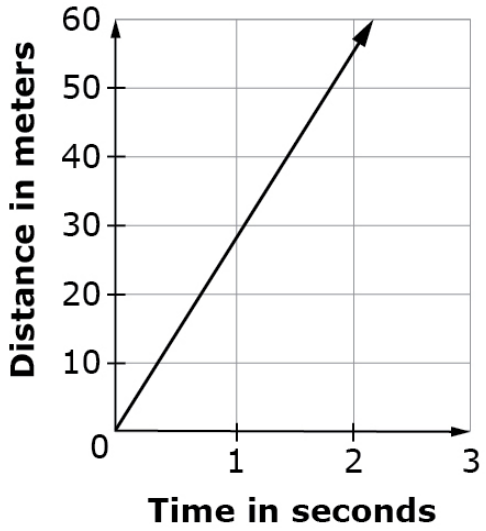
Response Type: Multiple Choice, single correct response

Commentary: In Grade 8, students should also be interpreting the x- and y-intercepts as well as the slope of linear relationships.

Example Item 2C.2d (Grade 7):
Primary Target 2A (Content Domain RP), Secondary Target 1A (CCSS 7.RP.A), Tertiary Target 2C, Quaternary Target 2D

A car is traveling on the highway. The distance, in meters, it has traveled over a two-second interval is shown in the graph. A crow can fly up to 32 meters per second. Would it be possible for a crow to pass the car?

A. Yes, it is possible for a crow to pass the car.
B. No, it is not possible for a crow to pass the car.
C. The speed of the car and the maximum speed of the crow are too close to tell.
D. There is not enough information to answer the question.



Rubric: (1 point) The student selects the correct answer choice (e.g., A).

Response Type: Multiple choice, single correct response

Target 2D: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas).

Target 2D identifies a key step in the modeling cycle, and is thus frequently present in problems with real-world contexts. Note that Target 2D is never the primary target for an item, but is frequently a Tertiary or Quaternary Target for an item with primary alignment to 2A, 2B, or 2C; see, for example, items in Task Models 2A.1, 2A.3, and 2C.2 and Example Items 2B.1a, 2B.2c, and 2C.1a.

General Task Model Expectations for Target 2D

- Students are presented with a mathematical problem in a real-world context where the quantities of interest are not named explicitly, are named but represented in different ways, or the relationship between the quantities is not immediately clear.
- The student is asked to solve a problem that may require the integration of concepts and skills from multiple domains.

Grades 6-8 Mathematics Item Specification Claim 3	
This claim refers to a recurring theme in the CCSSM content and practice standards: the ability to construct and present a clear, logical, convincing argument. For older students this may take the form of a rigorous deductive proof based on clearly stated axioms. For younger students this will involve more informal justifications. Assessment tasks that address this claim will typically present a claim or a proposed solution to a problem and will ask students to provide, for example, a justification, an explanation, or counter-example. (*Mathematics Content Specifications, p.63*) Communicating mathematical reasoning is not just a requirement of the Standards for Mathematical Practice—it is also a recurrent theme in the Standards for Mathematical Content. For example, many content standards call for students to explain, justify, or illustrate.	
Primary Claim 3: Communicating Reasoning: Students clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.	
Secondary Claim(s): Items/tasks written primarily to assess Claim 3 will necessarily involve Claim 1 content targets. Related Claim 1 targets should be listed below the Claim 3 targets in the item form. If Claim 2 or Claim 4 targets are also directly related to the item/task, list those following the Claim 1 targets in order of prominence.	
Primary Content Domain: Each item/task should be classified as having a primary, or dominant, content focus. The content should draw upon the knowledge and skills articulated in the progression of standards leading up to and including the targeted grade within and across domains.	
Secondary Content Domain(s): While tasks developed to assess Claim 3 will have a primary content focus, components of these tasks will likely produce enough evidence for other content domains that a separate listing of these content domains needs to be included where appropriate. The standards in the NS domain in grades 6-8 can be used to construct higher difficulty items for the adaptive pool. The integration of the RP, EE, F, and G domains with NS allows for higher content limits within the grade level than might be allowed when staying within the primary content domain.	
DOK Levels Target(s)	2, 3, 4
Allowable Response Types	**Response Types:** Multiple-Choice, single correct response (MC); Multiple Choice, multiple correct response (MS); Equation/Numeric (EQ); Drag and Drop, Hot Spot, and Graphing (GI); Matching Tables (MA); Fill-in Table (TI) No more than six choices in MS and MA items. Short Text - Performance tasks and Target B only **Scoring:** Scoring rules and answer choices will focus students' ability to use the appropriate reasoning. For some problems, multiple correct responses are possible. • MC will be scored as correct/incorrect (1 point) • If MS and MA items require two skills, scored as:

	○ All correct choices (2 points); at least ½ but less than all correct choices (1 point) ○ Justification[1] for more than 1 point **must be** clear in the scoring rules ○ Where possible, include a "disqualifier" option that if selected would result in a score of 0 points, whether or not the student answered ½ correctly • EQ, GI, and TI items will be scored as: ○ Single requirement items will be scored as correct/incorrect (1 point) ○ Multiple requirement items: All components correct (2 points); at least ½ but less than all correct (1 point) ○ Justification for more than 1 point **must be** clear in the scoring rules
Allowable Stimulus Materials	Effort must be made to minimize the reading load in problem situations. Use tables, diagrams with labels, and other strategies to lessen reading load. Use simple subject-verb-object (SVO) sentences; use contexts that are familiar and relevant to all or most students at the targeted grade level. Target specific stimuli will be derived from the Claim 1 targets used in the problem situation.
Construct-Relevant Vocabulary	Refer to the Claim 1 specifications to determine construct-relevant vocabulary associated with specific content standards.
Allowable Tools	Any mathematical tools appropriate to the problem situation and the Claim 1 target(s). Some tools are identified in Standard for Mathematical Practice 5 and others can be found in the language of specific standards.
Target-Specific Attributes	CAT items should take from 2 to 5 minutes to solve; Claim 3 items that are part of a performance task may take 3 to 10 minutes to solve.
Accessibility Guidance	Item writers should consider the following Language and Visual Element/Design guidelines[2] when developing items. Language Key Considerations: • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level • Avoid ambiguous or obscure words, idioms, jargon, unusual names and references Visual Elements/Design Key Considerations: • Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context • Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary

[1] For a CAT item to score multiple points, either distinct skills must be demonstrated that earn separate points or distinct levels of understanding of a complex skill must be tied directly to earning one or more points.

[2] For more information, refer to the General Accessibility Guidelines at: http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf

	• Avoid crowding of details and graphics Items are selected for a student's test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.[3]
Development Notes	• Items and task assessing Claim 3 may involve application of more than one standard. The focus is on communicating reasoning rather than demonstrating mathematical concepts or simple applications of mathematical procedures. • Targeted content standards for Claim 3 should belong to the major work of the grade (reference table of standards shown below). • Claim 1 *Specifications* that cover the following standards should be used to help inform an item writer's understanding of the difference between how these standards are measured in Claim 1 versus Claim 3. Development notes have been added to many of the Claim 1 specifications that call out specific topics that should be assessed under Claim 3. • Claim 3 items that require any degree of hand scoring must be written to primarily assess Target B. At least 80% of the items written to Claim 3 should primarily assess the standards and clusters listed in the table that follows.

Grade 6	Grade 7	Grade 8
6.RP.A	7.RP.A.2	8.EE.A.1
6.RP.A.3	7.NS.A	8.EE.B.5
6.NS.A	7.NS.A.1	8.EE.B.6
6.NS.A.1	7.NS.A.2	8.EE.C.7a
6.NS.C	7.EE.A.1	8.EE.C.7b
6.NS.C.5	7.EE.A.2	8.EE.C.8a
6.NS.C.6		8.F.A.1
6.NS.C.7		8.F.A.2
6.EE.A		8.F.A.3
6.EE.A.3		8.G.A.1
6.EE.A.4		8.G.A.2
6.EE.B		8.G.A.4
6.EE.B.6		8.G.A.5
6.EE.C.9		8.G.B.6
		8.G.B.8

[3] For more information about student accessibility resources and policies, refer to http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

Assessment Targets: Any given item/task should provide evidence for several of the following assessment targets; each of the following targets should not lead to a separate task. Multiple targets should be listed in order of prominence as related to the item/task.

Target A: Test propositions or conjectures with specific examples. (DOK 2)

Tasks used to assess this target should ask for specific examples to support or refute a proposition or conjecture (e.g., An item might begin, "Provide 3 examples to show why/how…").

Target B: Construct, autonomously[4], chains of reasoning that will justify or refute propositions or conjectures[5]. (DOK 3, 4)

Tasks used to assess this target should ask students to develop a chain of reasoning to justify or refute a conjecture. Tasks for Target B might include the types of examples called for in Target A as part of this reasoning, but should do so with a lesser degree of scaffolding than tasks that assess Target A alone. Some tasks for this target will ask students to formulate and justify a conjecture.

Target C: State logical assumptions being used. (DOK 2, 3)

Tasks used to assess this target should ask students to use stated assumptions, definitions, and previously established results in developing their reasoning. In some cases, the task may require students to provide missing information by researching or providing a reasoned estimate.

Target D: Use the technique of breaking an argument into cases. (DOK 2, 3)

Tasks used to assess this target should ask students to determine under what conditions an argument is true, to determine under what conditions an argument is not true, or both.

Target E: Distinguish correct logic or reasoning from that which is flawed and—if there is a flaw in the argument—explain what it is. (DOK 2, 3, 4)

Tasks used to assess this target present students with one or more flawed arguments and ask students to choose which (if any) is correct, explain the flaws in reasoning, and/or correct flawed reasoning.

Target F: Base arguments on concrete referents such as objects, drawings, diagrams, and actions. (DOK 2, 3)

In earlier grades, the desired student response might be in the form of concrete referents. In later grades, concrete referents

[4] By "autonomous" we mean that the student responds to a single prompt, without further guidance within the task.

[5] At the secondary level, these chains may take a successful student 10 minutes to construct and explain. Times will be somewhat shorter for younger students, but still giving them time to think and explain. For a minority of these tasks, subtasks may be constructed to facilitate entry and assess student progress towards expertise. Even for such "apprentice tasks," part of the task will involve a chain of autonomous reasoning that takes at least 5 minutes.

will often support generalizations as part of the justification rather than constituting the entire expected response.

Target G: At later grades, determine conditions under which an argument does and does not apply. (For example, area increases with perimeter for squares, but not for all plane figures.) (DOK 3, 4)

Tasks used to assess this target will ask students to determine whether a proposition or conjecture always applies, sometimes applies, or never applies and provide justification to support their conclusions. Targets A, B, C, and D will likely be included also in tasks that collect evidence for Target G.

