Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.	
Content Domain: **Operations and Algebraic Thinking**	
Target A [m]: Use the four operations with whole numbers to solve problems. (DOK 1, 2) Tasks for this target will require students to use the four operations to solve straightforward, one-step or multi-step contextual word problems, including problems where the remainder must be interpreted.	
Standards: 4.OA.A, 4.OA.A.1, 4.OA.A.2, 4.OA.A.3	**4.OA.A Use the four operations with whole numbers to solve problems.** **4.OA.A.1** Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. **4.OA.A.2** Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. **4.OA.A.3** Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies, including rounding.
Related Below-Grade and Above-Grade Standards for Purposes of Planning for Vertical Scaling: 3.OA.A, 3.OA.A.1, 3.OA.A.2, 3.OA.A.3, 3.OA.A.4, 3.OA.C, 3.OA.C.7, 3.OA.D, 3.OA.D.8 5.OA.A, 5.OA.A.1, 5.OA.A.2, 5.NBT.B, 5.NBT.B.5, 5.NBT.B.6	**Related Grade 3 Standards** **3.OA.A Represent and solve problems involving multiplication and division.** **3.OA.A.1** Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. **3.OA.A.2** Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. **3.OA.A.3** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. **3.OA.A.4** Determine the unknown whole number in a multiplication or division equation relating three whole numbers.

	3.OA.C Multiply and divide within 100. **3.OA.C.7** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers. **3.OA.D Solve problems involving the four operations, and identify and explain patterns in arithmetic.** **3.OA.D.8** Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies, including rounding. **Related Grade 5 Standards** **5.OA.A Write and interpret numerical expressions.** **5.OA.A.1** Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. **5.OA.A.2** Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. **5.NBT.B Perform operations with multi-digit whole numbers and with decimals to hundredths.** **5.NBT.B.5** Fluently multiply multi-digit whole numbers using the standard algorithm. **5.NBT.B.6** Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
DOK Levels:	1, 2
Achievement Level Descriptors:	
RANGE Achievement Level Descriptor (Range ALD) Target A: Use the four operations with whole numbers to solve problems.	**Level 1** Students should be able to use the four operations (add, subtract, multiply, and divide) to solve one-step problems involving equal groups and arrays.
	Level 2 Students should be able to use the four operations to solve one-step problems involving an unknown number. They should be able to realize that it is appropriate to multiply or divide in order to solve familiar multiplicative comparison problems.
	Level 3 Students should be able to use the four operations (add, subtract, multiply, and divide) to solve one-step problems involving

	equal groups and arrays, including problems where the remainder must be interpreted. They should be able to find an unknown number and represent problems using equations with a symbol representing the unknown quantity.
	Level 4 Students should be able to assess the reasonableness of answers using mental computation and estimation strategies, including rounding.
Evidence Required:	1. The student solves contextual problems involving multiplicative comparisons, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. 2. The student solves straightforward, contextual problems using the four operations.
Allowable Response Types:	Multiple Choice, single correct response; Equation/Numeric
Allowable Stimulus Materials:	multiplication equations, verbal statements of multiplicative comparison, contextual problems involving multiplicative comparison, one-step contextual word problems, measurements limited to: kilometers (km), meters (m), centimeters (cm), kilograms (kg), grams (g), pounds (lb), ounces (oz), liters (L), milliliters (mL), hours (hr), minutes (min), seconds (s), money (whole number $ or ¢ only), yards (yd), feet (ft), inches (in), gallons (gal), quarts (qt), pints (pt), cups
Construct-Relevant Vocabulary:	Remainder, sum, difference, quotient, product, equation, times as much, times as many, equation
Allowable Tools:	None
Target-Specific Attributes:	Numbers used in this target must be whole numbers. In describing a multiplicative comparison, the language “times as much” or “times as many” is preferable to “times more than.”
Non-Targeted Constructs:	
Accessibility Guidance:	Item writers should consider the following Language and Visual Element/Design guidelines[1] when developing items. Language Key Considerations: • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level • Avoid ambiguous or obscure words, idioms, jargon, unusual names and references Visual Elements/Design Key Considerations: • Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the

[1] For more information, refer to the General Accessibility Guidelines at: http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf

	context • Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary • Avoid crowding of details and graphics Items are selected for a student's test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.[2]
Development Notes:	Interpreting multiplication equations as multiplicative comparisons and representing verbal statements of multiplicative comparisons as multiplication equations (4.OA.1) will be assessed in Claim 4. Items asking students to solve a word problem by using an equation with an unknown number to represent the problem (4.OA.A.2) will be covered in Claim 4. Items asking students to solve multi-step word problems, interpret a remainder, and/or assess reasonableness of answers (4.OA.A.3) will be covered in Claim 2. Interpreting remainders in context by having students explain or justify why a quotient was rounded to the next whole or why the solution has a fraction remainder (4.OA.A.3) may be assessed in Claim 3.

[2] For more information about student accessibility resources and policies, refer to http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

Task Model 1a **Response Type: Equation / Numeric** **DOK Level 1** **4.OA.A.2** Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. **Evidence Required:** 1. The student solves contextual problems involving multiplicative comparisons, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. **Tools:** None	**Prompt Features:** The student is prompted to solve a contextual problem involving multiplicative comparison. **Stimulus Guidelines:** • Numbers should fit in the parameters of up to 4-digit by 1-digit, or 2-digit by 2-digit multiplication problems, and up to 4-digit divided by 1-digit division problems. • All quantities should be whole numbers. • Problems may involve measurements, limited to non-conversion items, using o kilometers (km), meters (m), centimeters (cm); o kilograms (kg), grams (g); o pounds (lb), ounces (oz); o liters (L), milliliters (mL); o hours (hr), minutes (min), seconds (s); o money (whole number $ or ¢ only); o yards (yd), feet (ft), inches (in); or o gallons (gal), quarts (qt), pints (pt), or cups. • Item difficulty can be adjusted via these example methods: o Using multiplication facts in the context o Using non-math facts in the context **TM1a** **Stimulus:** The student is presented with a contextual problem involving multiplicative comparison with an unknown product. **Example Stem:** A cat has 4 times as many toys as a puppy. The puppy has 12 toys. How many toys does the cat have? Enter your answer in the response box. **Rubric:** (1 point) The student solves for an unknown and enters the correct number (e.g., 48). **Response Type:** Equation/Numeric

Task Model 1b-c **Response Type: Equation / Numeric** **DOK Level 2** **4.OA.A.2** Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. **Evidence Required:** 1. The student solves contextual problems involving multiplicative comparisons, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. **Tools:** None **Version 3 update:** Replaced example stem in TM1c to update context.	**Prompt Features:** The student is prompted to solve a contextual problem involving multiplicative comparison. **Stimulus Guidelines:** Same as for TM1a. **TM1b** **Stimulus:** The student is presented with a contextual problem involving multiplicative comparison with an unknown factor. The unknown is a quantity of objects or measurement quantity. **Example Stem:** A cat has 2 times as many toys as a puppy. The cat has 10 toys. How many toys does the puppy have? Enter your answer in the response box. **TM1c** **Stimulus:** The student is presented with a contextual problem involving multiplicative comparison that solves for an unknown factor. The unknown is the multiplier that describes how many times more one quantity is than the other. **Example Stem:** Josh and Aaron are collecting shells at the beach. Josh collects 9 shells and Aaron collects 36 shells. How many times more shells does Aaron collect than Josh? Enter your answer in the response box. **Rubric:** (1 point) The student solves for an unknown and enters the correct number (e.g., 5; 4). **Response Type:** Equation/Numeric

Task Model 2 **Response Type: Equation / Numeric** **DOK Level 2** **4.OA.A** Use the four operations with whole numbers to solve problems. **Evidence Required:** 2. The student solves straightforward, contextual problems using the four operations. **Tools:** None	**Prompt Features:** The student is prompted solve straightforward word problems using the four operations. **Stimulus Guidelines:** • Numbers should fit in the parameters of up to 4-digit by 1-digit, or 2-digit by 2-digit, multiplication problems. • All quantities should be whole numbers. • Problems may involve measurements, limited to non-conversion items, using o kilometers (km), meters (m), centimeters (cm); o kilograms (kg), grams (g); o pounds (lb), ounces (oz); o liters (L), milliliters (mL); o hours (hr), minutes (min), seconds (s); o money (whole number $ or ¢ only); o yards (yd), feet (ft), inches (in); or o gallons (gal), quarts (qt), pints (pt), or cups. • Item difficulty can be adjusted via these example methods: o Using numbers less than 100 o Using numbers greater than 100, but less than 1,000 o Using numbers greater than 1,000, but less than 1,000,000 (for addition and subtraction only) **TM2** **Stimulus:** The student is presented with a contextual problem using any of the four operations. **Example Stem 1:** Tanya ran 400 meters on Tuesday. She ran 800 meters on Wednesday. What is the total number of meters Tanya ran these two days? **Example Stem 2:** A container holds 750 milliliters of water. Jess drank 90 milliliters of the water. How many milliliters of water remain in the container? **Rubric:** (1 point) The student correctly solves the word problem (e.g., 1200; 660). **Response Type:** Equation/Numeric

Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.	
Content Domain: **Operations and Algebraic Thinking**	
Target B [s]: Gain familiarity with factors and multiples. (DOK 1, 2) Tasks for this target will ask students to find factor pairs and determine whether a whole number (1–100) is a multiple of a given one-digit number and whether a whole number (1–100) is prime or composite.	
Standards: 4.OA.B, 4.OA.B.4	**4.OA.B Gain familiarity with factors and multiples.** **4.OA.B.4** Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.
Related Below-Grade and Above-Grade Standards for Purposes of Planning for Vertical Scaling: 3.OA.A, 3.OA.A.4, 3.OA.B, 3.OA.B.5, 3.OA.B.6, 3.OA.C, 3.OA.C.7, 3.NBT.A, 3.NBT.A.3 5.NBT.B, 5.NBT.B.6, 5.NF.A, 5.NF.A.1	**Related Grade 3 Standards** **3.OA.A Represent and solve problems involving multiplication and division.** **3.OA.A.4** Determine the unknown whole number in a multiplication or division equation relating three whole numbers. **3.OA.B Understand properties of multiplication and the relationship between multiplication and division.** **3.OA.B.5** Apply properties of operations as strategies to multiply and divide. *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)* **3.OA.B.6:** Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.* **3.OA.C Multiply and divide within 100.** **3.OA.C.7** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers. **3.NBT.A Use place value understanding and properties of operations to perform multi-digit arithmetic.**

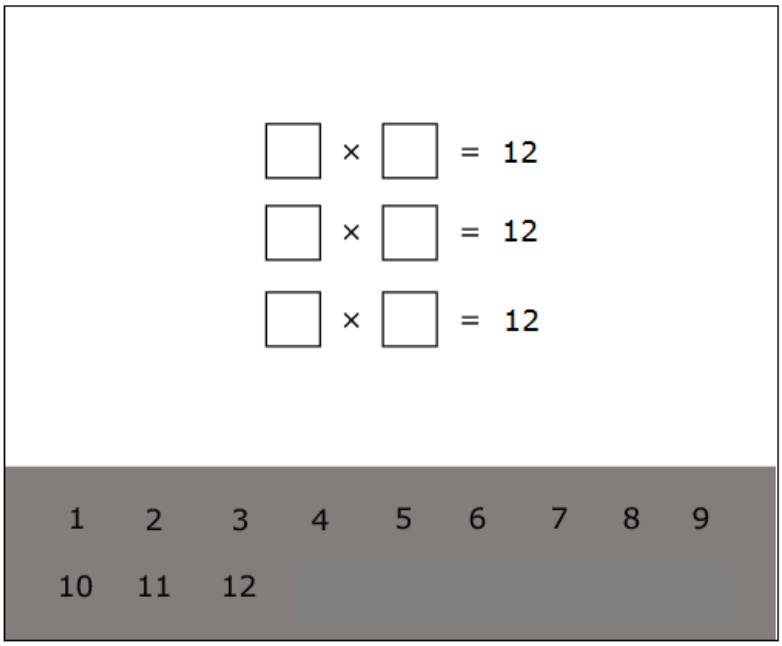
	3.NBT.A.3 Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations. **Related Grade 5 Standards** **5.NBT.B Perform operations with multi-digit whole numbers and with decimals to hundredths.** **5.NBT.B.6** Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **5.NF.A Use equivalent fractions as a strategy to add and subtract fractions.** **5.NF.A.1** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12.* (In general, $a/b + c/d = (ad + bc)/bd$.) Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies, including rounding.
DOK Levels:	1, 2
Achievement Level Descriptors:	
RANGE Achievement Level Descriptor (Range ALD) Target B: Gain familiarity with factors and multiples.	**Level 1** Students should be able to recognize that a whole number is a multiple of each of its factors.
	Level 2 Students should be able to find factor pairs for whole numbers in the range of 1–100 that are multiples of 2 or 5 and determine whether a given whole number in the range of 1–100 is a multiple of a given one-digit number.
	Level 3 Students should be able to find all factor pairs for whole numbers in the range of 1–100 and determine whether a given whole number in the range of 1–100 is prime or composite.
	Level 4 No Descriptor
Evidence Required:	1. The student determines one or more factors or factor pairs for a given whole number (from 1 to 100). 2. The student recognizes that a whole number (from 1 to 100) is a multiple of each of its factors. 3. The student determines if a whole number (from 1 to 100) is a multiple of a given one-digit number. 4. The student determines if a whole number (from 1 to 100) is prime or composite.