Grade 6 standards that lend themselves to communicating reasoning	**The following standards can be effectively used in various combinations in Grade 6 Claim 3 items:** **Ratios and Proportional Relationships (RP)** **6.RP.A: Understand ratio concepts and use ratio reasoning to solve problems.** **6.RP.A.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. **The Number System (NS)** **6.NS.A: Apply and extend previous understandings of multiplication and division to divide fractions by fractions.** **6.NS.A.1** Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?* **6.NS.C: Apply and extend previous understandings of numbers to the system of rational numbers.** **6.NS.C.5** Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. **6.NS.C.6** Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. **6.NS.C.7** Understand ordering and absolute value of rational numbers. **Expressions and Equations (EE)** **6.EE.A: Apply and extend previous understandings of arithmetic to algebraic expressions.**

	6.EE.A.3 Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression 3 (2 + *x*) to produce the equivalent expression 6 + 3*x*; apply the distributive property to the expression 24*x* + 18*y* to produce the equivalent expression 6(4*x* + 3*y*); apply properties of operations to *y* + *y* + *y* to produce the equivalent expression 3*y*. **6.EE.A.4** Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions *y* + *y* + *y* and 3*y* are equivalent because they name the same number regardless of which number *y* stands for. **6.EE.B: Reason about and solve one-variable equations and inequalities.** **6.EE.B.6** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number or, depending on the purpose at hand, any number in a specified set. **6.EE.C: Represent and analyze quantitative relationships between dependent and independent variables.** **6.EE.C.9** Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation *d* = 65*t* to represent the relationship between distance and time.
Grade 7 standards that lend themselves to communicating reasoning	**The following standards can be effectively used in various combinations in Grade 7 Claim 3 items:** **Ratios and Proportional Relationships (RP)** **7.RP.A: Analyze proportional relationships and use them to solve real-world and mathematical problems.** **7.RP.A.2** Recognize and represent proportional relationships between quantities. **The Number System (NS)** **7.NS.A: Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.** **7.NS.A.1** Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

	7.NS.A.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. **Expressions and Equations (EE)** **7.EE.A: Use properties of operations to generate equivalent expressions.** **7.EE.A.1** Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. **7.EE.A.2** Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, $a + 0.05a = 1.05a$ means that increase by 5% is the same as multiply by 1.05.
Grade 8 standards that lend themselves to communicating reasoning	**The following standards can be effectively used in various combinations in Grade 8 Claim 3 items:** **Expressions and Equations (EE)** **8.EE.A: Work with radicals and integer exponents** **8.EE.A.1** Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$. **8.EE.B: Understand the connections between proportional relationships, lines, and linear equations.** **8.EE.B.5** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* **8.EE.B.6** Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b. **8.EE.C: Analyze and solve linear equations and pairs of simultaneous linear equations.** **8.EE.C.7** Solve linear equations in one variable. **a.** Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). **b.** Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting

	like terms. **8.EE.C.8a** Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. **Functions (F)** **8.F.A: Define, evaluate, and compare functions.** **8.F.A.1** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. **8.F.A.2** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions)…. **8.F.A.3** Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear…. **Geometry (G)** **8.G.A: Understand congruence and similarity using physical models, transparencies, or geometry software.** **8.G.A.1** Verify experimentally the properties of rotations, reflections, and translations: **8.G.A.2** Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. **8.G.A.4** Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. **8.G.A.5** Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles…. **8.G.B: Understand and apply the Pythagorean Theorem.** **8.G.B.6** Explain a proof of the Pythagorean Theorem and its converse. **8.G.B.8** Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Range ALDs – Claim 3 Grades 6 - 8	**Level 1** Students should be able to base arguments on concrete referents such as objects, drawings, diagrams, and actions and identify obvious flawed arguments in familiar contexts.
	Level 2 Students should be able to find and identify the flaw in an argument by using examples or particular cases. Students should be able to break a familiar argument given in a highly scaffolded situation into cases to determine when the argument does or does not hold.
	Level 3 Students should be able to use stated assumptions, definitions, and previously established results and examples to test and support their reasoning or to identify, explain, and repair the flaw in an argument. Students should be able to break an argument into cases to determine when the argument does or does not hold.
	Level 4 Students should be able to use stated assumptions, definitions, and previously established results to support their reasoning or repair and explain the flaw in an argument. They should be able to construct a chain of logic to justify or refute a proposition or conjecture and to determine the conditions under which an argument does or does not apply.

Target 3A: Test propositions or conjectures with specific examples.

General Task Model Expectations for Target 3A

- Items for this target should focus on the core mathematical work that students are doing around ratios and proportional relationships, the rational number system, and equations and expressions in grades 6-7 and equations, functions, and geometry in grade 8.
- In response to a claim or conjecture, the student should:
  - Find a counterexample if the claim is false,
  - Find examples and non-examples if the claim is sometimes true, or
  - Provide supporting examples for a claim that is always true without concluding that the examples establish that truth, unless there are only a finite number of cases and all of them are established one-by-one. The main role for using specific examples in this case is for students to develop a hypothesis that the conjecture or claim is true, setting students up for work described in Claim 3B.
- False or partially true claims that students are asked to find counterexamples for should draw upon frequently held mathematical misconceptions whenever possible.
- Note: When asking students for a single example, take care to avoid mathematical language that suggests a single example proves a conjecture.
- Tasks have DOK Level 2.

Task Model 3A.1

- The student is presented with a proposition or conjecture and asked to give
  - a counterexample if the claim is false,
  - examples and non-examples if the claim is sometimes true, or
  - one or more supporting examples for a claim that is always true without concluding that the example(s) establish that truth.

Example Item 3A.1a (Grade 6)

Primary Target 3A (Content Domain NS), Secondary Target 1D (CCSS 6.NS.C), Tertiary Target 3G

Linh said, "The opposite of 5 is −5. The opposite of $\frac{2}{3}$ is $-\frac{2}{3}$. I think the opposite of a number is always negative."

Linh's claim is **not** true. Give an example of a number whose opposite is **not** a negative number.

Enter your answer in the response box.

Rubric: (1 point) The student enters a negative number or 0 in the response box.

Response Type: Equation/Numeric

Example Item 3A.1b (Grade 7)
Primary Target 3A (Content Domain NS), Secondary Target 1B (CCSS 6.NS.A), Tertiary Target 3G

When you divide 100 by a positive whole number, the result is always less than or equal to 100. This is not always true when you divide by a positive fraction.

Give an example of a fraction $\frac{a}{b}$ where $100 \div \frac{a}{b} < 100$
Enter your fraction in the first response box.

Give an example of a fraction $\frac{c}{d}$ where $100 \div \frac{c}{d} > 100$
Enter your fraction in the second response box.

Rubric: (1 point) The student enters appropriate fractions in the response boxes ($\frac{a}{b} > 1$ and $\frac{c}{d} < 1$)

Response Type: Equation/Numeric

Task Model 3A.2

- The student is presented with one or more propositions or conjectures and several examples and asked which examples support or refute one or more of the propositions.
- Items in this task model should cover all cases and not be unintentionally misleading about the truth status of a particular proposition or conjecture.

Example Item 3A.2a (Grade 6)
Primary Target 3A (Content Domain NS), Secondary Target 1D (CCSS 6.NS.C), Tertiary Target 3G

Gina said, "For every possible value of n, we know that $|-n| = n$."

Marco said, "Sometimes $|-n| = -n$."

Who is correct?

A. Gina
B. Marco

Select **all** the values for n shown below that support the correct claim.

B. $n = 12$
C. $n = 4.5$
D. $n = \frac{1}{2}$
E. $n = -4.5$
F. $n = -100$

Rubric: (1 point) The student selects the correct student (B, Marco) and all of the correct values that support Marco's claim (E and F).

Response Type: Multiple Choice, multiple select response

Example Item 3A.2b (Grade 8)
Primary Target 3A (Content Domain NS), Secondary Target 1B (CCSS 7.NS.A), Tertiary Target 3G

Franco said that for any values a, b, and c the equation $a^2 + b^2 = c^2$ is always true. Mary disagrees.

Which of the following values for a, b, and c support Mary's claim? Select **all** that apply.

A. $a = 6, b = 8, c = 10$
B. $a = 2, b = 4, c = 6$
C. $a = b = c = 0$
D. $a = -2, b = 2, c = 0$

Rubric: (1 point) The student selects all of the correct values that support Mary's claim (B, D).

Response Type: Multiple choice, multiple select response

Target 3B: Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures.

General Task Model Expectations for Target 3B

- Items for this target should focus on the core mathematical work that students are doing around ratios and proportional relationships, the rational number system, and equations and expressions in grades 6-7 and equations, functions, and geometry in grade 8 with mathematical content from other domains playing a supporting role in setting up the reasoning contexts.
- Items for this target can probe a key mathematical structure such as that found in expressions and equations, ratios and proportional relationships, and the rational number system.
- Items for this target can require students to solve a multi-step, well-posed problem involving the application of mathematics to a real-world context. The difference between items for Claim 2A and Claim 3B is that the focus in 3B is on communicating the reasoning process in addition to getting the correct answer.
- Note that in grades 6-8, items provide less structure than items for earlier grades to focus on justifying or refuting a proposition or conjecture.
- Many machine-scorable items for these task models can be adapted to increase the autonomy of student's reasoning process but would require hand-scoring.
- Tasks have DOK Level 3, 4.

Task Model 3B.1

- The student is presented with a proposition or conjecture. The student is asked to identify or construct reasoning that justifies or refutes the proposition or conjecture.
- Items in this task model often address more generalized reasoning about a class of problems or reasoning that generalizes beyond the given problem context even when it is presented in a particular case.

Example Item 3B.1a (Grade 6)

Primary Target 3B (Content Domain NS), Secondary Target 1D (CCSS 6.NS.C, 4.G.A), Tertiary Target 3C

Lola said, "If n is a positive number, then the points $P = (n, n)$, $Q = (-n, n)$, $R = (-n, -n)$, and $S = (n, -n)$ are the vertices of a square in the coordinate plane."

Select **all** of the statements that support Lola's claim that the figure is a square.

A. The number n is a whole number.
B. The angles at P, Q, R and S, are all 90 degrees.
C. The distances between P and Q, Q and R, R and S, and S and P are n units.
D. The distances between P and Q, Q and R, R and S, and S and P are $2n$ units.

Rubric: (1 point) The student selects all of the statements that support Lola's claim (B and D).

Response Type: Multiple Choice, multiple select response

Example Item 3B.1b (Grade 8)
Primary Target 3B (Content Domain NS), Secondary Target 1B (CCSS 7.NS.A), Tertiary Target 3C

The numbers *a*, *b*, and *c* are **not** zero and $a \bullet b = c$.

Part A
Click on the equation below that **must** also be true.

A. $-a \bullet b = c$
B. $a \bullet -b = c$
C. $-a \bullet -b = c$
D. $-a \bullet -b = -c$

Part B
Choose **four** statements that support your claim.

A. $-a = (-1) \bullet a$
B. $-b = (-1) \bullet b$
C. $-c = (-1) \bullet c$
D. $(-1) \bullet (-1) = 1$
E. $(-1) \bullet (1) = -1$
F. You can multiply numbers in any order.

Rubric: (2 point) The student selects the correct equation (C) and selects four statements that support the claim (A, B, D, and F).
(1 point) The student does one or the other.

Response Type: Multiple choice, single correct response and multiple choice, multiple select response

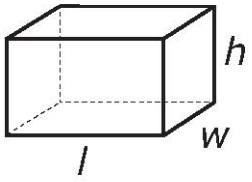
Task Model 3B.2

- The student is asked a mathematical question and is asked to identify or construct reasoning that justifies his or her answer.
- Items in this task model often address more generalized reasoning about a class of problems or reasoning that generalizes beyond the given problem context even when it is presented in a particular case.

Example Item 3B.2a (Grade 6)

Primary Target 3B (Content Domain G), Secondary Target 1H (CCSS 6.G.A), Tertiary Target 3A

A right rectangular prism has a height of 5 centimeters. Is it possible that the volume of the prism is 42 cubic centimeters?



(Not drawn to scale)

If it is possible:

Enter a possible length and width, in cm, of a prism with a height of 5 cm in two response boxes.

If it is **not** possible:

Enter a possible volume (in cubic centimeters) and the corresponding length and width (in centimeters) in the response boxes.

Rubric: (1 point) The student enters dimensions that are possible (e.g., any two numbers whose product is 8.4).

Response Type: Equation/Numeric (2 response boxes)

Commentary: This item addresses the misconception that the side-lengths of a right rectangular prism must be whole numbers or the related misconception that if the product of two numbers is a whole number then each factor must also be a whole number. Sixth grade is the year where students address the key related concepts most directly.

Example Item 3B.2b (Grade 7)
Primary Target 3B (Content Domain RP), Secondary Target 1A (CCSS 7.RP.A)

A robot moves at a constant speed. It travels *n* miles in *t* minutes. The robot's pace is the number of minutes it takes to travel one mile.

Part A

A. What is the robot's speed in miles per minute?
B. What is the robot's pace in minutes per mile?

Part B

If the robot's speed is greater than 1, then the pace is

A. Greater than 1.
B. Equal to 1.
C. Less than 1.
D. Cannot be determined.

Explain your reasoning.

Rubric: (2 points) The student enters the correct speed (n/t) in the first response box and the correct pace (t/n) in the second response box and selects the correct statement about the pace (C) and enters a correct explanation (see Examples below).
(1 point) The student gets Part A right or Part B right, but not both.

Example 1
If the speed a/b is greater than 1, then the pace b/a must be less than one. The speed and the pace are reciprocals. If a number is greater than 1, then its reciprocal is less than one and vice-versa.

Example 2
The speed is greater than 1, so a/b>1. If we multiply both sides by b we get a>b. If we divide both sides by a, we get 1>b/a, which is the pace. So the pace is less than 1.

Response Type: Equation/numeric, multiple choice single correct response, hand-scored text box.

Note: Functionality for this item type does not currently exist, but the item could be implemented with a single text box.

Example Item 3B.2c (Grade 8)
Primary Target 3B (Content Domain EE), Secondary Target 1D (CCSS 8.EE.C), Tertiary Target 3F, Quaternary Target 3G

Part A
Is it possible for three linear equations in *x* and *y* to have a solution common to all three? [drop-down choices: yes, no]

Part B
[If "yes" is selected] Use the Arrow tool to draw the graphs of three equations that have a common solution. Add a point that represents the common solution.

[If "no" is selected] Explain why this is not possible in the response box.

Interaction: The student has to select yes or no before seeing Part B. If the student selects "yes" then he/she sees the graphing tools and is asked to graph the system. If he/she selects "no" there is a text box that asks for an explanation as to it is not possible. The student can change his/her mind.

Rubric: (1 point) The student selects "yes" and draws three lines that intersect in a single point and places a point at the intersection of the three lines (it is allowable for the lines to coincide, but they have to draw three graphs).

Response Type: Drop-Down Menu[6] and Graphing/Short-Text

Note: Functionality for this item type does not currently exist but it could be implemented by showing Parts A and B simultaneously. When possible, the point of having a student try to explain his or her incorrect reasoning is that in the process of trying to construct an argument, he or she may self-correct.

Task Model 3B.3

- Items for this target require students to solve a multi-step, well-posed problem involving the application of mathematics to a real-world context.
- The difference between Claim 2 task models and this task model is that the student needs to provide some evidence of his/her reasoning. The difference between Claim 4 task models and this task model is that the problem is completely well posed and no extraneous information is given.

[6] Drop-Down Menu response type is not yet available in the Smarter Balanced item authoring tool, but it is a scheduled enhancement by 2017.

Example Item 3B.3a (Grade 6)
Primary Target 3B (Content Domain RP), Secondary Target 1A (CCSS 6.RP.A), Tertiary Target 3C

Clark biked 4 miles in 20 minutes. How far can he go in 2 hours if he bikes at this rate?

Enter your answer in the first response box.

Show how you would solve this problem with a table or an equation (choose one option).

Option 1: Table

Enter values in the table so that it shows the number of miles, *m*, Clark can bike in 2 hours at this rate.

Miles (*m*)							
Minutes							
Hours							

Option 2: Equation

Enter an equation that can be solved to find the number of miles, *m*, Clark can bike in 2 hours at this rate in the second response box.

Rubric: (2 points) The student enters the correct number of miles (24) and fills in the table with at least two columns, one of which contains the correct answer, or enters an equation that can be solved to find the answer (see examples below of each). (1 point) The student does one of these parts correctly.

Example for Option 1

Miles	4	8	12	16	20	24	
Minutes	20	40	60	80	100	120	
Hours	1/3	2/3	1	4/3	5/3	2	

Example for Option 2

$2 \cdot 3 \cdot 4 = m$ *or* $4/20 = m/120$ or equivalent equation.

Response Type: Equation/Numeric and Fill-in Table

Note: The functionality for this kind of combination of item types does not currently exist, but is a scheduled enhancement for 2017.

Example Item 3B.3b (Grade 7)
Primary Target 3B (Content Domain EE), Secondary Target 1D (7.EE.B), Tertiary Target 3C

In February, the price of a gallon of gasoline increased by 23% from the price in January. In March, the price decreased by 11% from the price in February. In March, gas cost $2.63 per gallon.

How much did a gallon of gasoline cost in January, in dollars? Round your answer to the nearest cent. Enter your answer in the response box.

Which equation shown can be solved to find x, the cost of gas in January?

A. $(0.11)(0.23)x = 2.63$
B. $(1.11)(1.23)x = 2.63$
C. $(0.89)(1.23)x = 2.63$
D. $(1.11)(0.77)x = 2.63$

Rubric: (2 points) The student enters the correct cost of a gallon of gas (2.40) and selects the correct equation (C).
(1 point) The student does one of these parts correctly.

Response Type: Equation/numeric and multiple choice, single correct response

Note: Current functionality doesn't allow for mixing equation/numeric and multiple choice, so in the meantime the first part could be made multiple choice.

Example Item 3B.3c (Grade 8)
Primary Target 3B (Content Domain RP), Secondary Target 1A (CCSS 7.RP.A), Tertiary Target 4F

A car is traveling at a constant speed and drove 75 miles in 1.5 hours. One mile is approximately 1.6 kilometers. Approximately how fast is the car traveling in kilometers per hour?

Explain or show clear steps for how you determined your answer.

Rubric: (2 points) The student includes the correct numeric value in the response (80) and provides a coherent, complete explanation or sequence of computations that shows where this comes from (see Examples).
(1 point) The student enters the correct numeric value but does not provide a coherent explanation OR the student provides an incorrect speed and includes an explanation that shows an understanding of how the answer could be found, but with some computational errors or a small misstep in reasoning.

Example 1

Going 75 miles in 1.5 hours is the same as going 50 miles per hour.
50 miles is 50*1.6=80 km.
A car driving 50 miles per hour is driving 80 kilometers per hour.

Example 2

75 miles in 1.5 hours is 75/1.5 = 50 mi/hr.
50 mi/hr * 1.6 km/mi =80 km/hr.
The car is traveling at 80 kilometers per hour.

Response Type: Short Text (handscored)

Target 3C: State logical assumptions being used.

General Task Model Expectations for Target 3C

- Items for this target should focus on the core mathematical work that students are doing around ratios and proportional relationships, the rational number system, and equations and expressions in grades 6-7 and equations, functions, and geometry in grade 8.
- For some items, the student must explicitly identify assumptions that
    - Make a problem well-posed, or
    - Make a particular solution method viable.
- When possible, items in this target should focus on assumptions that are commonly made implicitly and can cause confusion when left implicit.
- For some items, the student will be given a definition and be asked to reason from that definition.
- Tasks are DOK Level 2, 3.

Task Model 3C.1

- The student is asked to identify an unstated assumption that would make the problem well-posed or allow them to solve a problem using a given method.

Example Item 3C.1a (Grade 6)

Primary Target 3C (Content Domain RP), Secondary Target 1A (CCSS 6.RP.A), Tertiary Target 3G

Lyla flew her radio-controlled airplane 500 feet in 20 seconds. She claims that the speed of her airplane was 25 feet per second during the flight. What assumption must Lyla make for her claim to be true?

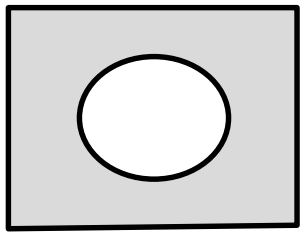
A. The airplane flew in a circle.
B. The airplane flew in a straight line.
C. The airplane flew at a constant speed.
D. The airplane flew faster at the end of the flight than at the beginning.

Rubric: (1 point) The student selects the correct statement (C).

Response Type: Multiple Choice, single correct response

Example Item 3C.1b (Grade 7)
Primary Target 3C (Content Domain G), Secondary Target 1F (CCSS 7.G.B), Tertiary Target 3G

Glenn saw the figure below and said,
"If I find the length (*l*), width (*w*), and radius (*r*), then the area (*A*) of the shaded region is $A = l \cdot w - \pi r^2$."



Which assumptions must Glenn be making in order for his equation to give the correct area of the shaded region? Select **all** that apply.

A. The quadrilateral is a rhombus.
B. The quadrilateral is a rectangle.
C. The curved figure in the center is a circle.
D. The curved figure in the center is a sphere.

Rubric: (1 point) The student selects the correct assumptions (B and C).

Response Type: Multiple Choice, single correct response

Task Model 3C.2

- The student will be given one or more definitions or assumptions and be asked to reason from that set of definitions and assumptions.

Example Item 3C.2a (Grade 7)
Primary Target 3C (Content Domain NS), Secondary Target 1B (CCSS 7.NS.A), Tertiary Target 3C

A **perfect square** is a number s that is the product of an integer, n, and itself, so that $s = n^2$.

Examples of perfect squares include 25 because it is equal to 5^2 and 81 because it is equal 9^2.

Can a perfect square be negative?

A. Yes; an example is –25.
B. No; a square of any integer is always positive.
C. Sometimes Yes, sometimes No; it depends on the value of n.
D. There is not enough information to tell.

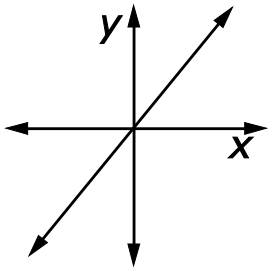
Rubric: (1 point) The student selects the correct statement (B).

Response Type: Multiple Choice, single correct response

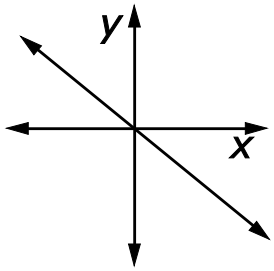
Example Item 3C.2b (Grade 8)
Primary Target 3C (Content Domain EE), Secondary Target 1C (CCSS 8.EE.B), Tertiary Target 3F

A proportional relationship between x and y is one that can be represented by the equation $y = k \cdot x$, where k is a positive number.

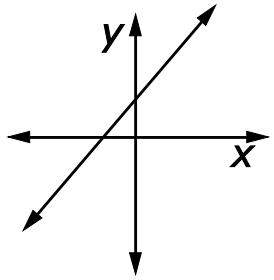
Consider these four graphs.



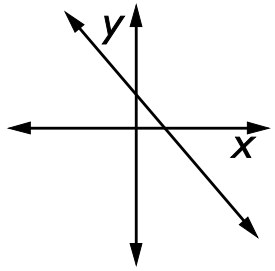
Graph A



Graph B



Graph C



Graph D

Based on this definition, identify whether or not each graph could represent a proportional relationship. Answer "Yes" if it does represent a proportional relationship and "No" if it does not.

	Yes	No
Graph A		
Graph B		
Graph C		
Graph D		

Rubric: (1 point) The student identifies the correct graphs (YNNN).

Response Type: Matching Table

Target 3D: Use the technique of breaking an argument into cases.

General Task Model Expectations for Target 3D

- Items for this target should focus on the core mathematical work that students are doing around ratios and proportional relationships, the rational number system, and equations and expressions in grades 6-7 and equations, functions, and geometry in grade 8.
- The student is given
  - a problem that has a finite number of possible solutions, some of which work and some of which don't, or
  - a proposition that is true in some cases but not others.
- Items for Claim 3 Target D should either present an exhaustive set of cases to consider or expect students to consider all possible cases in turn in order to distinguish it from items in other targets.
- Items have DOK Level 2, 3.

Task Model 3D.1

- The student is given a problem that has a finite number of possible solutions, some of which work and some of which don't.

Example Item 3D.1a (Grade 7)

Primary Target 3D (Content Domain RP), Second Target 1A (CCSS 7.RP.A), Tertiary Target 3G

Green paint can be made by mixing yellow paint with blue paint. Two mixtures make the same shade of green if the ratio of yellow to blue is the same. Assume n is a positive number.

Identify **one or more** of the mixtures below that will make the same shade of paint as a mixture of 10 liters of yellow paint and 15 liters of blue paint. Answer "Yes" if it will make the same shade of paint, answer "No" if it will not.

	Liters of Yellow Paint	Liters of Blue Paint	Yes	No
Mixture 1	$2n$	3		
Mixture 2	2	$3n$		
Mixture 3	$2n$	$3n$		

Rubric: (1 point) The student identifies the correct mixture (NNY).

Response Type: Matching Table

Note: A drag-and-drop version of this could allow students to determine the equivalent mixtures themselves.

Example Item 3D.1b (Grade 8)
Primary Target 3D (Content Domain G), Secondary Target 1G (CCSS 8.G.A), Tertiary Target 3G

Select **all** of the following situations that show that Figure *P* is congruent to Figure *Q*.

A. There is a translation that takes Figure *P* to Figure *Q*.
B. There is a rotation that takes Figure *P* to Figure *Q*.
C. There is a reflection that takes Figure *P* to Figure *Q*.
D. There is a dilation that takes Figure *P* to Figure *Q*.

Rubric: (1 point) The student selects the correct transformations (A, B, and C).

Response Type: Multiple choice, multiple selection response

Task Model 3D.2

- The student is given a proposition and asked to determine in which cases the proposition is true.

Example Item 3D.2a (Grade 7)
Primary Target 3D (Content Domain NS), Secondary Target 1B (CCSS 7.NS.A), Tertiary Target 3C

Given x and y are rational numbers, when is $|x + y| = |x| + |y|$ true?

A. This is never true.
B. This is always true.
C. This is true when x and y have opposite signs.
D. This is true when x and y have the same sign.

Rubric: (1 point) The student selects the correct statement (D).