Allowable Response Types:	Multiple Choice, single correct response; Drag and Drop; Matching Tables; Hot Spot; Fill-in Table
Allowable Stimulus Materials:	whole numbers from 1 to 100, tables, lists, sets, 100s chart
Construct-Relevant Vocabulary:	whole number, prime, composite, factor, factor pair, multiple
Allowable Tools:	None
Target-Specific Attributes:	Numbers used in this target must be in the range 1 to 100.
Non-Targeted Constructs:	None
Accessibility Guidance:	Item writers should consider the following Language and Visual Element/Design guidelines[1] when developing items. Language Key Considerations: • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level • Avoid ambiguous or obscure words, idioms, jargon, unusual names and references Visual Elements/Design Key Considerations: • Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context • Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary • Avoid crowding of details and graphics Items are selected for a student's test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.[2]
Development Notes:	None

[1] For more information, refer to the General Accessibility Guidelines at: http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf

[2] For more information about student accessibility resources and policies, refer to http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

Task Model 1a-b **Response Type: Multiple Choice, single correct response** **DOK Level 1** **4.OA.B.4** Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite. **Evidence Required:** 1. The student determines one or more factors or factor pairs for a given whole number (from 1 to 100). **Tools:** None	**Prompt Features:** The student is prompted to identify factors of a whole number. **Stimulus Guidelines:** • All of the number's factors (or factor pairs) should be included in the key; distractors will include the same number of entries and will include numbers that are not factors (or factor pairs). • Numbers used are whole numbers within the range of 1–100. • Item difficulty can be adjusted via these example methods: o Changing the range of numbers used o Using prime numbers o Using numbers outside the 10 × 10 table **TM1a** **Stimulus:** The student is presented with a whole number and lists of whole number factors. **Example Stem:** Which list has **all** of the factors of 36? A. 1, 2, 3, 4, 6, 9, 12, 18, 36 B. 1, 2, 3, 6, 8, 9, 12, 16, 36 C. 1, 2, 4, 6, 8, 9, 16, 18, 36 D. 1, 2, 3, 5, 6, 9, 12, 18, 36 **TM1b** **Stimulus:** The student is presented with a whole number and lists of pairs of whole number factors. **Example Stem:** Which list has **all** of the factor pairs of 36? A. 1 × 36, 2 × 18, 3 × 12, 4 × 9, 6 × 6 B. 1 × 36, 2 × 16, 3 × 12, 4 × 9, 6 × 8 C. 1 × 36, 2 × 18, 3 × 6, 4 × 8, 6 × 12 D. 1 × 36, 2 × 12, 3 × 9, 4 × 8, 6 × 6 **Rubric:** (1 point) The student identifies the correct list (e.g., A; A). **Response Type:** Multiple Choice, single correct response

Task Model 1c **Response Type:** **Drag and Drop** **DOK Level 1** **4.OA.B.4** Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite. **Evidence Required:** 1. The student determines one or more factors or factor pairs for a given whole number (from 1 to 100). **Tools:** None **Accessibility Note:** Drag and Drop items are not currently able to be Brailled. Minimize the number of items developed to this TM.	**Prompt Features:** The student is prompted to identify factor pairs of a whole number. **Stimulus Guidelines:** • Numbers used in the presets are whole numbers within the range of 2–20. • The whole number presented in the stem is **not** 4, 9 or 16. **TM1c** **Stimulus:** The student is presented with a whole number in the range of 2–20 (excluding 4, 9, and 16) and a set of multiplication equations with boxes for all possible factor pairs, each having the given whole number as the product. **Example Stem:** Drag numbers into the boxes to make factor pairs of 12.  **Rubric:** (1 point) The student identifies all of the correct factor pairs (e.g., 1 × 12, 2 × 6, 3 × 4). **Response Type:** Drag and Drop

Task Model 1d **Response Type: Multiple Choice, single correct response** **DOK Level 1** **4.OA.B.4** Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite. **Evidence Required:** 1. The student determines one or more factors or factor pairs for a given whole number (from 1 to 100). **Tools:** None	**Prompt Features:** The student is prompted to identify a common factor of two numbers. **Stimulus Guidelines:** • Numbers used are whole numbers within the range of 1–100. **TM1d** **Stimulus:** The student is presented with two whole numbers within the range of 1–100. **Example Stem:** Which number is a factor of **both** 16 and 20? A. 3 B. 4 C. 5 D. 8 **Rubric:** (1 point) The student identifies a common factor (e.g., B). **Response Type:** Multiple Choice, single correct response

Task Model 1e **Response Type: Multiple Choice, single correct response** **DOK Level 1** **4.OA.B.4** Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite. **Evidence Required:** 1. The student determines one or more factors or factor pairs for a given whole number (from 1 to 100). **Tools:** None	**Prompt Features:** The student is prompted to identify common factors of two numbers. **Stimulus Guidelines:** • Numbers used are whole numbers within the range of 1–100. • Answer choices should be in the form of lists of three to four whole numbers. • Item difficulty can be adjusted via these example methods: o Changing the range of numbers used o Using numbers outside the 10 × 10 table **TM1e** **Stimulus:** The student is presented with two whole numbers. **Example Stem:** Which numbers are factors of **both** 18 and 45? A. 1, 2, 8 B. 1, 3, 9 C. 1, 4, 8 D. 1, 5, 9 **Rubric:** (1 point) The student identifies common factors of two whole numbers (e.g., B). **Response Type:** Multiple Choice, single correct response

Task Model 1f

Response Type:
Fill-in Table

DOK Level 1

4.OA.B.4
Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

Evidence Required:
1. The student determines one or more factors or factor pairs for a given whole number (from 1 to 100).

Tools: None

Prompt Features: The student is prompted to identify factor pairs of a whole number.

Stimulus Guidelines:

- A maximum of four equations/factor pairs should be required.
- Item difficulty can be adjusted via these example methods:
  - Number of factor pairs required
  - Familiarity with the number
  - Changing the range of numbers used
  - Using numbers outside the 10 × 10 table

TM1f
Stimulus: The student is presented with a whole number in the range of 1–100 and multiplication equations equaling that whole number with unknown factors in a fill-in table.

Example Stem: Enter numbers into the boxes to make four different factor pairs of 54.

	×		= 54
	×		= 54
	×		= 54
	×		= 54

Rubric: (1 point) The student identifies four different factor pairs of the number (e.g., 1 and 54, 2 and 27, 3 and 18, 6 and 9).

Note: Numbers may be in either order (e.g., 1 and 54 **or** 54 and 1).

Response Type: Fill-in Table

Task Model 1g-h **Response Types: Multiple Choice, single correct response and Equation / Numeric** **DOK Level 1** **4.OA.B.4** Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite. **Evidence Required:** 1. The student determines one or more factors or factor pairs for a given whole number (from 1 to 100). **Tools:** None **Version 3 Update:** Added two new task models TM1g and TM1h.	**Prompt Features:** The student is prompted to identify factors of a whole number. **Stimulus Guidelines:** • All of the number's factors (or factor pairs) should be included in the key; distractors will include the same number of entries and will include numbers that are not factors (or factor pairs). • Numbers used are whole numbers within the range of 1–100. • Item difficulty can be adjusted via these example methods: o Changing the range of numbers used o Using prime numbers o Using numbers outside the 10 × 10 table **TM1g** **Stimulus:** The student is presented with a whole number and lists of whole number factors with one factor missing. **Example Stem:** Which factor of 12 is missing in this list of numbers? 1, 2, 3, 4, __, 12 A. 5 B. 6 C. 8 D. 10 **Rubric:** (1 point) The student identifies the correct missing factor (e.g., B). **Response Type:** Multiple Choice, single correct response **TM1h** **Stimulus:** The student is presented with a whole number and lists of whole number factors with one factor missing. **Example Stem:** Which factor of 24 is missing in this list of numbers? 1, 2, 3, 4, 6, __, 12, 24 **Rubric:** (1 point) The student identifies the correct missing factor (e.g., 8). **Response Type:** Equation/Numeric

Task Model 2

Response Type: Hot Spot / Matching Tables

DOK Level 2

4.OA.B.4
Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

Evidence Required:
2. The student recognizes that a whole number (from 1 to 100) is a multiple of each of its factors.

Tools: None

Accessibility Note: Hot Spot items are not currently able to be Brailled, but the only current way to assign partial credit. Minimize the number of items developed to this TM using Hot Spot. An accessible alternate to Hot Spot for this TM is to use the Matching Tables format when partial credit scoring becomes available for this format.

Prompt Features: The student is prompted to match factors and multiples.

Stimulus Guidelines:

- All numbers are whole numbers within the range of 1–100.
- Item must include at least one number for each column (e.g., cannot have a column that should be left blank).
- Item difficulty can be adjusted via these example methods:
  - Using factors of 1, 2, or 5
  - Using factors of 3 or 4
  - Using factors of 6, 7, 8, or 9
  - Using prime numbers
  - Using numbers outside the 10 × 10 table

TM2
Stimulus: The student is presented with a one-digit whole number and six additional whole numbers.

Example Stem: Decide whether each number is a multiple of 6, a factor of 6, or neither. Each number may be matched to more than one description. Click in the table to respond.

	Multiple of 6	Factor of 6	Neither a Multiple Nor a Factor of 6
1			
2			
3			
6			
8			
12			

Rubric: (3 points) The student correctly fills in all three columns (e.g., Multiple column: 6, 12; Factor column: 1, 2, 3, 6; Neither column: 8), with no incorrect fill-ins.
(2 points) Partial credit is possible for correctly filling in two of the three columns (e.g., Multiple column and Neither column filled in correctly, but Factor column not filled in correctly), with no incorrect fill-ins on the two columns (e.g., Multiple and Neither).
(1 point) Partial credit is possible for correctly filling in one of the three columns (e.g., Multiple column filled in correctly but Factor and Neither column not filled in correctly), with no incorrect fill-ins on the one column (e.g., Multiple).