Response Type: Multiple Choice, single correct response

Example Item 3D.2b (Grade 8)
Primary Target 3D (Content Domain EE), Secondary Target 1B (CCSS 8.EE.A), Tertiary Target 3C

Maggie claims that when you raise a whole number to a power, the result is always a greater number. That is, $s^n > s$. For example:

$4^3 > 4$
$5^4 > 5$
$10^9 > 10$

Maggie's claim is **not** true for all values of *n* and *s*. For what values of *n* and *s* is Maggie's claim true? Complete the inequalities.

s > [ ]

n > [ ]

Rubric: (1 point) The student enters the correct values in the response boxes (1 and 1).

Response Type: Equation/Numeric (two response boxes, label the boxes with *s* > and *n* >, respectively.)

Target 3E: Distinguish correct reasoning from flawed reasoning

General Task Model Expectations for Target 3E

- Items for this target should focus on the core mathematical work that students are doing around ratios and proportional relationships, the rational number system, and equations and expressions in grades 6-7 and equations, functions, and geometry in grade 8.
- The student is presented with valid or invalid reasoning and told it is flawed or asked to determine its validity. If the reasoning is flawed, the student identifies, explains, and/or corrects the error or flaw.
- The error should be more than just a computational error or an error in counting, and should reflect an actual error in reasoning.
- Analyzing faulty algorithms is acceptable so long as the algorithm is internally consistent and it isn't just a mechanical mistake executing a standard algorithm.
- Items have DOK Level 2, 3, 4.

Task Model 3E.1

- Some flawed reasoning or student work is presented and the student identifies and/or corrects the error or flaw.
- The student is presented with valid or invalid reasoning and asked to determine its validity. If the reasoning is flawed, the student will explain or correct the flaw.

Example Item 3E.1a (Grade 6)
Primary Target 3E (Content Domain EE), Secondary Target 1F (CCSS 6.EE.B), Tertiary Target 3C

Emma was solving the equation $t - 4 = 16$. She said, "I'm looking for a number t that is 4 less than 16. So $t = 12$."

Which statement best describes the flaw in Emma's reasoning?

A. Emma's answer is right but she should just subtract 4 from both sides of the equation.
B. Emma's answer is wrong but she thought about the equation correctly.
C. Emma is confused about which number the 4 is being subtracted from.
D. Emma should subtract the 16 from the 4 instead of 4 from the 16.

Rubric: (1 point) The student selects the correct analysis of the flaw in reasoning (C).

Response Type: Multiple choice, single correct response

Example Item 3E.1b (Grade 7)
Primary Target 3E (Content Domain RP), Secondary Target 1A (CCSS 7.RP.A), Tertiary Target 3C

Dena is trying to solve this problem:

> A store has a sale where every item has a sale price that is 20% less than the regular price. Write an expression that represents the sale price of an item if the regular price is p dollars.

Dena said, "To find 20% of a number, I should multiply by 0.20. So the sale price of an item will be $0.20p$."

Which statement best describes Dena's reasoning?

A. Dena is correct.
B. Dena needs to subtract 0.20p from the regular price, p.
C. Dena should calculate the sale price as $20p$ and then divide by 100.
D. Dena is trying to solve an impossible problem because it doesn't say what the regular price is.

Rubric: (1 point) The student selects the statement that represents correct reasoning (B).

Response Type: Multiple choice, single correct response

Task Model 3E.2

- Two or more approaches or chains of reasoning are given and the student is asked to identify the correct method and justification OR identify the incorrect method/reasoning and the justification.

Example Item 3E.2a (Grade 7)

Primary Target 3E (Content Domain NS), Secondary Target 1B (CCSS 6.NS.A), Tertiary Target 3C

Clyde and Lily were solve the equation $\frac{8}{9} \div \frac{1}{2} = x$.

Clyde said, "I can think of this division problem as a multiplication problem." Then he wrote:		Lily said, "You need to invert and multiply." Then she wrote:	
Step 1.	$\frac{8}{9} \div \frac{1}{2} = x$	Step 1.	$\frac{8}{9} \div \frac{1}{2} = x$
Step 2.	$\frac{1}{2}x = \frac{8}{9}$	Step 2.	$\frac{8}{9} = 2 \bullet x$
Step 3.	$2\left(\frac{1}{2}x\right) = 2\left(\frac{8}{9}\right)$	Step 3.	$\frac{1}{2}(2x) = \left(\frac{1}{2}\right) \bullet \left(\frac{8}{9}\right)$
Step 4.	$x = \frac{16}{9}$	Step 4.	$x = \frac{8}{18}$

Who solved the problem correctly?

A. Only Clyde solved the equation correctly.
B. Only Lily solved the equation correctly.
C. They both solved the equation correctly.
D. Neither one solved the equation correctly.

Rubric: (1 point) The student selects the correct characterization of these two approaches (A).

Response Type: Multiple choice, single correct response

Example Item 3E.2b (Grade 8)
Primary Target 3E (Content Domain EE), Secondary Target 1D (CCSS 8.EE.C), Tertiary Target 3C, Quaternary Target 3F

The students in Mr. Martin's class are learning about linear equations. Kenny made a claim and two supporting claims about the possible number of solutions to a system of linear equations. Rhonda made a different claim with two supporting claims.

Indicate whether each claim is valid or not valid.

Kenny's Claims	Valid	Not Valid
Claim 1. A system of two linear equations can only have zero solutions or one solution.		
Claim 1a. If the corresponding lines are distinct and parallel, then there are no solutions.		
Claim 1b. If the corresponding lines are distinct and intersect, then there is one solution.		

Rhonda's Claims	Valid	Not Valid
Claim 2. A system of two linear equations can have more than one solution.		
Claim 2a. If the corresponding lines intersect in exactly two places, then there will be exactly two solutions.		
Claim 2b. If the corresponding lines completely coincide, then there are an infinite number of solutions.		

Rubric: (1 point) The student selects the correct claims (NVV, VNV).

Response Type: Matching Table

Target 3F: Base arguments on concrete referents such as objects, drawings, diagrams, and actions

Task Model 3F.1

- The student uses concrete referents to help justify or refute an argument.
- In grade 6, items in this task model should focus on the use of number lines. In grade 7, they should focus on number lines and graphs of proportional relationships. In grade 8, they should focus on graphs of linear equations and systems of linear equations and geometric contexts related to transformations of the plane or the Pythagorean Theorem.
- Items have DOK Level 2, 3.

Example Item 3F.1a (Grade 7)
Primary Target 3F (Content Domain NS), Secondary Target 1D (CCSS 6.NS.C), Tertiary Target 3D

P and T are numbers and $P + T = 0$.

Select **all** of the statements about P and Q that could be true.

A. $P = 0$ and $T = 0$

B. $P = 0$ or $T = 0$, but not both.

C. P can be any positive number and T can be any negative number.

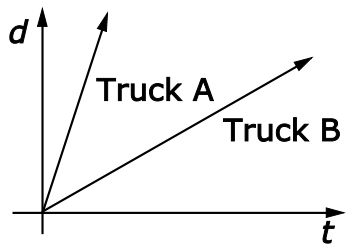
D. P and T are on opposite sides of zero and equally distant from zero on the number line.

Rubric: (1 point) The student selects the correct statements (A, D).

Response Type: Multiple Choice, multiple correct response

Example Item 3F.1b (Grade 7)
Primary Target 3F (Content Domain NS), Secondary Target 1A (CCSS 7.RP.A), Tertiary Target 3D

Two trucks are traveling on a highway at a constant speed. The graphs of their distances, *d*, over time, *t*, are shown.



Which truck is traveling faster, and how do you know?

Truck [drop-down menu choices: A, B] is traveling faster because the graph is [drop-down menu choices: steeper, less steep, longer, shorter].

Rubric: (1 point) The student chooses the correct truck (A) and the correct reason (steeper).

Response Type: Drop-down menu

Note: Functionality for this item type does not currently exist, but could be implemented with two-part multiple choice.

Example Item 3F.1c (Grade 8)
Primary Target 3F (Content Domain G), Secondary Target 1H (CCSS 8.G.B), Tertiary Target 3B

The Pythagorean Theorem states that if a right triangle has legs of length a and b and hypotenuse of length c, then $a^2 + b^2 = c^2$.

Figures 1 and 2 represent the key ideas in a proof of the Pythagorean Theorem.

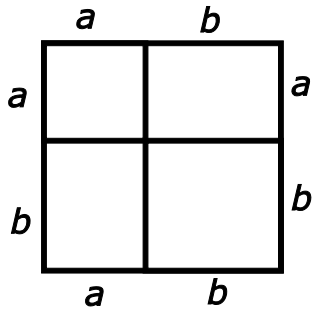


Figure 1

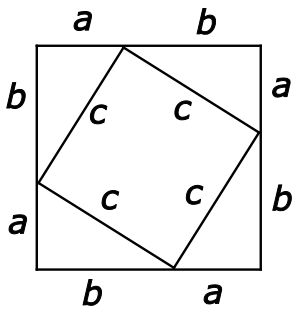


Figure 2

Create an outline a proof for the Pythagorean Theorem based on Figures 1 and 2, by dragging the seven statements shown into a logical sequence.

A right triangle has legs of length a and b and hypotenuse of length c.

1.

2.

3.

4.

5.

6.

7.

Thus, $a^2 + b^2 = c^2$

Subdivide the large square in Figure 1 into a square with side-length a, a square with side-length b, and two rectangles with side-lengths a and b.

Subdivide the large square in Figure 2 into four right triangles with legs a and b and a square with side-length c.

The total area of the large square in Figure 1 is $a^2 + b^2 + ab + ab$.

The total area of the large square in Figure 2 is $c^2 + 4(\frac{1}{2}ab)$.

Start with two large squares with sides of length $a + b$.

$a^2 + b^2 + ab + ab = c^2 + 4(\frac{1}{2}ab)$

The two large squares have the same area because they are congruent.

Rubric: (2 points) The student drags the steps of the proof into a logical order. Note that 1 must be first and 7 must be last and 2 must precede 5 and 3 must precede 6, but any other permutations are allowed as long as they are consistent with these constraints).
(1 point) The student gets the steps in an order consistent with the constraints described above, but has at most one step out of order.

Exemplar (more solutions are possible as noted above)

1. Start with two large squares with sides of length $a + b$.
2. Subdivide the large square in Figure 1 into a square with side-length a, a square with side length b, and two rectangles with side-lengths a and b.
3. Subdivide the large square in Figure 2 into four right triangles with legs a and b and a square with side-length c.
4. The two large squares have the same area because they are congruent.
5. The total area of the large square in Figure 1 is $a^2 + b^2 + ab + ab$.
6. The total area of the large square in Figure 2 is $c^2 + 4(\frac{1}{2}ab)$.
7. $a^2 + b^2 + ab + ab = c^2 + 4(\frac{1}{2}ab)$.

Response Type: Drag and Drop

Target 3G: Determine conditions under which an argument does and does not apply

Target 3G is a closely related extension of the expectations in Targets 3A, 3B, 3C, and 3D, and as with those targets, is often a tertiary alignment for items in those targets. Students often test propositions and conjectures with specific examples (as described in Target 3A) for the purpose of formulating conjectures about the conditions under which an argument does and does not apply. Students then must explicitly describe those conditions (as in Target 3C). Expectations for Target 3D include determining conditions under which an argument is true given cases—the next step is articulating those cases autonomously (Target 3B).