Response Type: Hot Spot or Matching Tables

Task Model 3 **Response Type: Multiple Choice, single correct response** **DOK Level 1** **4 OA.B.4.** Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite. **Evidence Required:** 3. The student determines if a whole number (from 1 to 100) is a multiple of a given one-digit number. **Tools:** None	**Prompt Features:** The student is prompted to identify multiples of a one-digit whole number. **Stimulus Guidelines:** • All numbers are whole numbers within the range of 1–100. • Answer choices should be a list of whole numbers that are multiples of the given number. • Item difficulty can be adjusted via these example methods: o Varying the factors (factors of 1, 2, or 5), (factors of 3 or 4), and (factors of 6, 7, 8, or 9) o Using correct answers that are less than or equal to ten times the given factor o Using correct answers that are greater than ten times the given factor **TM3a** **Stimulus:** The student is presented with a one-digit whole number. **Example Stem:** Select the list of numbers that are **all** multiples of 8. A. 8, 18, 24, 44 B. 8, 26, 44, 62 C. 16, 32, 48, 64 D. 40, 48, 54, 76 **Rubric:** (1 point) The student correctly identifies the multiples of a one-digit number (e.g., C). **Response Type:** Multiple Choice, single correct response

Task Model 3 **Response Type: Multiple Choice, single correct response** **DOK Level 1** **4.OA.B.4** Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite. **Evidence Required:** 3. The student determines if a whole number (from 1 to 100) is a multiple of a given one-digit number. **Tools:** None	**Prompt Features:** The student is prompted to identify common multiples of two one-digit whole numbers. **Stimulus Guidelines:** • All numbers are whole numbers within the range of 1–100. • Item difficulty can be adjusted via these example methods: o Varying the factors (factors of 1, 2, or 5), (factors of 3 or 4), and (factors of 6, 7, 8, or 9) o Using correct answers that are less than or equal to ten times the given factor o Using correct answers that are greater than ten times the given factor **TM3b** **Stimulus:** The student is presented with two one-digit whole numbers. **Example Stem:** Which number is a multiple of **both** 2 and 5? A. 7 B. 14 C. 40 D. 52 **Rubric:** (1 point) The student correctly identifies the multiple of two one-digit numbers (e.g., C). **Response Type:** Multiple Choice, single correct response

Task Model 4

Response Type: Matching Tables

DOK Level 2

4.OA.B.4
Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

Evidence Required:
4. The student determines if a whole number (from 1 to 100) is prime or composite.

Tools: None

Prompt Features: The student is prompted to identify numbers as prime or composite.

Stimulus Guidelines:

- All numbers are whole numbers within the range of 1–100.
- Item difficulty can be adjusted via these example methods:
  - Prime numbers less than vs. greater than 50
  - Composite numbers with factors that are less than 10 vs. greater than 10

TM4
Stimulus: The student is presented with three whole numbers.

Example Stem: Decide whether each number is prime or composite. Click in the table to respond.

	Prime	Composite
17		
52		
87		

Rubric: (1 point) The student identifies all numbers as prime or composite (e.g., P, C, C).

Response Type: Matching Tables

Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.	
Content Domain: **Operations and Algebraic Thinking**	
Target C [a]: Generate and analyze patterns. (DOK 2, 3) Tasks for this target will ask students to generate and analyze number and shape patterns. Analyses should include explanations of features of the pattern (other than the rule itself).	
Standards: 4.OA.C, 4.OA.C.5	**4.OA.C Generate and analyze patterns.** **4.OA.C.5** Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*
Related Below-Grade and Above-Grade Standards for Purposes of Planning for Vertical Scaling: 3.OA.D, 3.OA.D.9 5.OA.B, 5.OA.B.3	**Related Grade 3 Standards** **3.OA.D Solve problems involving the four operations, and identify and explain patterns in arithmetic.** **3.OA.D.9** Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.* **Related Grade 5 Standards** **5.OA.B Analyze patterns and relationships.** **5.OA.B.3** Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*
DOK Levels:	2, 3
Achievement Level Descriptors:	
RANGE Achievement Level Descriptor (Range ALD) Target C: Generate and analyze patterns.	**Level 1** Students should be able to extend a number or shape pattern that follows a given rule.
	Level 2 Students should be able to generate a number or shape pattern that follows a given rule.
	Level 3 Students should be able to analyze a pattern for apparent features that are not explicit in the rule itself.
	Level 4 No Descriptor.

Evidence Required:	1. The student generates number patterns. 2. The student generates shape patterns. 3. The student analyzes a number pattern or shape pattern, showing understanding of the pattern rule and features other than the pattern rule.
Allowable Response Types:	Drag and Drop; Hot Spot; Fill-in Table; Matching Tables; Equation/Numeric; Multiple Choice, single correct response
Allowable Stimulus Materials:	whole number patterns using all four operations, fraction patterns using addition and subtraction with like denominators (limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12 and 100), shape patterns with two-dimensional figures or pictures of objects Limitations on numbers in number patterns: • Multiplication should never exceed 4-digit by 1-digit or 2-digit by 2-digit. • Multiplication should never involve decimals, but may involve multiplying a whole number by a fraction. • Division should never exceed 4-digit by 1-digit. • Division should never involve fractions or decimals.
Construct-Relevant Vocabulary:	pattern
Allowable Tools:	None
Target-Specific Attributes:	None
Non-Targeted Constructs:	None
Accessibility Guidance:	Item writers should consider the following Language and Visual Element/Design guidelines[1] when developing items. Language Key Considerations: • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level • Avoid ambiguous or obscure words, idioms, jargon, unusual names and references Visual Elements/Design Key Considerations: • Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context • Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary • Avoid crowding of details and graphics

[1] For more information, refer to the General Accessibility Guidelines at: http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf

	Items are selected for a student's test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.[2]
Development Notes:	None

[2] For more information about student accessibility resources and policies, refer to http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

Task Model 1

Response Type:
Fill-in Table

DOK Level 2

4.OA.C.5
Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

Evidence Required:
1. The student generates number patterns.

Tools: None

Prompt Features: The student is prompted to generate up to five terms of a number pattern.

Stimulus Guidelines:

- Pattern rules fit the form of "[basic operation] [number]" (e.g., "add 2," "multiply by 3").
- Item difficulty can be adjusted via these example methods:
  - Addition or subtraction patterns involving whole numbers
  - Addition or subtraction patterns involving fractions
  - Multiplication or division patterns involving whole numbers
  - Multiplication patterns involving fractions (multiplying a fraction by a whole number)

TM1
Stimulus: The student is presented with a number pattern rule and starting number.

Example Stem: A pattern is generated using this rule: Start with the number 7 as the first term and add 5.

Enter numbers into the boxes to complete the table.

Term	Number
First	7
Second	
Third	
Fourth	
Fifth	

Rubric: (1 point) The student enters the correct numbers (e.g., 12, 17, 22, 27).

Response Type: Fill-in Table

Task Model 2a

Response Type: Matching Tables

DOK Level 2

4.OA.C.5
Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

Evidence Required:
2. The student generates shape patterns.

Tools: None

Prompt Features: The student is prompted to identify up to six terms of a shape pattern.

Stimulus Guidelines:

- Item difficulty can be adjusted via these example methods:
  - A simple repetitive shape pattern involving no more than four shapes
  - Shapes that grow, diminish, or rotate
  - A combination shape/number pattern involving a shape rule (as described above) and a number rule (as described in TM1)

TM2a
Stimulus: The student is presented with a shape pattern rule.

Example Stem: A shape pattern is generated by repeating the pattern of "Star, Circle, Circle."
Click in the table to show the first six terms of the pattern, starting with Star.

	Star	Circle
First term		
Second term		
Third term		
Fourth term		
Fifth term		
Sixth term		

Rubric: (1 point) The student correctly identifies the first six terms of the pattern (e.g., Star, Circle, Circle, Star, Circle, Circle).

Response Type: Matching Tables

Task Model 2b

Response Type: Multiple choice, single correct response

DOK Level 2

4.OA.C.5
Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

Evidence Required:
2. The student generates shape patterns.

Tools: None

Prompt Features: The student is prompted to identify a specific term of a shape pattern.

Stimulus Guidelines: Same as for TM2a.

TM2b
Stimulus: The student is presented with a shape pattern rule.

Example Stem: A shape pattern is generated by repeating the pattern of "Star, Circle, Square, Triangle" as shown.

							
First term	Second term	Third term	Fourth term	Fifth term	Sixth term	Seventh term	Eighth term

This pattern continues for 100 terms. Select the shape that represents the 98th term.

A. 
B. 
C. 
D. 

Rubric: (1 point) The student correctly identifies the indicated term (e.g., B).

Response Type: Multiple Choice, single correct response

Task Model 3a-b

Response Type: Matching Tables

DOK Level 3

4.OA.C.5
Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

Evidence Required:
3. The student analyzes a number pattern or shape pattern, showing understanding of the pattern rule and features other than the pattern rule.

Tools: None

Prompt Features: The student is prompted to identify features of a number pattern.

Stimulus Guidelines:

- Pattern rules should fit the form of "[*basic operation*] [*number*]" (e.g., "add 2," "multiply by 3").
- Item difficulty can be adjusted via this example method:
  - Whether the answer choice statements refer to terms in general or to specific terms
- Items should be equally distributed across these types:
  - Addition or subtraction patterns involving whole numbers
  - Addition or subtraction patterns involving fractions
  - Multiplication or division patterns involving whole numbers
  - Multiplication patterns involving fractions (multiplying a fraction by a whole number)

TM3a
Stimulus: The student is presented with a number pattern rule and starting number.

Example Stem: A pattern is generated using this rule: Start with the number 5 as the first term and add 2.
Select True or False for each statement about the pattern.

	True	False
The terms alternate between even and odd numbers.		
Each term is greater than the term before it.		
All possible multiples of 5 are terms in the pattern.		

Rubric: (1 point) The student correctly identifies all three statements as True or False (e.g., F, T, F).

Response Type: Matching Tables

Task Model 3a-b

Response Type: Matching Tables

DOK Level 3

4.OA.C.5
Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

Evidence Required:
3. The student analyzes a number pattern or shape pattern, showing understanding of the pattern rule and features other than the pattern rule.

Tools: None

Version 3 Update:
Edited TM3b to show four terms of the pattern and changed the questions in the table.

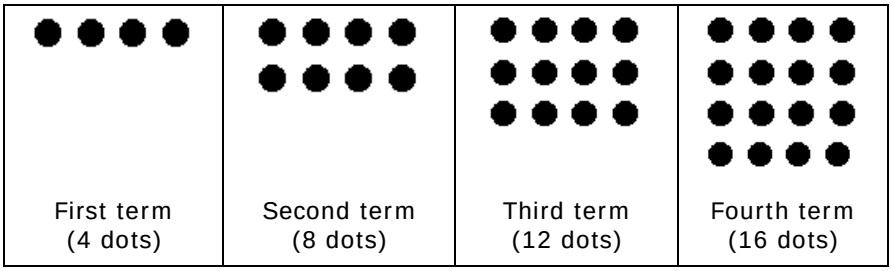
Prompt Features: The student is prompted to identify features of a shape pattern.

Stimulus Guidelines:

- Statements describe features of the pattern, not the rule of the pattern.
- True statements will name a strategy that can be used to find the *n*th term that is *not* a direct application of the rule itself.
- Item difficulty can be adjusted via this example method:
  - Whether the answer choice statements refer to terms in general or to specific terms
- Items should be equally distributed across these types:
  - Simple repetitive shape pattern involving no more than three shapes
  - Shapes that grow, diminish, or rotate
  - A combination shape/number pattern involving a shape rule (as described above) and a number rule (as described in TM3a)

TM3b
Stimulus: The student is presented with a shape pattern rule.

Example Stem: The first four terms of a shape pattern are shown. Each term is generated by following the same rule.



First term (4 dots)	Second term (8 dots)	Third term (12 dots)	Fourth term (16 dots)

Decide whether each statement can be used to describe the dot pattern shown. Select Yes or No for each statement.

	Yes	No
The difference between the number of dots in each term is 8.		
The number of dots in the 7^{th} term is 28.		
The digit in the ones place of the number of dots repeats in the following pattern: 4, 8, 2, 6, 0.		

Rubric: (1 point) The student correctly selects yes or no for each method (e.g., N, Y, Y).

Response Type: Matching Tables

Task Model 3c **Response Type: Equation / Numeric** **DOK Level 3** **4.OA.C.5** Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.* **Evidence Required:** 3. The student analyzes a number pattern or shape pattern, showing understanding of the pattern rule and features other than the pattern rule. **Tools:** None	**Prompt Features:** The student is prompted to analyze features of a number pattern. **Stimulus Guidelines:** • All items are limited to addition or multiplication of whole numbers only. • Item difficulty can be adjusted via this example method: o Whether the student is prompted to consider the pattern's terms in general or specific terms in the pattern • Number pattern rules fit the form of "[*basic operation*] [*number*]" (e.g., "add 2," "multiply by 3"). **TM3c** **Stimulus:** The student is presented with a rule and starting number. **Example Stem:** A pattern is generated using this rule: Start with 42 and add 5. Enter one number in each response box that makes this sentence correct: The ones digit for every term in the pattern is either ___ or ___. **Rubric:** (1 point) The student correctly names the values between which the identified place's digits alternate (e.g., 2, 7). **Response Type:** Equation/Numeric (2 response boxes)

Task Model 3d

Response Types: Drag and Drop; Hot Spot

DOK Level 3

4.OA.C.5
Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

Evidence Required:
3. The student analyzes a number pattern or shape pattern, showing understanding of the pattern rule and features other than the pattern rule.

Tools: None

Accessibility Note:
Drag and Drop and Hot Spot items are not currently able to be Brailled. Minimize the number of items developed to this TM.

Prompt Features: The student is prompted to analyze features of a number pattern.

Stimulus Guidelines:

- All items are limited to addition or multiplication of whole numbers only.
- Item difficulty can be adjusted via this example method:
  - Whether the student is prompted to consider the pattern's terms in general or specific terms in the pattern
- Number pattern rules fit the form of "[*basic operation*] [*number*]" (e.g., "add 2," "multiply by 3").

TM3d
Stimulus: The student is presented with a rule and starting number.

Example Stem: A pattern is generated using this rule: Start with the number 7 as the first term and add 5.

Part A: Drag numbers into the boxes to show the next six terms of this pattern.

7						
First term	Second term	Third term	Fourth term	Fifth term	Sixth term	Seventh term

Part B: Based on what you observe about the first seven terms, which numbers below are also in the pattern? Select **all** of the numbers that are in the pattern.

377 955 1022 9992

Interaction: In Part A, the student drags digits 0-9 to boxes to create the next six terms in the number pattern. In Part B, the student clicks on the numbers that are in the pattern.

Rubric:
Part A: (1 point) The student correctly names the next six terms of the pattern (e.g., 12, 17, 22, 27, 32, 37).

Response Type: Drag and Drop

Part B: (1 point) The student correctly identifies other terms from the pattern (e.g., 377, 1022, 9992).

Response Type: Hot Spot

Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.	
Content Domain: **Numbers and Operations in Base Ten**	
Target D [m]: Generalize place value understanding for multi-digit whole numbers. (DOK 1, 2) Tasks for this target will ask students to compare multi-digit numbers using >, =, and <. Tasks should tap into students' understanding of place value (e.g., by asking students to give a possible digit for the empty box in 4357 < 43□9 that would make the inequality true). A smaller number of these tasks will incorporate student understanding of rounding (e.g., explaining why rounding to a certain place would change the symbol < or > to =). In Claims 2–4, students should see contextual problems associated with this target that highlight issues with precision, including problems in Claim 3 that ask students to explain how improper estimation can create unacceptable levels of precision and/or lead to flawed reasoning.	
Standards: 4.NBT.A, 4.NBT.A.1, 4.NBT.A.2, 4.NBT.A.3	**4.NBT.A Generalize place value understanding for multi-digit whole numbers.** **4.NBT.A.1** Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.* **4.NBT.A.2** Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. **4.NBT.A.3** Use place value understanding to round multi-digit whole numbers to any place.
Related Below-Grade and Above-Grade Standards for Purposes of Planning for Vertical Scaling: 3.NBT.A, 3.NBT.A.1 5.NBT.A, 5.NBT.A.1	**Related Grade 3 Standards** **3.NBT.A Use place value understanding and properties of operations to perform multi-digit arithmetic.** **3.NBT.A.1** Use place value understanding to round whole numbers to the nearest 10 or 100. **Related Grade 5 Standards** **5.NBT.A Understand the place value system.** **5.NBT.A.1** Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*
DOK Levels:	1, 2

Achievement Level Descriptors:	
RANGE Achievement Level Descriptor (Range ALD) Target D: Generalize place value understanding for multi-digit whole numbers.	**Level 1** Students should be able to read and write multi-digit whole numbers less than or equal to 1000 using base-ten numerals, number names, and expanded form; compare multi-digit numbers up to 1000 using <, >, and =; and round multi-digit whole numbers up to 1000 to any place.
	Level 2 Students should look for and use repeated reasoning to generalize place value understanding to be able to read and write multi-digit whole numbers less than or equal to 100,000 using base-ten numerals, number names, and expanded form; compare multi-digit numbers up to 100,000 using <, >, and =; and round multi-digit whole numbers up to 100,000 to any place.
	Level 3 Students should look for and use repeated reasoning to generalize place value understanding to be able to read and write multi-digit whole numbers less than or equal to 1,000,000 using base-ten numerals, number names, and expanded form; compare multi-digit numbers up to 1,000,000 using <, >, and =; round multi-digit whole numbers up to 1,000,000 to any place; and recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.
	Level 4 No Descriptor
Evidence Required:	1. The student compares two multi-digit whole numbers in the same form using >, <, and = symbols. 2. The student rounds multi-digit whole numbers to any place. 3. The student identifies multi-digit whole numbers that, when rounded to a given place value, will be closest to a given number. 4. The student compares two multi-digit whole numbers in different forms. 5. The student explains the difference between the values of a numeral in the tens and the ones place, the hundreds place and the tens place, or the thousands place and the hundreds place in mathematical situations.
Allowable Response Types:	Multiple Choice, single correct response; Multiple choice, multiple correct responses; Matching Table; Equation/Numeric
Allowable Stimulus Materials:	• Multi-digit whole numbers less than or equal to 1,000,000 in any of these forms: o Numeric form (e.g., 427) o Expanded form (e.g., 400 + 20 + 7) o "Expanded word" form (e.g., 4 hundreds + 2 tens + 7 ones) • Comparisons using <, >, or = • Numbers that include a 0 in one or more place values

Construct-Relevant Vocabulary:	nearest ten, nearest hundred, nearest thousand, nearest ten thousand, nearest hundred thousand, ones, tens, hundreds, thousands, ten thousands, hundred thousands, millions
Allowable Tools:	None
Target-Specific Attributes:	Items will include multi-digit whole numbers less than or equal to 1,000,000.
Non-Targeted Constructs:	None
Accessibility Guidance:	Item writers should consider the following Language and Visual Element/Design guidelines[1] when developing items. Language Key Considerations: • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level • Avoid ambiguous or obscure words, idioms, jargon, unusual names and references Visual Elements/Design Key Considerations: • Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context • Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary • Avoid crowding of details and graphics Items are selected for a student's test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.[2]
Development Notes:	In Claims 2–4, students should see contextual problems associated with this target that highlight issues with precision, including problems in Claim 3 that ask students to explain how improper estimation can create unacceptable levels of precision and/or lead to flawed reasoning. A small number of Claim 3 tasks will incorporate student understanding of rounding (e.g., explaining why rounding to a certain place would change the symbol < or > to =).

[1] For more information, refer to the General Accessibility Guidelines at: http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf

[2] For more information about student accessibility resources and policies, refer to http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

Task Model 1a

Response Type: Matching Table

DOK Level 1

4.NBT.A.2
Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

Evidence Required:
1. The student compares two multi-digit whole numbers in the same form using >, <, and = symbols.

Tools: None

Version 3 Update:
Changed TM1a from an equation/numeric response type to a matching table response type. Updated the stimulus and stem to match the new format. Retired TM1b (redundant due to new response type).

Prompt Features: The student is prompted to compare place values in two pairs of whole numbers.

Stimulus Guidelines:

- Items should be equally distributed across these number bands: small numbers (up to 1,000), medium numbers (from 1,000 up to 100,000), and large numbers (from 100,000 up to 1,000,000).
- Items should be equally distributed across these item types:
  - Numbers given in numeric form
  - Numbers given in expanded form
- Item difficulty can be adjusted by changing the number of digits in the numbers being compared.

TM1a
Stimulus: The student is presented with two pairs of multi-digit whole numbers and directed to compare them using (<, >, or =).

Example Stem:
Select the symbol (<, >, or =) that correctly compares each pair of numbers.

	<	>	=
6,285 ☐ 6,258			
47,385 ☐ 47,299			

Rubric: (1 point) The student selects the correct symbols (e.g., >, >).

Response Type: Matching Table

Task Model 1c
 	
Response Type: Equation / Numeric
 	
DOK Level 2
 	
4.NBT.A.2
Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
 	
Evidence Required:
1. The student compares two multi-digit whole numbers in the same form using >, <, and = symbols.
 	
Tools: None
 	
Version 3 Update:
Clarified the wording of the stem in TM1c.
	Prompt Features: The student is prompted to compare two numbers.
	
	Stimulus Guidelines:
	• In any comparison, numbers have the same number of place values.
	• Items should be equally distributed across these number bands: small numbers (up to 1,000), medium numbers (from 1,000 up to 100,000), and large numbers (from 100,000 up to 1,000,000).
	• Digits in place values to the left of the unknown digit should be the same. Digits to the right of the unknown digit may be different.
	• Numbers should be chosen so that there are no more than three correct digits possible.
	• Item difficulty can be adjusted via these example methods:
	  o The location of the unknown digit within the number
	  o Which number has the unknown digit
	
	TM1c
	Stimulus: The student is presented with two multi-digit whole numbers in numeric form. One number has a box to represent an unknown digit.
	
	Example Stem: Identify a digit that, when placed in the box (□), makes this comparison true.
	
	524,9□7 < 524,932
	
	Enter the digit in the response box.
	
	Rubric: (1 point) The student enters a digit to create a correct comparison (e.g., 2, 1, or 0).
	
	Response Type: Equation/Numeric

Task Model 1d **Response Type: Equation / Numeric** **DOK Level 2** **4.NBT.A.2** Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. **Evidence Required:** 1. The student compares two multi-digit whole numbers in the same form using >, <, and = symbols. **Tools:** None **Version 3 Update:** Added new TM1d.	**Prompt Features:** The student is prompted to generate a number using given digits that is larger or smaller than a given number. **Stimulus Guidelines:** • In any comparison, numbers have the same number of place values. • Digits used should be unique and create only one correct possibility. • Items should be equally distributed across these number bands: small numbers (up to 1,000), medium numbers (from 1,000 up to 100,000), and large numbers (from 100,000 up to 1,000,000). **TM1d** **Stimulus:** The student is prompted to generate a multi-digit number from a given set of digits to create a larger number for the given context. **Example Stem:** Joe and Kate were playing a number game with the following four cards. 3 1 7 6 The winner of the game is the person that makes the number with the greatest value. Joe made the number 6731. Using the same cards, what number could Kate make to win the game? **Rubric:** (1 point) The student enters a number greater than 6731 (e.g., 7631, 7613, 7361, 7316, 7163, 7136, etc.). **Response Type:** Equation/Numeric

Task Model 2 **Response Type: Equation / Numeric** **DOK Level 1** **4.NBT.A.3** Use place value understanding to round multi-digit whole numbers to any place. **Evidence Required:** 2. The student rounds multi-digit whole numbers to any place. **Tools:** None	**Prompt Features:** The student is prompted to round a whole number. **Stimulus Guidelines:** • Items should be equally distributed across these number bands: small numbers (up to 1,000), medium numbers (from 1,000 up to 100,000), and large numbers (from 100,000 up to 1,000,000). • Item difficulty can be adjusted via these example methods: o Whether the focus is on rounding to the greatest place value, the second greatest place value, or any other place value in the number o The value of digits beyond the next place value to the right (e.g., when rounding 56,489 to the nearest thousand, a student must consider only the 4, which will round it down, despite all of the other digits being 5 or greater) **TM2** **Stimulus:** The student is presented with a multi-digit whole number in numeric form and the name of a place value to which the number should be rounded. **Example Stem 1:** Round 4108 to the nearest thousand. Enter your answer in the response box. **Example Stem 2:** Round 658,749 to the nearest ten thousand. Enter your answer in the response box. **Rubric:** (1 point) The student enters the number correctly rounded to the nearest designated place value (e.g., 4000; 660,000). **Response Type:** Equation/Numeric

Task Model 3a **Response Type: Equation / Numeric** **DOK Level 2** **4.NBT.A.3** Use place value understanding to round multi-digit whole numbers to any place. **Evidence Required:** 3. The student identifies multi-digit whole numbers that, when rounded to a given place, will be closest to a given number. **Tools:** None	**Prompt Features:** The student is prompted to identify the smallest or largest number that rounds to a given number. **Stimulus Guidelines:** • Items should be equally distributed across these number bands: small numbers (up to 1,000), medium numbers (from 1,000 up to 100,000), and large numbers (from 100,000 up to 1,000,000). • Item difficulty can be adjusted via this example method: o Whether the focus is on rounding to the greatest place value, the second greatest place value, or any other place value in the number **TM3a** **Stimulus:** The student is presented with a multi-digit whole number in numeric form and the name of a place value. **Example Stem:** When rounding to the nearest thousand, what is the least whole number that rounds to 16,000? Enter your answer in the response box. **Rubric:** (1 point) The student enters the correct number (e.g., 15,500). **Response Type:** Equation/Numeric

Task Model 3b-c

Response Types: Matching Tables, Multiple Choice, multiple correct responses

DOK Level 1

4.NBT.A.3
Use place value understanding to round multi-digit whole numbers to any place.