Grades 6-8 Mathematics Item Specification Claim 4	
"Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decision-making." (p.72, CCSSM)	
Primary Claim 4: Modeling and Data Analysis Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.	
Secondary Claim(s): Items/tasks written primarily to assess Claim 4 will necessarily involve some Claim 1 content targets. Related Claim 1 targets should be listed below the Claim 4 targets in the item form. If Claim 2 or Claim 3 targets are also directly related to the item/task, list those following the Claim 1 targets in order of prominence.	
Primary Content Domain: Each item/task should be classified as having a primary, or dominant, content focus. The content should draw upon the knowledge and skills articulated in the progression of standards leading up to and including the targeted grade with strong emphasis on the major work of previous grades.	
Secondary Content Domain(s): While items/tasks developed to assess Claim 4 will have a primary content focus, components of these tasks will likely produce enough evidence for other content domains that a separate listing of these content domains needs to be included where appropriate. The standards in the NS domain in grades 6-8 can be used to construct higher difficulty items for the adaptive pool. The integration of the RP, EE, SP, F, and G domains with NS allows for higher content limits within the grade level than might be allowed when staying within the primary content domain.	
DOK Levels	1, 2, 3, 4
Allowable Response Types	**Response Types:** Multiple Choice, single correct response (MC); Multiple Choice, multiple correct response (MS); Equation/Numeric (EQ); Drag and Drop, Hot Spot, and Graphing (GI); Matching Tables (MA), Fill-in Table (TI) No more than six choices in MS and MA items. Short Text –CAT items for Targets B, E and Performance Tasks **Scoring:** Scoring rules and answer choices will focus students' ability to use the appropriate reasoning. For some problems, multiple correct responses and/or strategies are possible. • MC and MS items will be scored as correct/incorrect (1 point). • If MA items require two skills, they will be scored as:

	○ All correct choices (2 points); at least but less than all correct choices (1 point) ○ Justification[1] for more than 1 point **must be** clear in the scoring rules ○ Where possible, include a "disqualifier" option that if selected would result in a score of 0 points, whether or not the student answered correctly. • EQ, GI, and TI items will be scored as: ○ Single requirement items: will be scored as correct/incorrect (1 point) ○ Multiple requirement items: All components correct (2 points); at least ½ but less than all correct (1 point) ○ Justification for more than 1 point **must be** clear in the scoring rules
Allowable Stimulus Materials	Effort must be made to minimize the reading load in problem situations. Use tables, diagrams with labels, and other strategies to lessen reading load. Use simple subject-verb-object (SVO) sentences; use contexts that are familiar and relevant to students at the targeted grade level. Target-specific stimuli will be derived from the Claim 1 targets used in the problem situation. All real-world problem contexts will be relevant to the age of the students. Stimulus guidelines specific to task models are given below.
Construct-Relevant Vocabulary	Refer to the Claim 1 specifications to determine construct-relevant vocabulary associated with specific content standards.
Allowable Tools	Any mathematical tools appropriate to the problem situation and the Claim 1 target(s). Some tools are identified in Standard for Mathematical Practice 5 and others can be found in the language of specific standards.
Target-Specific Attributes:	CAT Items should take from 3 to 8 minutes to solve; Claim 4 items that are part of a performance task may take 5 to 15 minutes.
Accessibility Guidance	Item writers should consider the following Language and Visual Element/Design guidelines[2] when developing items. Language Key Considerations: • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level • Avoid ambiguous or obscure words, idioms, jargon, unusual names and references Visual Elements/Design Key Considerations: • Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context

[1] For a CAT item to score multiple points, either distinct skills must be demonstrated that earn separate points or distinct levels of understanding of a complex skill must be tied directly to earning one or more points.

[2] For more information, refer to the General Accessibility Guidelines at: http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf

	• Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary • Avoid crowding of details and graphics Items are selected for a student's test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.[3]
Development Notes	CAT items/tasks generating evidence for Claim 4 in a given grade will draw upon knowledge and skills articulated in the progression of standards up through that grade, though more complex problem-solving tasks may draw upon knowledge and skills from lower grade levels. Claim 1 *Specifications* that cover the following standards should be used to help inform an item writer's understanding of the difference between how these standards are measured in Claim 1 versus Claim 4. Development notes have been added to many of the Claim 1 specifications that call out specific topics that should be assessed under Claim 4. Distinguishing between Claim 4 and Claims 1 and 2: • In early grades when equations are still new to students, an important distinction between Claim 2 and Claim 4 is requiring a model that would lead to a problem's solution. • In Claim 2 problems are well posed, while in Claim 4 they may have extraneous or missing information. • In Claims 1 and 2, measurements of objects or figures can be accurately determined. In Claim 4, modeling is used to make approximations. • In Claim 1, data analysis is straightforward procedural. In Claim 4, the analysis should be tied to some useful purpose in the real-world. At least 80% of the items written to Claim 4 should primarily assess the standards and clusters listed in the table that follows.

[3] For more information about student accessibility resources and policies, refer to http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

Grade 6	Grade 7	Grade 8
6.RP.A	7.RP.A	8.EE.A.3
6.NS.A	7.NS.A	8.EE.A.4
6.NS.C	7.EE.B	8.EE.B
6.EE.B	7.G.A*	8.EE.C
6.EE.C	7.G.B*	8.F.B*
6.G.A*	7.SP.A*	8.G.B
6.SP.A*	7.SP.B*	8.G.C*
6.SP.B*	7.SP.C*	8.SP.A*

* Denotes additional and supporting clusters

REMINDER: Claim 4 tasks may also ask students to apply content from prior grades in sophisticated applications.

Assessment Targets: Any given item/task should provide evidence for two or more Claim 4 assessment targets. Each of the following targets should not lead to a separate task. It is in *using* content from different areas, including work studied in earlier grades, that students demonstrate their problem-solving proficiency. Multiple targets should be listed in order of prominence as related to the item/task.

Target A: Apply mathematics to solve problems arising in everyday life, society, and the workplace. (DOK 2, 3)
Problems used to assess this target for Claim 4 should not be completely formulated (as they are for the same target in Claim 2), and require students to extract relevant information from within the problem and find missing information through research or the use of reasoned estimates.

Target B: Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem. (DOK 2, 3, 4)
Items that require the student to make decisions about the solution path needed to solve a problem are aligned with this target. Target B is not intended to be the primary target for an item, but should be a secondary, tertiary, or quaternary target for an item with primary alignment to other targets.

Target C: State logical assumptions being used. (DOK 1, 2)
Tasks used to assess this target ask students to use stated assumptions, definitions, and previously established results in developing their reasoning. In some cases, the task may require students to provide missing information by researching or providing a reasoned estimate.

Target D: Interpret results in the context of a situation. (DOK 2, 3)
Tasks used to assess this target should ask students to link their answer(s) back to the problem's context (See Claim 2, Target C, for further explication.)

Target E: Analyze the adequacy of and make improvements to an existing model or develop a mathematical model of a real phenomenon. (DOK 3, 4)
Tasks used to assess this target ask students to investigate the efficacy of existing models (e.g., develop a way to analyze the claim that a child's height at age 2 doubled equals his/her adult height) and suggest improvements using their own or provided data.

Other tasks for this target will ask students to develop a model for a particular phenomenon (e.g., analyze the rate of global ice melt over the past several decades and predict what this rate might be in the future).
Longer constructed-response items and extended performance tasks should be used to assess this target.

Target F: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas). (DOK 1, 2, 3)

Unlike Claim 2, where this target might appear as a separate target of assessment (see Claim 2, Target D), it will be embedded in a larger context for items/tasks in Claim 4. The mapping of relationships should be part of the problem posing and solving related to Claim 4, Targets A, B, E, and G.

Target G*: Identify, analyze, and synthesize relevant external resources to pose or solve problems. (DOK 3, 4)
Especially in extended performance tasks, students should have access to external resources to support their work in posing and solving problems (e.g., finding or constructing a set of data or information to answer a particular question or looking up measurements of a structure to increase precision in an estimate for a scale drawing). Constructed-response items should incorporate "hyperlinked" information to provide additional detail (both relevant and extraneous) for solving problems in Claim 4.

*Measured in Performance Tasks only; functionality of linking to external resources is planned for future enhancements.

What sufficient evidence looks like for Claim 4 (Modeling and Data Analysis)[4]:

"A key feature of items and tasks in Claim 4 is that the student is confronted with a contextualized, or 'real world' situation and must decide which information is relevant and how to represent it. As some of the examples provided below illustrate, 'real world' situations do not necessarily mean questions that a student might really face; it means that mathematical problems are embedded in a practical application context. In this way, items and tasks in Claim 4 differ from those in Claim 2, because while the goal is clear, the problems themselves are not yet fully formulated (well-posed) in mathematical terms.

"Items/tasks in Claim 4 assess student expertise in choosing appropriate content and using it effectively in formulating models of the situations presented and making appropriate inferences from them. Claim 4 items and tasks should sample across the content domains, with many of these involving more than one domain. Items and tasks of this sort require students to apply mathematical concepts at a significantly deeper level of understanding of mathematical content than is expected by Claim 1. Because of the high strategic demand that substantial non-routine tasks present, the technical demand will be lower—normally met by content first taught in earlier grades, consistent with the emphases described under Claim 1. Although most situations faced by students will be embedded in longer performance tasks, within those tasks, some selected-response and short constructed-response items will be appropriate to use.

"Modeling and data analysis in the Common Core State Standards trace a visible arc of growing prominence across the grades, showing low prominence in grades K-5, higher prominence in grades 6-8 (which is when the Statistics and Probability domain first appears), and highest prominence in High School (which is when Modeling appears as a content category with the full modeling cycle). Therefore to align to the Standards, Claim 4 will be more important on the assessment in high school, less important in grades 6-8, and the least important in grades 3-5. Again, to align to the Standards, Claim 4 tasks will be most sophisticated and complete in high school (cf. the modeling cycle in CCSSM pp. 72, 73), less sophisticated/more tied to specific content in middle school, and least sophisticated/most tied to specific content in grades 3-5."

[4] Text excerpted from the Smarter Balanced Mathematics Content Specifications (p. 72-73).

Grade 6 Content Combinations:	**Primary emphases for Claim 4 Items: Ratios and Proportional Relationships, The Number System, and Expressions and Equations** **The following standards can be effectively used in various combinations in Grade 6 Claim 4 items:** **Ratios and Proportional Relationships (RP)** **6.RP.A: Understand ratio concepts and use ratio reasoning to solve problems.** **6.RP.A.1** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* **6.RP.A.2** Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."* **6.RP.A.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. **a.** Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. **b.** Solve unit rate problems including those involving unit pricing and constant speed. *For example, "If it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?"* **c.** Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. **d.** Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities **The Number System (NS)** **6.NS.A: Apply and extend previous understanding of multiplication and division to divide fractions by fractions.** **6.NS.A.1** Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ac/bd.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide*

	is a rectangular strip of land with length 3/4 mi and area 1/2 square mi? **6.NS.C: Apply and extend previous understandings of numbers to the system of rational numbers.** **6.NS.C.5** Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. **6.NS.C.6** Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. **a.** Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite. **b.** Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. **c.** Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. **6.NS.C.7** Understand ordering and absolute value of rational numbers. **a.** Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.* **b.** Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3°C > –7°C to express the fact that –3°C is warmer than –7°C.* **c.** Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.* **d.** Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.* **6.NS.C.8** Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate. **Expressions and Equations (EE)** **6.EE.B: Reason about and solve one-variable equations and inequalities.** **6.EE.B.5** Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine

	whether a given number in a specified set makes an equation or inequality true. **6.EE.B.6** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. **6.EE.B.7** Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q, and x are all nonnegative rational numbers. **6.EE.B.8** Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams. **6.EE.C: Represent and analyze quantitative relationships between dependent and independent variables.** **6.EE.C.9** Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.* **Standards to integrate with the primary emphases:** **Geometry (G)** **6.G.A: Solve real-world and mathematical problems involving area, surface area, and volume.** **6.G.A.1** Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. **6.G.A.2** Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. **6.G.A.3** Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. **6.G.A.4** Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

	Statistics and Probability (SP) **6.SP.A: Develop understanding of statistical variability.** **6.SP.A.1** Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.* **6.SP.A.2** Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. **6.SP.A.3** Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. **6.SP.B Summarize and describe distributions.** **6.SP.B.4** Display numerical data in plots on a number line, including dot plots, histograms, and box plots. **6.SP.B.5** Summarize numerical data sets in relation to their context, such as by: **a.** Reporting the number of observations. **b.** Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. **c.** Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. **d.** Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.
Grade 7 Content Combinations:	**Primary emphases for Claim 4 Items at Grade 7: Ratios and Proportional Relationships, The Number System, and Expressions and Equations** **The following standards can be effectively used in various combinations in Grade 7 Claim 4 items:** **Ratios and Proportional Relationships (RP)** **7.RP.A: Analyze proportional relationships and use them to solve real-world and mathematical problems.** **7.RP.A.1** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.* **7.RP.A.2** Recognize and represent proportional relationships between quantities. **a.** Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent

	ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. **b.** Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. **c.** Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.* **d.** Explain what a point *(x, y)* on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r)* where *r* is the unit rate. **7.RP.A.3** Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.* **The Number System (NS)** **7.NS.A: Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.** **7.NS.A.1** Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. **a.** Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.* **b.** Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. **c.** Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. **d.** Apply properties of operations as strategies to add and subtract rational numbers. **7.NS.A.2** Apply and extend previous understandings of multiplication and division of fractions to multiply and divide rational numbers. **a.** Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(–1)(–1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. **b.** Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with a non-zero divisor) is a rational number. If p and q are integers, then $–(p/q) = (–p)/q = p/(–q)$. Interpret quotients of rational numbers by describing real-world contexts. **c.** Apply properties of operations as strategies to multiply and divide rational numbers.