Evidence Required:
3. The student identifies multi-digit whole numbers that, when rounded to a given place, will be closest to a given number.

Tools: None

Version 3 Update:
Added new TM3c.

Prompt Features: The student is prompted to identify the numbers that round to a given number.

Stimulus Guidelines: Same as for TM3a.

TM3b
Stimulus: The student is presented with a multi-digit whole number in numeric form.

Example Stem: When rounding to the nearest thousand, which numbers round to 16,000?

Select Yes if the number rounds to 16,000. Select No if the number does **not** round to 16,000.

	Yes	No
15,179		
16,523		
15,545		

Rubric: (1 point) The student correctly identifies whether three numbers round to a given number (e.g., N, N, Y).

Response Type: Matching Tables

TM3c
Stimulus: The student is presented with a multi-digit whole number in numeric form.

Example Stem: When rounding to the nearest thousand, select **all** numbers that round to 16,000.

A. 16,204
B. 15,179
C. 16,523
D. 15,545

Rubric: (1 point) The student correctly identifies the numbers that round to a given number (e.g., A, D).

Response Type: Multiple choice, multiple correct responses

Task Model 4a **Response Type: Matching Tables** **DOK Level 2** **4.NBT.A.2** Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. **Evidence Required:** 4. The student compares two multi-digit whole numbers in different forms. **Tools:** None	**Prompt Features:** The student is prompted to compare two numbers in different forms. **Stimulus Guidelines:** • Items should be equally distributed across these number bands: small numbers (up to 1,000), medium numbers (from 1,000 up to 100,000), and large numbers (from 100,000 up to 1,000,000). • Within each comparison, multi-digit numbers are represented in two of these forms: o Numeric form (e.g., 625) o Expanded form (e.g., 600 + 20 + 5) o "Expanded word" form with a mix of numerals and place value names (e.g., 6 hundreds + 2 tens + 5 ones) • Items should be equally distributed across these types: o The numbers require distinguishing between small and large numbers in different place values (e.g., 398 and 400 + 20 + 5). o The numbers have one place value that differs (e.g., 3 hundreds + 8 tens + 5 ones and 300 + 90 + 5). o The numbers contain the same digits in different place value locations (e.g., 3 hundreds + 8 ones + 9 tens and 300 + 80 + 9). • Item difficulty can be adjusted via these example methods: o Whether numbers are represented in numeric form, expanded form, or "expanded word" form o In expanded and "expanded word" forms, whether the place values are presented in order ▪ In place value order (e.g., 600 + 20 + 5, 6 hundreds + 2 tens + 5 ones) ▪ Not in place value order (e.g., 20 + 600 + 5, 2 tens + 6 hundreds + 5 ones) o In "expanded word" form, how precisely the number is presented ▪ Precise (e.g., 6 hundreds + 2 tens + 5 ones) ▪ Low degree of imprecision (e.g., 5 hundreds + 12 tens + 5 ones) ▪ High degree of imprecision (e.g., 4 hundreds + 18 tens + 45 ones)

Task Model 4a

Response Type: Matching tables

DOK Level 2

4.NBT.A.2
Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

Evidence Required:
4. The student compares two multi-digit whole numbers in different forms.

Tools: None

Version 3 Update:
Retired TM4b.

TM4a

Stimulus: The student is presented with three comparisons of two multi-digit whole numbers in different forms.

Example Stem: Select True or False for each comparison.

	True	False
5 hundreds + 4 tens > 50 + 400		
524 < 50 + 200 + 4		
50 tens + 20 ones = 520		

Rubric: (1 point) The student correctly identifies three comparisons as true or false (e.g., T, F, T).

Response Type: Matching Tables

Task Model 5a **Response Type: Multiple Choice, single correct response** **DOK Level 1** **4.NBT.A.1** Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.* **Evidence Required:** 5. The student explains the difference between the values of a numeral in the tens and the ones place, the hundreds place and the tens place, or the thousands place and the hundreds place in mathematical situations. **Tools:** None	**Prompt Features:** The student is prompted to explain the difference between the values of the same digit in different place values. **Stimulus Guidelines:** • Follow any stated guidelines on allowable number ranges. • In each item, the numbers presented differ by a factor of 10 (e.g., 8 and 80, or 1725 and 17,250). • Items should be equally distributed across these number bands: small numbers (up to 1,000), medium numbers (from 1,000 up to 100,000), and large numbers (from 100,000 up to 1,000,000). • Answer choices should be in the form of sentences that use multiplication to explain how the values of the numbers differ. • Item difficulty can be adjusted via this example method: o Whether the student is prompted to consider the same digit in the tens and ones places, the hundreds and tens places, or the thousands and hundreds places (e.g., the 7 in 720 vs. the 7 in 72) **TM5a** **Stimulus:** The student is presented with two multi-digit whole numbers. **Example Stem:** Select the statement that explains how the values of the numbers 420 and 4200 are different. A. 4200 is 1000 times as large as 420. B. 4200 is 100 times as large as 420. C. 4200 is 10 times as large as 420. D. 4200 is 1 time as large as 420. **Rubric:** (1 point) The student selects the correct statement (e.g., C). **Response Type:** Multiple Choice, single correct response

Task Model 5b **Response Type: Equation / Numeric** **DOK Level 1** **4.NBT.A.1** Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.* **Evidence Required:** 5. The student explains the difference between the values of a numeral in the tens and the ones place, the hundreds place and the tens place, or the thousands place and the hundreds place in mathematical situations. **Tools:** None **Version 3 Update:** Added new TM5b.	**Prompt Features:** The student is prompted to determine difference between the values of the same digit in different place values. **Stimulus Guidelines:** Same as TM5a. **TM5b** **Stimulus:** The student is presented with two multi-digit whole numbers in context. **Example Stem:** Jim and Tom collected empty soda cans to raise money. Jim collected 70 cans and Tom collected 700 cans. The number of cans Tom collected is how many times greater than the number of cans Jim collected? Enter your answer in the response box. **Rubric:** (1 point) The student enters the correct value (e.g., 10). **Response Type:** Equation/Numeric

Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.	
Content Domain: **Numbers and Operations in Base Ten**	
Target E [m]: Use place value understanding and properties of operations to perform multi-digit arithmetic. (DOK 1, 2) Tasks for this target will ask students to add and subtract multi-digit whole numbers; multiply whole numbers (up to and including four digits by one digit or two digits by two digits); and find whole number quotients and remainders (up to four-digit dividends and one-digit divisors). When possible, the focus should be on the strategies students use when solving multiplication and division problems. *Note: Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.	
Standards: 4.NBT.B, 4.NBT.B.4, 4.NBT.B.5, 4.NBT.B.6	**4.NBT.B Use place value understanding and properties of operations to perform multi-digit arithmetic.** **4.NBT.B.4** Fluently add and subtract multi-digit whole numbers using the standard algorithm. **4.NBT.B.5** Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **4.NBT.B.6** Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
Related Below-Grade and Above-Grade Standards for Purposes of Planning for Vertical Scaling: 3.OA.A, 3.OA.A.3, 3.OA.A.4, 3.OA.B, 3.OA.B.5, 3.OA.C, 3.OA.C.7, 3.OA.D, 3.OA.D.8, 3.NBT.A, 3.NBT.A.2, 3.NBT.A.3 5.NBT.B, 5.NBT.B.5, 5.NBT.B.6	**Related Grade 3 Standards** **3.OA.A Represent and solve problems involving multiplication and division.** **3.OA.A.3** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. **3.OA.A.4** Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = ☐ ÷ 3, 6 × 6 = ?.* **3.OA.B Understand properties of multiplication and the relationship between multiplication and division.**

	3.OA.B.5 Apply properties of operations as strategies to multiply and divide. *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)* **3.OA.C Multiply and divide within 100.** **3.OA.C.7** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers. **3.OA.D Solve problems involving the four operations, and identify and explain patterns in arithmetic.** **3.OA.D.8** Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. **3.NBT.A Use place value understanding and properties of operations to perform multi-digit arithmetic.** **3.NBT.A.2** Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. **3.NBT.A.3** Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations. **Related Grade 5 Standards** **5.NBT.B Perform operations with multi-digit whole numbers and with decimals to hundredths.** **5.NBT.B.5** Fluently multiply multi-digit whole numbers using the standard algorithm. **5.NBT.B.6** Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and

	explain the calculation by using equations, rectangular arrays, and/or area models.
DOK Levels:	1, 2
Achievement Level Descriptors:	
RANGE Achievement Level Descriptor (Range ALD) Target E: Use place value understanding and properties of operations to perform multi-digit arithmetic.	**Level 1** Students should be able to add and subtract one- and two-digit whole numbers using strategies based on place value; multiply two one-digit whole numbers based on place value and properties of operations; and find whole-number quotients with no remainders with up to two-digit dividends and one-digit divisors using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division.
	Level 2 Students should be able to use place value understanding to add and subtract two- and three-digit whole numbers using a standard algorithm; multiply whole numbers up to and including four digits by one digit based on place value and properties of operations; find whole-number quotients and remainders with up to two-digit dividends and one-digit divisors using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division; and illustrate multiplication and division by using equations, arrays, and/or area models.
	Level 3 Students should be able to fluently add and subtract multi-digit whole numbers using the standard algorithm; multiply whole numbers including two digits by two digits based on place value and properties of operations; find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors using strategies based on place value understanding, the properties of operations, and/or the relationship between multiplication and division; and explain multiplication and division using equations, arrays, and/or area models.
	Level 4 No Descriptor
Evidence Required:	1. The student adds or subtracts multi-digit whole numbers in non-contextual mathematics problems. 2. The student multiplies whole numbers (up to four digits by one digit or two digits by two digits) using strategies based on place value and the properties of operations. 3. The student finds whole numbers quotients and remainders (up to four-digit dividends and one-digit divisors) using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division.
Allowable Response Types:	Multiple Choice, single correct response; Equation/Numeric
Allowable Stimulus Materials:	Non-contextual problems in the four operations, equations, expressions, problems solved or partially solved reflecting different solution strategies
Construct-Relevant Vocabulary:	sum, difference, product, expression, equation, equal, partial product, quotient, partial quotient, remainder, multiple

Allowable Tools:	None
Target-Specific Attributes:	None
Non-Targeted Constructs:	None
Accessibility Guidance:	Item writers should consider the following Language and Visual Element/Design guidelines[1] when developing items. Language Key Considerations: • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level • Avoid ambiguous or obscure words, idioms, jargon, unusual names and references Visual Elements/Design Key Considerations: • Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context • Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary • Avoid crowding of details and graphics Items are selected for a student's test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.[2]
Development Notes:	Illustrating and explaining multiplication calculations (CCSS 4.NBT.B.5) and division calculations (CCSS 4.NBT.B.6) by using equations, rectangular arrays, and/or area models will be assessed in Claim 3.