	d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. **7.NS.A.3** Solve real-world and mathematical problems involving the four operations with rational numbers. **Expressions and Equations (EE)** **7.EE.B: Solve real-life and mathematical problems using numerical and algebraic expressions and equations.** **7.EE.B.3** Solve multi-step, real-life, and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.* **7.EE.B.4** Use variables to represent quantities in a real-world or mathematical problems, and construct simple equations and inequalities to solve problems by reasoning about the quantities. **a.** Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* **b.** Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Give an inequality for the number of sales you need to make, and describe the solutions.* **Standards to integrate with the primary emphases:** **Geometry (G)** **7.G.A: Draw, construct, and describe geometrical figures and describe the relationships between them.** **7.G.A.1** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. **7.G.A.2** Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the

	conditions determine a unique triangle, more than one triangle, or no triangle. **7.G.A.3** Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right-rectangular prisms and right-rectangular pyramids. **7.G.B Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.** **7.G.B.4** Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. **7.G.B.5** Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. **7.G.B.6** Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. **Statistics and Probability (SP)** **7.SP.A Use random sampling to draw inferences about a population.** **7.SP.A.1** Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences. **7.SP.A.2** Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.* **7.SP.B Draw informal comparative inferences about two populations.** **7.SP.B.3** Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.* **7.SP.B.4** Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.* **7.SP.C Investigate chance processes and develop, use, and evaluate probability models.** **7.SP.C.5** Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around ½ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

	7.SP.C.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.* **7.SP.C.7** Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. ***a.*** Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.* ***b.*** Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?* **7.SP.C.8** Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. **a.** Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. ***b.*** Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event. **c.** Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*

Grade 8 Content Combinations:	**Primary emphases for Grade 8 Claim 4 Items: Expressions and Equations and Geometry** **The following standards can be effectively used in various combinations in Grade 8 Claim 4 items:** **Expressions and Equations (EE)** **8.EE.A.3** Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3 × 108 and the population of the world as 7 × 109, and determine that the world population is more than 20 times larger.* **8.EE.A.4** Perform operations with numbers expressed in scientific notation, including problems where

	both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology. **8.EE.B: Understand the connections between proportional relationships, lines, and linear equations.** **8.EE.B.5** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* **8.EE.B.6** Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b. **8.EE.C: Analyze and solve linear equations and pairs of simultaneous linear equations.** **8.EE.C.7** Solve linear equations in one variable. **a.** Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). **b.** Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. **8.EE.C.8** Analyze and solve pairs of simultaneous linear equations. **a.** Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. **b.** Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.* **c.** Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line though the first pair of points intersects the line through and second pair.* **Geometry (G)** **8.G.B: Understand and apply the Pythagorean Theorem.** **8.G.B.6** Explain a proof of the Pythagorean Theorem and its converse. **8.G.B.7** Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real–world and mathematical problems in two and three dimensions. **8.G.B.8** Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

	Standards to integrate with the primary emphases **Functions (F)** **8.F.B: Use functions to model relationships between quantities.** **8.F.B.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x*, *y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. **8.F.B.5** Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally. **Geometry (G)** **8.G.C: Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.** **8.G.C.9** Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems. **Statistics and Probability (SP)** **8.SP.A Investigate patterns of association in bivariate data.** **8.SP.A.1** Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. **8.SP.A.2** Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. **8.SP.A.3** Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.* **8.SP.A.4** Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

Range ALDs – Claim 4 Grades 6 - 8	**Level 1** Students should be able to identify important quantities in the context of a familiar situation and translate words to equations or other mathematical formulation. When given the correct math tool(s), students should be able to apply the tool(s) to problems with a high degree of scaffolding.
	Level 2 Students should be able to identify important quantities in the context of an unfamiliar situation and to select tools to solve a familiar and moderately scaffolded problem or to solve a less familiar or a nonscaffolded problem with partial accuracy. Students should be able to provide solutions to familiar problems using an appropriate format (e.g., correct units, etc.). They should be able to interpret information and results in the context of a familiar situation.
	Level 3 Students should be able to map, display, and identify relationships, use appropriate tools strategically, and apply mathematics accurately in everyday life, society, and the workplace. They should be able to interpret information and results in the context of an unfamiliar situation.
	Level 4 Students should be able to analyze and interpret the context of an unfamiliar situation for problems of increasing complexity and solve problems with optimal solutions.

Target 4A: Apply mathematics to solve problems arising in everyday life, society, and the workplace.

General Task Model Expectations for Target 4A

- The student is asked to solve a problem arising in everyday life, society, or the workplace.
- Information needed to solve the problem has a level of complexity that is not present in items within Claim 2 Target A. For example, the student must
    - distinguish between relevant and irrelevant information, or
    - identify information that is not given in the problem and request it, or
    - make a reasonable estimate for one or more quantities and use that estimate to solve the problem.
- The student must select a mathematical model independently and is not directly told what arithmetic operation or geometric structure to use to solve the problem.
- Tasks in this model often have secondary alignments to other Claim 4 targets, in particular Target 4B, constructing autonomous chains of reasoning, Target 4D, requiring the student to interpret results in the context of the problem, and Target 4F, requiring students to identify quantities and map relationships between them.
- Problems in this model may have more than one possible solution.
- The student is often required to draw upon knowledge from different domains, including knowledge from earlier grade-levels.
- Tasks have Depth of Knowledge (DOK) Level 2 or 3.

Task Model 4A.1

Task Expectations

- The student solves a multi-step problem involving the four operations with rational numbers or solving equations.
- The student identifies needed information and chooses which operations to perform or which equation to solve. The student may
    - ignore irrelevant information,
    - request or conduct research to find missing information,
    - identify constraints that are not explicitly stated, and/or
    - Make an estimate for one or more quantities and use that estimate to solve the problem.
- Example items from Task Model 4A.1 for Grades 3–5 may be adapted to this task model by increasing the complexity of the numbers involved and introducing rational numbers and decimal fractions.

Example Item 4A.1a (Grade 6)
Primary Target 4A (Content Domain NS), Secondary Target 1B (CCSS 6.NS.A), Tertiary Target 4B, Quaternary Target 1A (CCSS 6.RP.A)

Juan has $7\frac{3}{4}$ cups of nuts. He wants to make either banana nut muffins or carrot muffins. The table shows how many cups of nuts are needed for each batch.

Amount of Nuts Needed Per Batch of Muffins

Muffin Type	Amount of Nuts per Batch
Banana nut	$\frac{1}{2}$ cup
Carrot	$\frac{5}{8}$ cup

Juan decided to make only carrot muffins. What is the maximum number of whole batches of carrot muffins Juan can make with $7\frac{3}{4}$ cups of nuts?

Enter your answer in the response box.

Rubric: (1 point) Student enters the correct number (12).

Response Type: Equation/Numeric

Commentary: The task could also ask about banana nut muffins, or about both for a 2-point item. A more cognitively demanding version of the task could ask how many whole batches can be made if he wants to make half banana nut and half carrot.

Example Item 4A.1a (Grade 6)
Primary Target 4A (Content Domain RP), Secondary Target 1A (CCSS 6.RP.A), Tertiary Target 4B, Quaternary Target 4F

Hummingbirds drink nectar from flowers and sugar water from bird feeders.

- Sugar water is made by mixing 50 grams of sugar with 200 grams of water.
- A hummingbird's favorite flower nectar is 21% sugar by mass.

The amount of food a hummingbird eats at one time is always the same whether it eats sugar water or flower nectar.

Part A
Will the hummingbird get more sugar from a meal of sugar water made according to the recipe, or from an equal-sized meal of flower nectar? [Drop down choices: sugar water, flower nectar]

Part B
How much more sugar, in grams, would a hummingbird get from 4 grams of the [fills in with student's choice for the more sugary food type from part A] than from 4 grams of the [fills in with student's choice for the less sugary food type from part A]?

Interaction: Once the student selects the more sugary food type in part A, part B populates with the student's choice. The student can go back and change the choice in part A, in which case the statement of part B changes as well. Title the response box in Part B "Grams of sugar."

Rubric: (2 points) The student selects the more sugary food item (flower nectar) and identifies the additional amount of sugar correctly (0.04).
(1 point) The student identifies the food made by the recipe and enters the difference as 0.16, which corresponds to assuming the recipe is 25% sugar by weight (a likely mistake) but then correctly computing the difference.

Response Type: Drop Down Menu[5] and Equation/Numeric

Note: Functionality for this item type does not currently exist, although the item could be modified to work with current technology by making Part A a hot Spot (choose between "Recipe" and "Flower Nectar") and by wording Part B, "How much more sugar, in grams, would a hummingbird get from 4 grams of the option you chose in Part A than from 4 grams of the other option?"

[5] Drop-Down Menu response type is not currently available, but is a planned enhancement to the test-authoring tool by 2017.

Example Item 4A1.b (Grade 7)
Primary Target 4A (Content Domain NS), Secondary Target 1B (CCSS 6.NS.A), Tertiary Target 4B, Quaternary Target 4D
[Adapted from Illustrative Mathematics task 50]

Alice, Raul, and Maria are baking cookies together.

They need $\frac{3}{4}$ cup of flour and $\frac{1}{3}$ cup of butter to make one batch of cookies.

They each brought the ingredients they had at home.

- Alice brought 2 cups of flour and $\frac{1}{4}$ cup of butter
- Raul brought 1 cup of flour and $\frac{1}{2}$ cup of butter
- Maria brought $1\frac{1}{4}$ cups of flour and $\frac{3}{4}$ cups of butter.

Assume the students have plenty of the other ingredients (sugar, salt, baking soda, etc.) they need to make the cookies.

What is the maximum number of whole batches of cookies they can make with the ingredients they brought from home?

Enter your answer in the second response box.

Response Type: Equation/Numeric

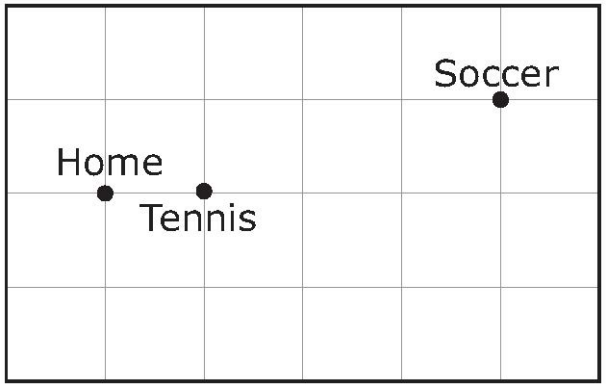
Commentary: Difficulty and grade level can be varied by varying the complexity of the numbers used. Item aligns with 4D because students must choose which fraction division limits the number of batches that can be made.

Example Item 4A.1c (Grade 6)
Primary Target 4A (Content Domain EE), Secondary Target 1F (CCSS 6.EE.B), Tertiary Target 4B, Quaternary Target 4F
Adapted from https://www.illustrativemathematics.org/illustrations/985

- Mrs. Jonas, her son Cody, and her daughter Laura drove from home to Cody's tennis practice.
- Mrs. Jonas then drove Laura to her soccer game and stayed to watch.
- After the game, mother and daughter picked up Cody from the tennis courts on the way home.
- Once home, Mrs. Jonas saw that they had driven 15 miles that day.

Mrs. Jonas took the shortest routes to and from each destination.

The figure shows the location of the Jonas family home, the tennis courts, and the soccer field. The gridlines in the figure represent the streets, and all distances between cross streets are approximately the same.



Part A:
Write an equation that can be used to find the distance, d, between the tennis courts and home.
Enter your answer in the first response box.

Part B:
What is the distance, in miles, between home and the tennis courts?
Enter your answer in the second response box.

Rubric: (2 points) Student correctly answers both parts ($10d = 15$, or $d + 4d + 4d + d = 15$ or equivalent equation for Part A; 1.5 or 1 ½ for Part B)

(1 point) Student correctly answers only one part.

Response Type: Equation/Numeric (Note: Label the two response boxes "Part A" and "Part B.")

Task Model 4A.2

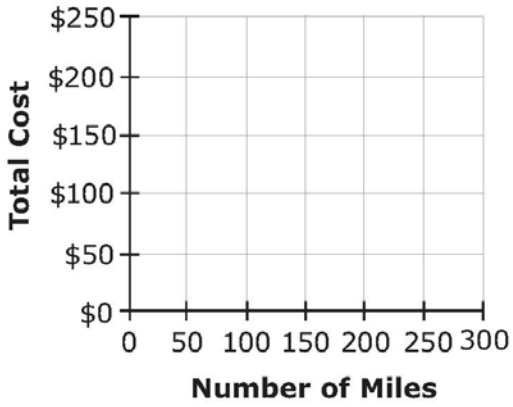
Task Expectations

- The student solves a problem involving ratios, proportional relationships, or linear functions.
- The student identifies needed information and chooses the ratio, proportional relationship, or linear function required to complete the problem. The problem should require the student to do one of the following:
    - ignore irrelevant information,
    - request or conduct research to find missing information,
    - identify constraints that are not explicitly stated, or
    - make an estimate for one or more quantities and use that estimate to solve the problem.