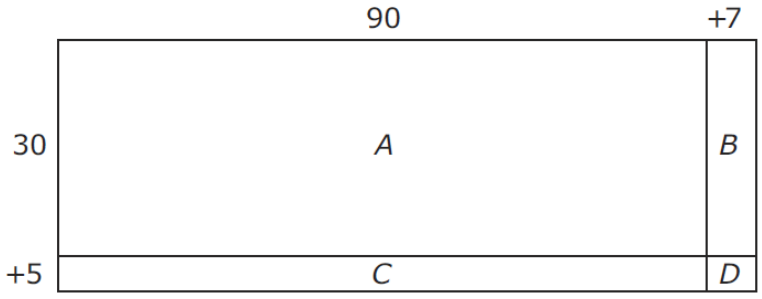
[1] For more information, refer to the General Accessibility Guidelines at: http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf

[2] For more information about student accessibility resources and policies, refer to http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

Task Model 1 **Response Type: Equation / Numeric** **DOK Level 1** **4.NBT.B.4** Fluently add and subtract multi-digit whole numbers using the standard algorithm. **Evidence Required:** 1. The student adds or subtracts whole numbers in non-contextual mathematics problems. **Tools:** None **Version 3 Update:** Added example stem 2 to TM1a.	**Prompt Features:** The student is prompted to add two or more multi-digit whole numbers, or to subtract two multi-digit whole numbers. **Stimulus Guidelines:** • Follow any stated guidelines on allowable number ranges. • At least one number in each item should be at least four digits. • Present numbers horizontally (4325+654=?), vertically, or with words (what is the difference between 4003 and 1486?). • Item difficulty can be adjusted via these example methods: o The number of times composing or decomposing is required while solving the problem o The absence/presence of zeros o More than 2 addends o Selecting numbers that are easier or harder to add/subtract (e.g., doubles + 1 are typically easier), numbers closer to 10 or 100 **TM1a** **Stimulus:** The student is presented with a non-contextual addition problem with two or more whole numbers. **Example Stem 1:** Enter the sum. 4325 + 654 **Example Stem 2:** Add together 33, 149, and 67. Enter the sum in the response box. **Rubric:** (1 point) The student enters the correct number (e.g., 4,979; 249). **Response Type:** Equation/Numeric

Task Model 1 **Response Type: Equation / Numeric** **DOK Level 1** **4.NBT.B.4** Fluently add and subtract multi-digit whole numbers using the standard algorithm. **Evidence Required:** 1. The student adds or subtracts whole numbers in non-contextual mathematics problems. **Tools:** None	**TM1b** **Stimulus:** The student is presented with a non-contextual subtraction problem. **Example Stem 1:** Enter the difference. 7529 - 382 **Example Stem 2:** Enter the difference. 4003 - 1486 **Rubric:** (1 point) The student enters the correct number (e.g., 7,147; 2,517). **Response Type:** Equation/Numeric

Task Model 2a **Response Type: Equation / Numeric** **DOK Level 1** **4.NBT.B.5** Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **Evidence Required:** 2. The student multiplies whole numbers (up to four digits by one digit or two digits by two digits) using strategies based on place value and the properties of operations. **Tools:** None **Version 3 Update:** Added example stem 2 to TM2a.	**Prompt Features:** The student is prompted to multiply two whole numbers. **Stimulus Guidelines:** • Follow any stated guidelines on allowable number ranges. • Item difficulty can be adjusted via these example methods: o One factor is a multiple of 10, 100, or 1000 o One or more partial products result from multiplying 5 by an even digit (e.g., multiplying 5 by 4 gives 20, but 5 by 40 gives 200 – the extra 0 seems to violate the pattern of "when you multiply ones by tens, just add a zero on the end") o Factors contain digits that are easier to multiply (e.g., multiplying by 2 or 5 is typically easier than multiplying by 6, 7, or 8). o Solving the problem requires composing/ decomposing o Presenting numbers horizontally vs. vertically; larger number first vs. smaller number first **TM2a** **Stimulus:** The student is presented with a non-contextual multiplication problem. **Example Stem 1:** Enter the product. 5327 × 4 **Example Stem 2:** Multiply 48 and 20. Enter the product in the response box. **Rubric:** (1 point) The student multiplies two whole numbers and enters the correct product (e.g., 21,308; 960). **Response Type:** Equation/Numeric

Task Model 2b **Response Type: Equation / Numeric** **DOK Level 2** **4.NBT.B.5** Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **Evidence Required:** 2. The student multiplies whole numbers (up to four digits by one digit or two digits by two digits) using strategies based on place value and the properties of operations. **Tools:** None **Version 3 Update:** Revised Example Stem 2 in TM2b to include an area model.	**Prompt Features:** The student is prompted to complete a multiplication strategy. **Stimulus Guidelines:** Same as for TM2a. **TM2b** **Stimulus:** The student is presented with a multiplication expression in which properties of operations have been used as strategies for multiplication, with one unknown number. **Example Stem 1:** Enter the unknown number that makes the equation true. $26 \times 74 = (20 + 6) \times (\square + 4)$ **Rubric:** (1 point) The student enters the unknown number that makes the equation true (e.g., 70). **Response Type:** Equation/Numeric **Example Stem 2:** In the area model shown, A = 2700 and D = 7. What are the values of B and C?  **Rubric:** (1 point) The student enters correct numbers for B and C (e.g., B = 210, C = 450). **Response Type:** Equation/Numeric (2 response boxes, labeled B = and C =, respectively).

Task Model 2c-d **Response Type: Multiple Choice, single correct response** **DOK Level 2** **4.NBT.B.5** Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **Evidence Required:** 2. The student multiplies whole numbers (up to four digits by one digit or two digits by two digits) using strategies based on place value and the properties of operations. **Tools:** None	**Prompt Features:** The student is prompted to select a multiplication strategy. **Stimulus Guidelines:** Same as for TM2a. **TM2c** **Stimulus:** The student is presented with a multiplication expression in the stem and expressions reflecting use of the distributive property or decomposition of factors in the answer choices. **Example Stem:** Which expression is equal to 36 × 94? A. (30 × 90) + (6 × 4) B. (30 + 6) × (90 + 4) C. (30 + 6) × 94 + (30 + 6) × 4 D. (30 × 90) + (30 × 6) + (90 × 6) + (90 × 4) **Rubric:** (1 point) The student selects a correct expression (e.g., B). **Response Type:** Multiple Choice, single correct response **TM2d** **Stimulus:** The student is presented with a multiplication problem and four vertically recorded partial solutions. **Example Stem:** Which strategy for multiplying 94 and 36 should result in the correct product? A. 94 × 36 = 24 + 540 + 120 + 270 B. 94 × 36 = 24 + 54 + 120 + 2700 C. 94 × 36 = 2700 + 540 + 120 + 240 D. 94 × 36 = 2700 + 540 + 120 + 24 **Rubric:** (1 point) The student selects a correct strategy (e.g., D). **Response Type:** Multiple Choice, single correct response

Task Model 3a **Response Type: Equation / Numeric** **DOK Level 1** **4.NBT.B.6** Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **Evidence Required:** 3. The student finds whole number quotients and remainders (up to four-digit dividends and one-digit divisors) using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. **Tools:** None **Version 3 Update:** Revised TM3a to focus on the relationship between multiplication and division with, or without, a remainder.	**Prompt Features:** The student is prompted to divide two whole numbers or determine the remainder when given two factors of a quotient plus an unknown number. **Stimulus Guidelines:** • Follow any stated guidelines on allowable number ranges. • Item difficulty can be adjusted via these example methods: o Using an easier divisor (e.g., dividing by 2 or 5 is typically easier than dividing by 6, 7, or 8) o Dividends that do or do not contain 0s (e.g., dividing 527 by 4 will be easier than dividing 507 by 4) o Divisors that are greater than, less than, or equal to the first digit of the dividend (e.g., dividing 839 by 7 will be easier than dividing 639 by 7) o Partial remainders of 0 (e.g., dividing 83 by 3 will be easier than dividing 83 by 4) o Partial dividends that are greater than the divisor (e.g., dividing 632 by 5 will be easier than dividing 632 by 6) o Problems that are “math facts” (e.g., dividing 64 by 8 will be easier than dividing 68 by 8) **TM3a** **Stimulus:** The student is presented with an equation that illustrates the relationship between multiplication and division with, or without, a remainder. (Note: In the case of a remainder as the unknown, be sure the number is less than the single digit factor.) **Example Stem 1:** Enter the unknown number that makes the equation true. 2571 ÷ 3 = □ **Example Stem 2:** Enter the unknown number that makes the equation true. 120 × 5 + □ = 603 **Rubric:** (1 point) The student enters the correct unknown number (857; 3). **Response Type:** Equation/Numeric

Task Model 3b **Response Type: Equation / Numeric** **DOK Level 2** **4.NBT.B.6** Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **Evidence Required:** 3. The student finds whole number quotients and remainders (up to four-digit dividends and one-digit divisors) using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. **Tools:** None	**Prompt Features:** The student is prompted to complete the decomposition of a dividend as a strategy to divide. **Stimulus Guidelines:** Same as for TM3a. **TM3b** **Stimulus:** The student is presented with a non-contextual division problem with a box to represent an unknown number. **Example Stem:** Enter the unknown number to make the equation true. 98 ÷ 5 = (□ ÷ 5) + (8 ÷ 5) **Rubric:** (1 point) The student enters the correct number (e.g., 90). **Response Type:** Equation/Numeric

Task Model 3c **Response Type: Multiple Choice, single correct response** **DOK Level 1** **4.NBT.B.6** Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **Evidence Required:** 3. The student finds whole number quotients and remainders (up to four-digit dividends and one-digit divisors) using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. **Tools:** None	**Prompt Features:** The student is prompted to select a multiplication equation that is the inverse of a given division equation. **Stimulus Guidelines:** Same as for TM3a. **TM3c** **Stimulus:** The student is presented with a division equation with a box representing an unknown number. **Example Stem:** Select the equation that has the same unknown number as 90 ÷ 5 = □. A. 5 × 90 = □ B. 90 × □ = 5 C. 5 × □ = 90 D. □ × 90 = 5 **Rubric:** (1 point) The student selects the related multiplication equation (e.g., C). **Response Type:** Multiple Choice, single correct response

Task Model 3d **Response Type: Equation / Numeric** **DOK Level 1** **4.NBT.B.6** Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **Evidence Required:** 3. The student finds whole number quotients and remainders (up to four-digit dividends and one-digit divisors) using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. **Tools:** None **Version 3 Update:** Added new TM3d.	**Prompt Features:** The student is prompted to determine the whole number quotient and remainder when given a contextual problem to solve. **Stimulus Guidelines:** Same as for TM3a. **TM3d** **Stimulus:** The student is presented with a contextual division problem where the student must identify the whole number quotient and remainder. **Example Stem:** A teacher has 1247 craft sticks. She divides them equally among 9 students. How many craft sticks does each student get? Enter your answer in the first response box. How many craft sticks are left over? Enter your answer in the second response box. **Rubric:** (1 point) The student enters the correct numbers in each response box (e.g., 138, 5). **Response Type:** Equation/Numeric, two response boxes

Claim 1: Concepts and Procedures
Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.

Content Domain: **Numbers and Operations—Fractions**

Target F [m]: Extend understanding of fraction equivalence and ordering. (DOK 1, 2)

Tasks for this target will ask students to recognize and generate equivalent fractions or compare fractions with different numerators and different denominators, sometimes using <, =, and >. These may include the use of visual fraction models or number lines to tap student understanding of equivalence and relative size with respect to benchmarks, such as 1/2.