Example Item 4A.2a (Grade 8)
Primary Target 4A (Content Domain EE), Secondary Target 1D (CCSS 8.EE.C), Tertiary Target 4D, Quaternary Target 4F

This table represents the cost of renting a truck from Moving Company X and Moving Company Y. Each company charges a one-time rental fee plus a charge for each mile driven.

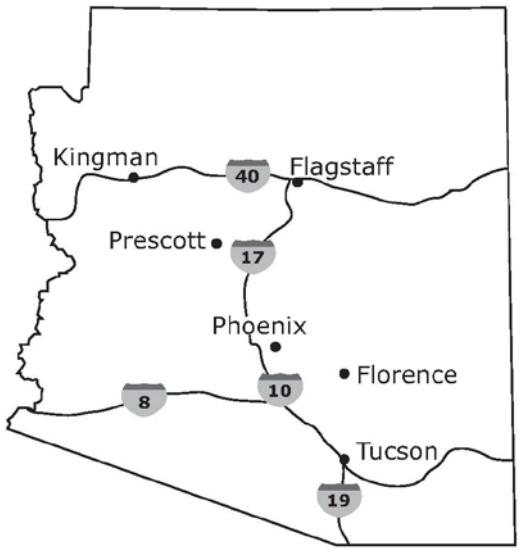
Moving Company	One-time Rental Fee	Charge per Mile
X	$150	$0.25
Y	$ 50	$0.75



Part A
Use the Add Arrow tool to graph two linear equations that represent the cost of using each moving company given a number of miles driven.

Part B
Select the moving company that will be the **least** expensive to move between the given cities. Refer to the map shown to determine the distances.

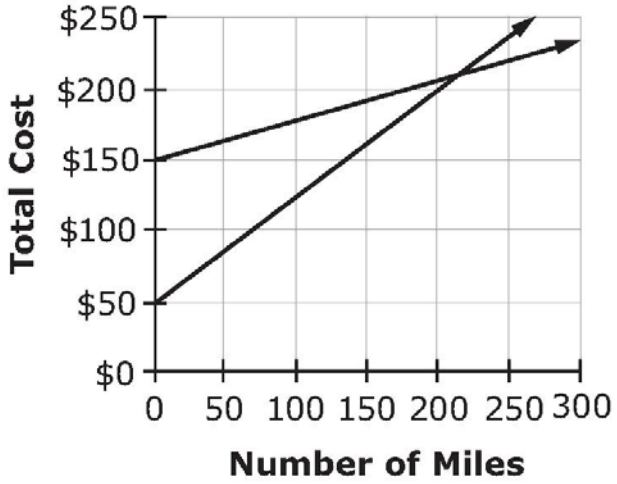
Cities	Company A	Company B
Tucson to Phoenix		
Phoenix to Flagstaff		
Tucson to Flagstaff		



Interaction: The student can use the ruler tool to measure distances on the map.

Rubric: Each part of this item is scored independently for a total of 2 points.
Part A (1 point) The student correctly graphs both functions.
Part B (1 point) The student selects the correct cells in the table.

Exemplar:



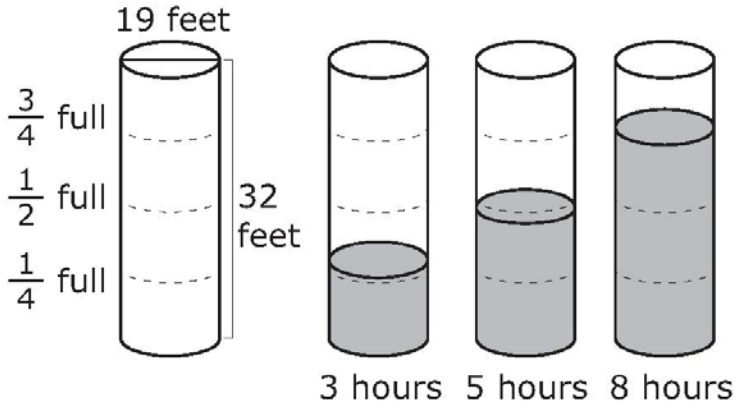
Cities	Company A	Company B
Tucson to Phoenix		
Phoenix to Flagstaff		
Tucson to Flagstaff		

Interaction: The Add Arrow tool will be available (with one arrow) to graph the lines, as well as Hot Spot to select the correct cells in the table. Also, the ruler tool needs to be active.

Response Type: Graphing and Hot Spot

Example Item 4A.2b (Grade 8)
Primary Target 4A (Content Domain G), Secondary Target 1I (CCSS 8.G.C), Tertiary Target 1A (CCSS 7.RP.A), Quaternary Target 4B

An empty tank in the shape of a cylinder is being filled with water. The tank is filled at a constant rate for a total of 10 hours. The figure shows the height of water in the tank at the given number of hours after filling started.



Enter the **percent** of the tank that is filled with water at 10 hours.

Rubric: (2 points) The student enters the correct numerical value for the percent (93.75–94).
(1 point) The student gives the height of water in the tank after 10 hours (30-30.1) OR the volume of water in the tank 10 hours (8500–8532), but forgets to find the percentage.

Response Type: Equation/Numeric (label the response box with %)

Commentary: The task can be done knowing only the information from the third picture (the height is 24 feet after 8 hours), so students who ignore extraneous information are rewarded. Notice that it is not necessary to compute the volume to find the percent, since it can be found by computing the ratio of the heights. Although it is not expected that many students will notice this, the task thus also rewards students with good modeling sense and geometric insight.

Target 4B: Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem.

Items that require the student to make decisions about the solution path needed to solve a problem are aligned with Target 4B. Note that Target 4B is never the primary target for an item, but is frequently a Tertiary or Quaternary Target for an item with primary alignment to other targets; see, for example, items in Task Models for 4A, 4C, and 4E.

General Task Model Expectations for Target 4B

- The student is presented with a multi-step problem with little or no scaffolding, or
- The student must make estimates or choose between different reasonable assumptions in order to solve the problem.

Target 4C: State logical assumptions being used.

General Task Model Expectations for Target 4C

- The student is presented with a problem arising in everyday life, society, or the workplace. The student either
  - identifies information or assumptions needed to solve the problem,
  - researches to provide information needed to solve the problem, or
  - provides a reasoned estimate of a quantity needed to solve the problem.

  It is not necessary that a student constructs a complete solution to the problem for this target.
- Tasks in this model generally have either more information than is needed solve the problem (and students must choose) or not enough information (and students must make a reasoned estimate).
- The student is often required to draw upon knowledge from different domains, including knowledge from earlier grade-levels.
- Tasks for this target may also assess Target 4F.
- Tasks have DOK Level 1 or 2

Task Model 4C.1

Task Expectations:

- Student chooses from a list of possible assumptions, or makes an estimate, and then solves a problem using the assumption or estimate.

Example Item 4C.1a (Grade 7)
Primary Target 4C (Content Domain SP), Secondary Target 1I (CCSS 7.SP.C), Tertiary Target 4B, Quaternary Target 4D

Ramos flips a coin 100 times and records the results in a table.

Results of 100 Coin Flips

Outcome of Flip	Number of Times
Heads	74
Tails	26

Part A
Select an assumption about the outcome of a single flip of this coin [heads and tails are equally likely; heads are 3 times as likely as tails]

Part B
Based on your assumption, which would be the most likely outcome for the next **2** flips?

A. two heads
B. two tails
C. one head and one tail

Interaction: The student must first select from the drop-down menu to make an assumption, and then select a correct option based on that assumption.

Rubric: (1 point) Student makes correct choice based on the assumption they choose (C for the first assumption, A for the second assumption).

Response Type: Drop-down Menu; Hotspot

Task Model 4C.2

Task Expectations:

- The student is given a problem with insufficient information and must indicate what information is needed to complete the solution to a problem.

Example Item 4C.2a (Grade 7)
Primary Target 4C (Content Domain RP), Secondary Target 1A (CCSS 7.RP.A), Tertiary Target 4F
[Adapted from Illustrative Mathematics task 1564.]

Chichén Itzá was a Mayan city in what is now Mexico. The picture shows El Castillo, also known as the pyramid of Kukulcán, which is located in the ruins of Chichén Itzá.

The pyramid is approximately 30 meters tall, and there are 91 steps leading up to a temple at the top.

What additional information do you need to know to estimate the height above the ground, in meters, of the 50th step? Select **all** that apply.

A. Each of the steps has approximately the same height.
B. The base of the pyramid is about 55 meters wide.
C. The height of the temple is about 6 meters.
D. The base of the pyramid is a square.

Rubric: (1 point) The student selects the correct options (A and C).

Response Type: Multiple Choice, multiple correct response

Target 4D: Interpret results in the context of a situation.

Target 4D identifies a key step in the modeling cycle, and is thus present in the majority of modeling problems that require students to find a numerical answer as well as many problems where students construct an equation or a graph.

General Task Model Expectations for Target 4D

- The student is presented with a problem situation in everyday life, society, or the workplace or a mathematical model of such a situation. The student interprets the solution to the problem in terms of the context, in terms of the model, or compares the results of the model with the real-world data it represents.
    - Item types with a primary alignment to 4D focus on interpreting results in terms of the model or comparing the results of the model with the real-world data it represents.
    - It is not necessary for a student to generate a complete solution for problems with a primary alignment to this target.
- Tasks in Targets 4A, 4C, 4E, and 4F frequently have this target as a tertiary or quaternary alignment because students must interpret their results in terms of the context.
- The student is often required to draw upon knowledge from different domains, including knowledge from earlier grade-levels.
- Tasks have DOK Level 2 or 3.

Task Model 4D.1

- The student is presented with a mathematical model of real-world data.
- The student interprets the solution to the problem in terms of the model or compares the results of the model with the real-world data it represents.

Example Item 4D.1a (Grade 8)
Primary Target 4D (Content Domain F), Secondary Target 1F (CCSS 8.F.B), Tertiary Target 4C

This graph shows the average number of words a child can say from birth to 36 months.

Which statement is the **most accurate** description of the growth in the number of words a child speaks based on the graph shown?

A. Children learn to say new words at a steady rate starting about 12 months of age.
B. Children are constantly learning to say new words from the moment they are born.
C. Children learn to say new words more slowly during their second year than during their third year.
D. Children begin learning to say words around 24 months and stop learning to say new words at 36 months.

Rubric: (1 point) The student chooses the best interpretation of the graph (C).

Note: To distinguish from Claim 1 items, interpretations should extend beyond simply looking at the graph and should help to evaluate whether students understand which interpretations are defensible. Item authors should be careful with language not to "overstate" a particular conclusion since all data based interpretations are subject to some error.

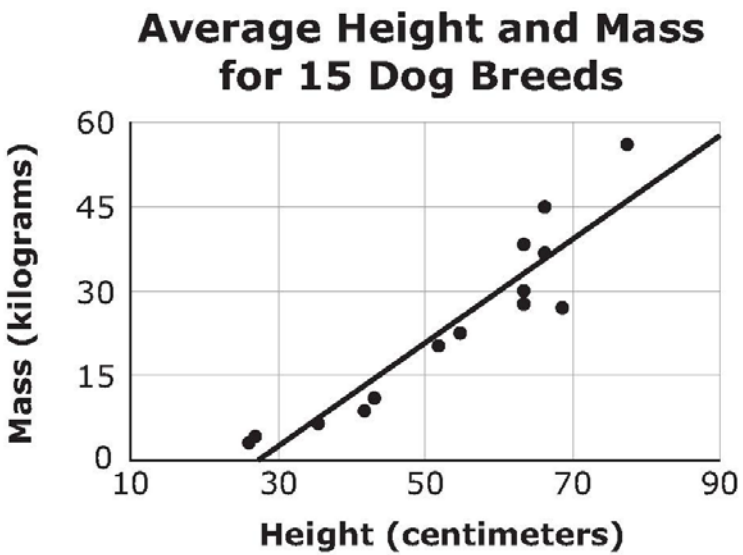
Response Type: Multiple Choice, single correct response

Example Item 4D.1b (Grade 8)
Primary Target 4D (Content Domain SP), Secondary Target 1J (CCSS 8.SP.A), Tertiary Target 4E

This scatter plot and line of best fit show the relationship between the height and mass of 15 different dog breeds.

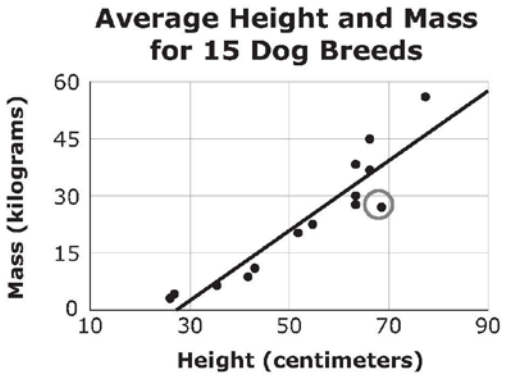
The mass of the Afghan Hound is less than would be predicted by the line of best fit, and the difference between the predicted mass and the actual mass is greater than for any other breed.

Click on the point in the scatterplot that corresponds to the Afghan Hound.



Rubric: (1 point) The student clicks the point that below and farthest away from the graph (see figure).

Response Type: Hot Spot



Target 4E: Analyze the adequacy of and make improvements to an existing model or develop a mathematical model of a real phenomenon.

General Task Model Expectations for Target 4E

- The student is presented with a problem arising in everyday life, society, or the workplace. The student either
  - Chooses between competing mathematical models to solve the problem (which may depend on different interpretations of the problem)
  - Evaluates a partial or complete (possibly incorrect) solution to the problem
  - Constructs a mathematical model to solve the problem

  It is not necessary that a student to generate a complete solution for problems in this target.
- Tasks in this model can also assess Target 4B (Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem). Thus some tasks should plausibly entail a chain of reasoning to complete the task (not just a single step). For example, it might be necessary for the student to construct a two-step arithmetic expression to evaluate a model or solution, or to try out a geometric shape and then perform a calculation to see if it satisfies the requirements.
- The student is often required to draw upon knowledge from different domains, including knowledge from earlier grade-levels.
- Tasks have DOK Level 3 or 4

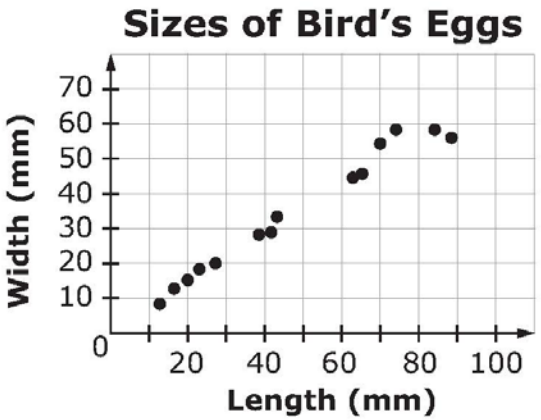
Task Model 4E.1

Task Expectations:

- Students construct an expression, equation, proportional relationship, linear function, or geometric figure that models a given problem.
- Models can be represented in symbolic or graphical form.
- The model is not explicitly given, but should be inferred from the situation.
- Students are expected to reason autonomously from a context to the model.

Example Item 4E.1a (Grade 8)
Primary Target 4E (Content Domain SP), Secondary Target 1J (CCSS 8.SP.A), Tertiary Target 4D, Quaternary 4B

This scatter plot shows the lengths and the widths (in millimetres) of the eggs of some American birds.



Use the information in the scatter plot to support each answer.

Part A
The scatter plot shows an association between the length of a bird egg and its width. Describe that association.

Part B
Fossils show that dinosaur eggs closely resemble the shape of bird eggs. One type of dinosaur (sauropods) grew from eggs that were 180 millimeters in length.

Assume that sauropod eggs were the same shape as bird eggs. What is the approximate width, in millimeters, of sauropod eggs? Explain how you determined your answer.

Rubric: (2 points) The student is able to answer both parts correctly and provide sufficient explanation/support for the answer to *Part B*.
(1 point) The student only answers one part correctly.

Exemplar[6]:
Part A: Typically, the greater the length of the egg, the greater the width.
Part B: The width is approximately 126 mm (accept values between 115 and 135 mm).
"I multiplied the length by about 0.7" or "The width is a little less than the length" or "I doubled the width of the egg that is 90 mm long."

Response Type: Short Text (handscored)

Example Item 4E.1b (Grade 8)
Primary Target 4E (Content Domain F), Secondary Target 1F (CCSS 8.F.B), Tertiary Target 4F, Quaternary Target 4D

Cory is buying copper for a construction project. He pays $1.85 per pound of copper for the first 100 pounds. He pays $1.75 per pound of copper for every pound over 100 pounds. Cory calculated that it would cost $228.75 to purchase 125 pounds of copper. He wrote an equation that allows him to determine the cost of copper for any number of pounds of copper over 100 pounds.

His equation is in the form $y = n(x - 100) + p$ where y is the amount of money, in dollars, Cory pays for x total pounds of copper when x is greater than 100. What are his values for n and p?

Enter the value of n in the first response box.

Enter the value of p in the second response box.

Rubric: (1 point) The student enters the correct values for n and p (1.75 and 185).

Response Type: Equation/Numeric (Note: Label each response box n = [box], p = [box])

[6] An exemplar response represents only one possible solution. Typically, many other solutions/responses may receive full credit. The full range of acceptable responses is determined during rangefinding and/or scoring validation.

Grades 6-8, Claim 4

Task Model 4E.2

Task Expectations:

- The student chooses between two or more different models to solve a given problem, between two or more problems that fit a given model, or between two or more different solutions to a given problem.
- Different models or solutions can depend on different (possibly incorrect) interpretations of the problem, but do not have to.
- The student assesses the fit of a particular model being used.

Example Item 4E.2a (Grade 8)
Primary Target 4E (Content Domain F), Secondary Target 1F (CCSS 8.F.B), Tertiary Target 4F, Quaternary Target 4D
(Source: Adapted from Illustrative Mathematics 8-F Modeling with a Linear Function)

Select **all** situations that can be modeled by the linear equation $y = 2x + 5$.

A. There are initially 5 rabbits on a farm. Each month thereafter the number of rabbits is 2 times the number in the month before. How many rabbits are there after x months?

B. Joe earns $2 for each magazine sale. He also earns $5 for each hour he spends trying to sell magazines. How much money will he earn after selling magazines for x hours?

C. Sandy charges $2 an hour for babysitting. Parents are charged $5 if they arrive home later than scheduled. Assuming the parents arrived home late, how much money does she earn for x hours?

D. The Reader's Club is a members-only audio book rental store. There is a $2 sign-up fee and a $5 per audio book rental fee. How much would Laney owe on her first visit if she becomes a member and rents x audio books?

E. Andre is saving money for a new CD player. He began saving with a $5 gift and will continue to save $2 each week. How much money will he have saved at the end of x weeks?

Rubric: (1 point) The student identifies all situations modeled by the equation (C and E).

Response Type: Multiple Choice, multiple correct response

Example Item 4E.2b (Grade 8)
Primary Target 4E (Content Domain F), Secondary Target 1F (CCSS 8.F.B), Tertiary Target 4D

The table shows the relationship between the average number of hours students studied for a mathematics test and their average grade.

Hours Studied	Average Grade
0	62
1	78
2	85
5	74

Which type of function is most likely to model these data?

A. linear function with positive rate of change
B. linear function with negative rate of change
C. non-linear function that decreases then increases
D. non-linear function that increases then decreases

Rubric: (1 point) The student recognized the function most likely to model the data (D).

Response Type: Multiple Choice, single correct response

Target 4F: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas).

Target 4F identifies a key step in the modeling cycle, and is thus present in the majority of modeling problems.

Task Model 4F.1

Task Model Expectations

- Students are presented with a mathematical problem in a real-world context where the quantities of interest are not named explicitly, are named but represented in different ways, or the relationship between the quantities is not immediately clear.
- The student is asked to solve a problem that may require the integration of concepts and skills from multiple domains.

Example Item 4F.1a (Grade 7)
Primary Target 4E (Content Domain EE), Secondary Target 1F (CCSS 6.EE.B), Tertiary Target 4F, Quaternary Target 4D

Megan has $2500. She spends money on the following:

- $800 on rent
- $400 on food
- $200 on utility services
- $250 on loan payments
- $$x$ on other expenses

Let y represent the amount of money in dollars Megan has left. Write an equation that represents the relationship between the amount of money Megan spends on other expenses and the amount of money Megan has left.

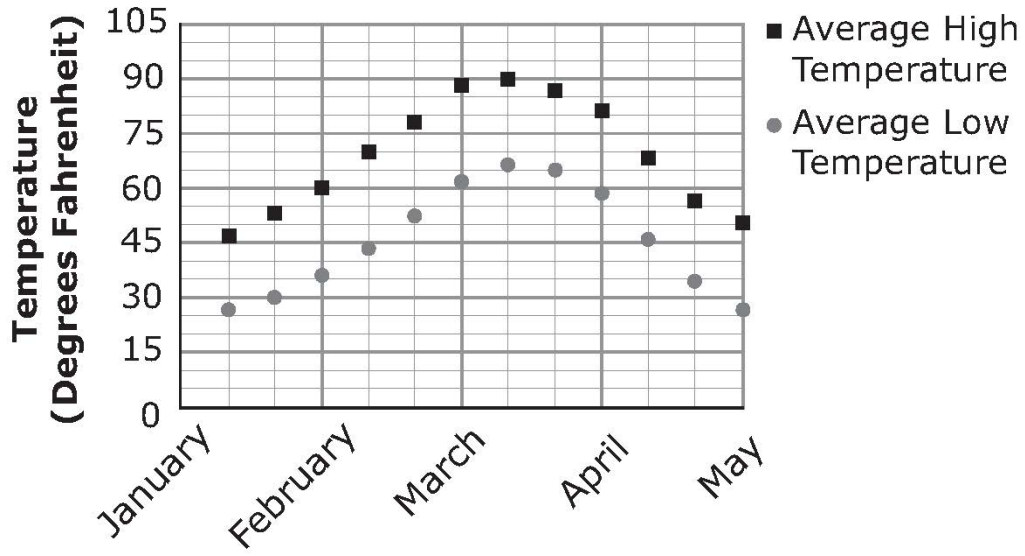
Rubric: (1 point) The student computes Megan's spending and represents the remaining money with an equation ($y = 850 - x$, or equivalent).

Response Type: Equation/Numeric

Example Item 4F.1b (Grade 6)
Primary Target 4F (Content Domain EE), Secondary Target 1G (CCSS 6.EE.C), Tertiary Target 4D

Part A
If you were going to plan a picnic, what temperature would you hope to have for the picnic?
Enter the temperature, in degrees Fahrenheit, you think would be best in the first response box. You may change your answer later if you wish.

Part B
The average monthly high and low temperatures for a town are shown in the graph below.



Select a month from the drop down menu where the temperature you chose would fall between the high and low temperatures for that month. [January, February,… December, no month will work]

Interaction: The student enters a temperature for a theoretical picnic in the first response box, then answers Part B with a drop down menu. The student can change his or her preferred temperature. The temperature a student chooses does not affect his or her score for the item except that the next choice must be consistent with it. When the student mouses over the points in the graph, the corresponding value appears (alternatively, there is a table of values as well).

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Average high in °F:	47	53	60	69	79	88	90	87	81	69	56	46
Average low in °F:	26	30	36	43	53	62	66	65	58	46	34	26

Rubric: (1 point) The student selects a month where the temperature he or she chose falls between the high and low temperatures for that (e.g., if the student selects 80, then they choose either June, July, August, or September).

Response Type: Equation/Numeric and Drop-down

Note: Functionality for this item type does not currently exist, but is planned for future enhancements to the item authoring tool in 2017.

Example Item 4F.1c (Grade 8)
Primary Target 4F (Content Domain F), Secondary Target 1F (CCSS 8.F.B), Tertiary Target 4D

The relationship between Jack's distance from home and the time since he left home is linear, as shown in the table.

Time (hrs)	Distance (mi)
0	7.5
2	17.5
4	27.5

Based on the values in the table, determine whether each statement is true. Select True or False for each statement.

Statement	True	False
Jack's initial distance from home is 7.5 miles.		
Jack's distance increases by 5 miles every 1 hour.		
Jack's distance from home at 3 hours is 23.5 miles.		

Rubric: (1 point) Student determines each statement as being either true or false (TTF).

Response Type: Matching Table