Standards: 4.NF.A, 4.NF.A.1, 4.NF.A.2	**4.NF.A Extend understanding of fraction equivalence and ordering.** **4.NF.A.1** Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. **4.NF.A.2** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.
Related Below-Grade and Above-Grade Standards for Purposes of Planning for Vertical Scaling: 3.NF.A, 3.NF.A.3, 3.NF.A.3a, 3.NF.A.3b, 3.NF.A.3d 5.NF.A, 5.NF.A.1, 5.NF.A.2	**Related Grade 3 Standards** **3.NF.A Develop understanding of fractions as numbers.** **3.NF.A.3** Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. **a**. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. **b**. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model. **d.** Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

	Related Grade 5 Standards **5.NF.A Use equivalent fractions as a strategy to add and subtract fractions.** **5.NF.A.1** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)* **5.NF.A.2** Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*
DOK Levels:	1, 2
Achievement Level Descriptors:	
RANGE Achievement Level Descriptor (Range ALD) Target F: Extend understanding of fraction equivalence and ordering.	**Level 1** Students should be able to recognize that fraction comparisons are valid only when the two fractions are referring to the same whole.
	Level 2 Students should be able to compare two fractions with different numerators and different denominators using <, >, and = by comparing to a benchmark fraction such as 1/2 and recognize equivalent fractions using visual models.
	Level 3 Students should be able to extend understanding to compare two fractions with different numerators and different denominators using <, >, and = by creating common denominators or numerators and recognize and generate equivalent fractions using visual models.
	Level 4 Students should be able to extend understanding to compare two fractions with different numerators and different denominators using <, >, and = and justify the conclusions using a visual fraction model.
Evidence Required:	1. The student recognizes when two or more fractions are equivalent. 2. The student generates equivalent fractions given an initial fraction or fraction model. 3. The student uses the symbols <, >, and = to compare fractions with different numerators and different denominators.
Allowable Response Types:	Matching Tables; Equation/Numeric; Multiple choice, multiple correct responses; Hot Spot
Allowable Stimulus Materials:	<, >, and = symbols, number lines, parts of whole visual models, parts of set visual models, tables
Construct-Relevant Vocabulary:	fraction, equivalent, divide, equal to, greater than, less than, digits, numerator, denominator
Allowable Tools:	None

Target-Specific Attributes:	The majority of items in this target should follow the CCSS limitations on denominators allowed at Grade 4 (2, 3, 4, 5, 6, 8, 10, 12, and 100). For the purposes of adaptive testing, however, some items may use denominators appropriate to 5th grade (multiples of 2, 3, 5, and/or 7 that are less than or equal to 100).
Non-Targeted Constructs:	None
Accessibility Guidance:	Item writers should consider the following Language and Visual Element/Design guidelines[1] when developing items. Language Key Considerations: • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level • Avoid ambiguous or obscure words, idioms, jargon, unusual names and references Visual Elements/Design Key Considerations: • Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context • Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary • Avoid crowding of details and graphics Items are selected for a student's test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.[2]
Development Notes:	Explaining why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ (CCSS 4.NF.A.1), will be assessed in Claim 3. Recognizing that comparisons are valid only when the two fractions refer to the same whole (CCSS 4.NF.A.2) will be assessed in Claim 3. Justifying the comparison of fractions (CCSS 4.NF.A.2) will be assessed in Claim 3.

[1] For more information, refer to the General Accessibility Guidelines at: http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf

[2] For more information about student accessibility resources and policies, refer to http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

Task Model 1a

Response Type: Matching Tables

DOK Level 1

4.NF.A.1
Explain why a fraction *a*/*b* is equivalent to a fraction (*n* × *a*)/(*n* × *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

Evidence Required:
1. The student recognizes when two or more fractions are equivalent.

Tools: None

Prompt Features: The student is prompted to identify equivalent fractions.

Stimulus Guidelines:

- The fractions in the table have different denominators than the given fraction.
- Item difficulty can be adjusted via these example methods:
  - Presenting fractions that are less than or greater than 1
  - Using denominators that are multiples of 2, 3, 4, 5, 6, 8, 10, 12, or 100, but not actually those numbers (e.g., 9, 15, or 18)

TM1a
Stimulus: The student is presented with a visual fraction model in the form $\frac{a}{b}$.

Example Stem: Figure A has $\frac{2}{3}$ of its whole shaded gray.

Figure A

Decide whether each fraction is equal to $\frac{2}{3}$. Select Yes or No for each fraction.

	Yes	No
$\frac{4}{6}$		
$\frac{1}{2}$		
$\frac{8}{12}$		

Rubric: (1 point) The student correctly identifies all of the fractions as equivalent or not equivalent (e.g., Y, N, Y).

Response Type: Matching Tables

Task Model 1b

Response Type: Matching Tables

DOK Level 1

4.NF.A.1
Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

Evidence Required:
1. The student recognizes when two or more fractions are equivalent.

Tools: None

Prompt Feature: The student is prompted to identify equivalent fractions.

Stimulus Guidelines:

- Equations show pairs of fractions with different numerators and denominators.
- At least one fraction of each pair should have a denominator of 2, 3, 4, 5, 6, 8, 10, 12, or 100.
- Item difficulty can be adjusted via these example methods:
  - Location of the fraction with a denominator of 2, 3, 4, 5, 6, 8, 10, 12, or 100 (left or right side of equation)
  - Using denominators that are multiples of 2, 3, 4, 5, 6, 8, 10, 12, or 100, but not actually those numbers (e.g., 9, 15, or 18)
  - Presenting fractions that are less than or greater than 1

TM1b
Stimulus: The student is presented with pairs of fractions in numeric form in the answer choices.

Example Stem: Select True if the equation is true. Select False if the equation is **not** true.

	True	False
$\frac{4}{6} = \frac{8}{12}$		
$\frac{50}{100} = \frac{3}{4}$		
$\frac{6}{8} = \frac{75}{100}$		

Rubric: (1 point) The student correctly identifies all fraction equivalencies as True or False (e.g., T, F, T).

Response Type: Matching Tables

Task Model 1c

Response Type: Matching Tables

DOK Level 1

4.NF.A.1
Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

Evidence Required:
1. The student recognizes when two or more fractions are equivalent.

Tools: None

Prompt Feature: The student is prompted to identify equivalent fractions.

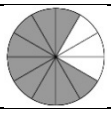
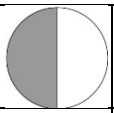
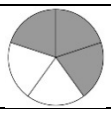
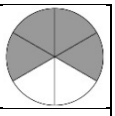
Stimulus Guidelines:

- All fractions used should have at least one equivalent fraction (e.g., there should be no fractions that do not have any matches).
- At least one fraction of each match should have a denominator of 2, 3, 4, 5, 6, 8, 10, 12, or 100.
- Item difficulty can be adjusted via these example methods:
  - Location of the fraction with a denominator of 2, 3, 4, 5, 6, 8, 10, 12, or 100 (along left side or top)
  - Using denominators that are multiples of 2, 3, 4, 5, 6, 8, 10, 12, or 100, but not actually those numbers (e.g., 9, 15, or 18)
  - Having more than one match per fraction
  - Presenting fractions that are less than or greater than 1

TM1c
Stimulus: The student is presented with four visual fraction models and four fractions in numeric form.

Example Stem: A fraction of the whole is shaded in each model.

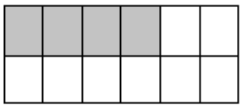
Click in the chart to match each fraction to the shaded part of the model that shows an equivalent fraction.

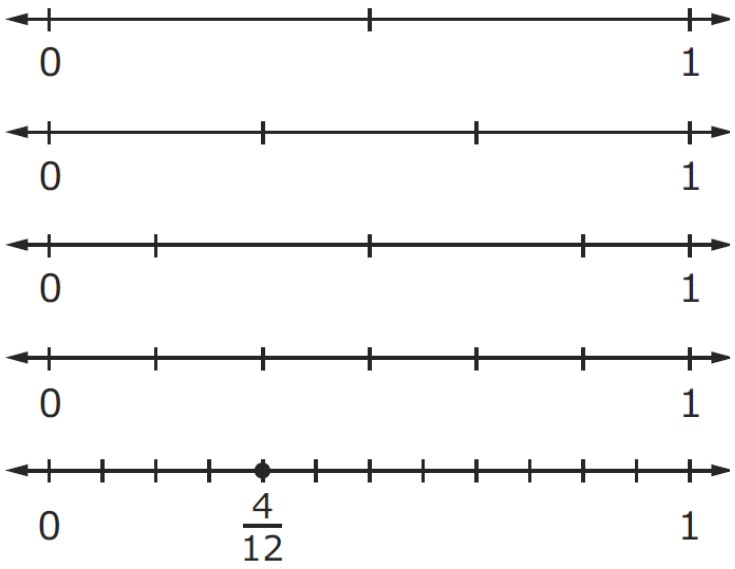
				
$\frac{2}{3}$				
$\frac{3}{4}$				
$\frac{4}{8}$				
$\frac{6}{10}$				

Rubric: (1 point) The student correctly matches all fractions to its model (e.g., $\frac{3}{4} \rightarrow$  , $\frac{4}{8} \rightarrow$  , $\frac{6}{10} \rightarrow$  , $\frac{2}{3} \rightarrow$ )

Response Type: Matching Tables

Task Model 1d **Response Type: Multiple Choice, multiple correct responses** **DOK Level 1** **4.NF.A.1** Explain why a fraction *a*/*b* is equivalent to a fraction (*n* × *a*)/(*n* × *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. **Evidence Required:** 1. The student recognizes when two or more fractions are equivalent. **Tools:** None **Version 3 Update:** Added new TM1d.	**TM1d** **Stimulus:** The student is presented with six fractions in numeric form. **Example Stem:** Select **all** fractions that are equal to $\frac{3}{4}$. A. $\frac{1}{2}$ B. $\frac{3}{5}$ C. $\frac{4}{6}$ D. $\frac{6}{8}$ E. $\frac{6}{10}$ F. $\frac{9}{12}$ **Rubric:** (1 point) The student selects all of the equivalent fractions (e.g., D, F). **Response Type:** Multiple choice, multiple correct responses

Task Model 2a **Response Type: Equation / Numeric** **DOK Level 1** **4.NF.A.1** Explain why a fraction *a*/*b* is equivalent to a fraction (*n* × *a*)/(*n* × *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. **Evidence Required:** 2. The student generates equivalent fractions given an initial fraction or fraction model. **Tools:** None	**Prompt Features:** The student is prompted to enter an equivalent fraction. **Stimulus Guidelines:** • The given fraction should have a denominator of 2, 3, 4, 5, 6, 8, 10, 12, or 100. • Fraction model must represent the given fraction (total shaded sections = numerator, total sections = denominator). • Item difficulty can be adjusted via these example methods: o Location of the shaded sections (e.g., all connected or scattered apart from each other) o Student familiarity with the denominator used o Presenting fractions that are less than or greater than 1 o Presenting fractions greater than 1 as improper fractions or mixed numbers **TM2a** **Stimulus:** The student is presented with a visual fraction model. **Example Stem:** Figure A has $\frac{4}{12}$ of its whole shaded.  **Figure A** Enter **another** fraction that is equal to $\frac{4}{12}$. **Rubric:** (1 point) The student enters a fraction equivalent to the given fraction (e.g., $\frac{1}{3}$; $\frac{8}{24}$, etc). **Scoring Note:** The fraction given in the stem (e.g., $\frac{4}{12}$) will not be accepted as a correct answer. **Response Type:** Equation/Numeric

Task Model 2b **Response Type: Equation / Numeric** **DOK Level 1** **4.NF.A.1** Explain why a fraction *a*/*b* is equivalent to a fraction (*n* × *a*)/(*n* × *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. **Evidence Required:** 2. The student generates equivalent fractions given an initial fraction or fraction model. **Tools:** None **Version 3 Update:** Added a new example stem for TM2b.	**TM2b** **Stimulus:** The student is presented with a fraction in numeric form, with or without a series of number lines with one fraction labeled. **Example Stem 1:** Enter **another** fraction that is equivalent to $\frac{4}{12}$. **Example Stem 2:** Figure B shows several number lines that divide 1 into equal parts. **Figure B**  Enter **another** fraction that is equal to $\frac{4}{12}$. **Rubric:** (1 point) The student enters a fraction equivalent to the given fraction (e.g., $\frac{1}{3}$ or $\frac{2}{6}$ or other equivalent fraction). **Scoring Note:** The fraction given in the stem (e.g., $\frac{4}{12}$) will not be accepted as a correct answer. **Response Type:** Equation/Numeric

Task Model 2c **Response Type:** **Hot Spot** **DOK Level 2** **4.NF.A.1** Explain why a fraction *a*/*b* is equivalent to a fraction (*n* × *a*)/(*n* × *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. **Evidence Required:** 2. The student generates equivalent fractions given an initial fraction or fraction model. **Tools:** None **Accessibility Note:** Hot Spot items are not currently able to be Brailled. Minimize the number of items developed to this TM using Hot Spot.	**Prompt Features:** The student is prompted to generate a fraction model that is equivalent to a given fraction or fraction model. **Stimulus Guidelines:** • The given fraction should have a denominator of 2, 3, 4, 5, 6, 8, 10, 12, or 100. • The number of sections of the fraction model should be a multiple or factor of the denominator of the given fraction (e.g., if given fraction is $\frac{4}{6}$, fraction model could be in thirds or twelfths). • Item difficulty can be adjusted via these example methods: o Using a fraction model with the number of sections other than 2, 3, 4, 5, 6, 8, 10, 12, or 100 o Student familiarity with the denominator used o Presenting fractions that are less than or greater than 1 o Presenting fractions greater than 1 as improper fractions or mixed numbers **TM2c** **Stimulus:** The student is presented with a fraction in numeric form. **Example Stem:** Click the spaces of the model to shade $\frac{3}{6}$ of Figure A.  **Figure A** **Rubric:** (1 point) The student builds a model of an equivalent fraction (e.g., $\frac{2}{4}$). **Response Type:** Hot Spot

Task Model 3a

Response Type: Matching Tables

DOK Level 2

4.NF.A.2
Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Evidence Required:
3. The student uses the symbols <, >, and = to compare fractions with different numerators and different denominators.

Tools: None

Version 3 Update:
Added more example methods for varying the item difficulty to the stimulus guidelines.

Prompt Feature: The student is prompted to compare two fractions.

Stimulus Guidelines:

- At least one of each pair of the given fractions should have a denominator of 2, 3, 4, 5, 6, 8, 10, 12, or 100.
- Item difficulty can be adjusted via these example methods:
  - Selecting fractions that are equivalent; have same denominator or same numerator; fractions that are not related such as 4/5 and 2/3
  - Selecting fractions that are close to benchmarks of 0, 1/2, or 1
  - Selecting fractions that are not near an easily recognized benchmark or are closer in value
  - Student familiarity with the denominator used
  - Presenting fractions that are less than or greater than 1
  - Presenting fractions greater than 1 as improper fractions or mixed numbers

TM3a
Stimulus: The student is presented with three fraction inequalities that compare two fractions each.

Example Stem: Select True if the comparison is true. Select False if the comparison is **not** true.

	True	False
$\frac{1}{4} < \frac{2}{12}$		
$\frac{2}{10} > \frac{3}{5}$		
$\frac{4}{6} > \frac{5}{12}$		

Rubric: (1 point) The student correctly identifies three fraction comparisons as either true or false (e.g., FFT).

Response Type: Matching Tables

Task Model 3b

Response Type: Matching Table

DOK Level 2

4.NF.A.2
Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Evidence Required:
3. The student uses the symbols <, >, and = to compare fractions with different numerators and different denominators.

Tools: None

Version 3 Update:
Changed TM3b from an equation/numeric response type to a matching table response type. Updated the stimulus and stem to match the new format.

Prompt Feature: The student is prompted to compare two fractions.

Stimulus Guidelines: Same as for TM3a.

TM3b

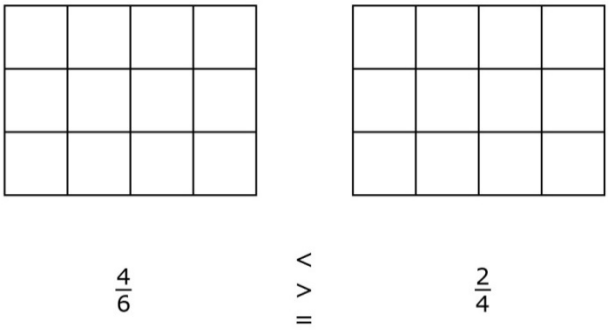
Stimulus: The student is presented with two pairs of fractions and directed to compare them using (<, >, or =).

Example Stem:
Select the symbol (<, >, or =) that correctly compares each pair of numbers.

	<	>	=
$\frac{2}{8}$ ☐ $\frac{1}{4}$			
$\frac{3}{5}$ ☐ $\frac{7}{8}$			

Rubric: (1 point) The student selects the correct symbols (e.g., =, <).

Response Type: Matching Table

Task Model 3c **Response Type:** **Hot Spot** **DOK Level 2** **4.NF.A.2** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. **Evidence Required:** 3. The student uses the symbols <, >, and = to compare fractions with different numerators and different denominators. **Tools:** None **Accessibility Note:** Hot Spot items are not currently able to be Brailled. Minimize the number of items developed to this TM.	**Prompt Features:** The student is prompted to compare fractions and justify the comparison with visual models. **Stimulus Guidelines:** • Fraction models should reflect a common multiple of the denominators of the fractions (not necessarily the least common denominator). **TM3c** **Stimulus:** The student is presented with two fractions and two blank fraction models. **Example Stem:** • Click on the squares in the rectangles that are needed to represent $\frac{4}{6}$ and $\frac{2}{4}$, as labeled below each large rectangle. • Choose the correct symbol to compare the fractions. Each large rectangle represents one whole.  **Rubric:** (2 points) The student correctly shades the fraction models and chooses the correct comparison symbol (e.g., the left model shows $\frac{8}{12}$, the right model shows $\frac{6}{12}$, and the symbol selected is >). (1 point) Partial credit is possible for either shading the fraction models correctly, or for choosing the correct comparison symbol. **Response Type:** Hot Spot

Claim 1: Concepts and Procedures
Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.

Content Domain: **Numbers and Operations—Fractions**

Target G [m]: Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. (DOK 1, 2)

Tasks for this target will ask students to identify and generate equivalent forms of a fraction *a*/*b* with *a*>1, including mixed numbers with like denominators. Some tasks should incorporate unit fractions and the operations addition and subtraction to express equivalent forms. Other tasks should represent *a*/*b* as multiplication of a whole number and unit fraction, with *a*/*b* sometimes expressed as the product of a whole number and fraction (e.g., $3 \times \left(\frac{2}{5}\right) = 6 \times \left(\frac{\square}{5}\right)$).

One-step, contextual word problems involving addition and subtraction of fractions referring to the same whole and having like denominators and those involving multiplication of a fraction by a whole number should also be included in this target.

Standards:	
4.NF.B, 4.NF.B.3, 4.NF.B.4	**4.NF.B Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.** **4.NF.B.3** Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*. **a.** Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. **b.** Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.* **c.** Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. **d.** Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem. **4.NF.B.4** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. **a.** Understand a fraction *a*/*b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).* **b.** Understand a multiple of *a*/*b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)* **c.** Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party*

	will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?
Related Below-Grade and Above-Grade Standards for Purposes of Planning for Vertical Scaling: 3.NF.A, 3.NF.A.1, 3.NF.A.2 5.NF.A, 5.NF.A.1, 5.NF.A.2, 5.NF.B, 5.NF.B.4, 5.NF.B.6	**Related Grade 3 Standards** **3.NF.A Develop understanding of fractions as numbers.** **3.NF.A.1** Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*. **3.NF.A.2** Understand a fraction as a number on the number line; represent fractions on a number line diagram. **a.** Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line. **b.** Represent a fraction *a*/*b* on a number line diagram by marking off *a* lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line. **Related Grade 5 Standards** **5.NF.A Use equivalent fractions as a strategy to add and subtract fractions.** **5.NF.A.1** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)* **5.NF.A.2** Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.* **5.NF.B Apply and extend previous understandings of multiplication and division to multiply and divide fractions.** **5.NF.B.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. **a.** Interpret the product (*a*/*b*) × *q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a* × *q* ÷ *b*. **b.** Find the area of a rectangle with fractional side lengths by tiling

	it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas. **5.NF.B.6** Solve-real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
DOK Levels:	1, 2
Achievement Level Descriptors:	
RANGE Achievement Level Descriptor (Range ALD): Target G: Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.	**Level 1** Students should be able to understand that a fraction *a*/*b* with *a* > 1 is the sum of its unit fractional parts by extending previous understandings of addition on whole numbers. They should be able to identify fractions using visual models.
	Level 2 Students should be able to understand that a fraction *a*/*b* is a multiple of 1/*b* by extending previous understanding of multiplication on whole numbers; solve one-step problems involving addition and subtraction of fractions referring to the same whole with like denominators; and use visual fraction models and/or equations to represent the problem.
	Level 3 Students should be able to identify and generate equivalent forms of a fraction including mixed numbers with like denominators and solve one-step problems involving multiplication of a fraction by a whole number.
	Level 4 No Descriptor
Evidence Required:	1. The student adds and subtracts fractions with like denominators by joining and separating parts referring to the same whole. 2. The student expresses an equivalent form of a fraction or mixed number by considering each as a sum of fractions with the same denominator. 3. The student solves contextual problems involving addition and subtraction of fractions referring to the same whole and having like denominators by using visual fraction models and equations to represent the problem. 4. The student represents a fraction *a*/*b* as a multiple of 1/*b*. 5. The student multiplies a fraction by a whole number. 6. The student solves contextual problems involving the multiplication of a fraction by a whole number by using visual fraction models and equations to represent the problem.
Allowable Response Types:	Multiple Choice, single correct response; Equation/Numeric; Matching Tables; Drag and Drop; Hot Spot
Allowable Stimulus Materials:	number lines, parts of whole visual models, parts of set visual models, equations, expressions
Construct-Relevant Vocabulary:	equation, expression, equal, fraction, model, product, numerator
Allowable Tools:	none

Target-Specific Attributes:	• Unless otherwise specified, no distinction is made between proper and improper fractions. • Unless otherwise specified, improper fractions and mixed numbers do not get special treatment. • The majority of items in this target should follow the CCSS limitations on denominators allowed at 4th grade (2, 3, 4, 5, 6, 8, 10, 12, and 100). For the purposes of adaptive testing, however, some items will use denominators appropriate to 5th grade (includes all denominators).
Non-Targeted Constructs:	none
Accessibility Guidance:	Item writers should consider the following Language and Visual Element/Design guidelines[1] when developing items. Language Key Considerations: • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level • Avoid ambiguous or obscure words, idioms, jargon, unusual names and references Visual Elements/Design Key Considerations: • Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context • Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary • Avoid crowding of details and graphics Items are selected for a student's test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.[2]
Development Notes:	Representing contextual problems with equations will be assessed in Claim 4. Multi-step items combining addition/subtraction and multiplication will be assessed in Claim 2. Determining between which two whole numbers the product of a whole number and a fraction lies will be assessed in Claim 2.

[1] For more information, refer to the General Accessibility Guidelines at: http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf

[2] For more information about student accessibility resources and policies, refer to http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

Task Model 1a **Response Type: Equation / Numeric** **DOK Level 1** **4.NF.B.3** Understand a fraction *a/b* with *a* > 1 as a sum of fraction *1/b*. **Evidence Required:** 1. The student adds and subtracts fractions with like denominators by joining and separating parts referring to the same whole. **Tools:** None	**Prompt Features:** The student is prompted to solve an addition or subtraction fraction problem. **Stimulus Guidelines:** • The student is presented with either an addition or subtraction fraction equation with a box for an unknown number. • In addition problems, the unknown will be the sum. • In subtraction problems, the unknown will be the difference. • Item difficulty can be adjusted via these example methods: o The location of the unknown in the equation (on the left or right side of the equal sign) o How "friendly" the numbers are to work with o Using mixed numbers that have to be broken into parts prior to doing an operation (e.g., $1\frac{2}{5}-\frac{4}{5}=\frac{5}{5}+\frac{2}{5}-\frac{4}{5}=\frac{7}{5}-\frac{4}{5}=\frac{3}{5}$) **TM1a** **Stimulus:** The student is presented with a fraction addition or subtraction equation with a box to represent an unknown result. **Example Stem 1:** Enter the unknown number that makes the equation true. $\frac{1}{8}+\frac{4}{8}=\square$ **Example Stem 2:** Enter the unknown number that makes the equation true. $\square=\frac{4}{8}-\frac{1}{8}$ **Rubric:** (1 point) The student shows understanding of addition and subtraction of fractions by entering the correct sum or difference of two fractions with like denominators (e.g., $\frac{5}{8}$; $\frac{3}{8}$). **Response Type:** Equation/Numeric

Task Model 1b **Response Type: Equation / Numeric** **DOK Level 2** **4.NF.B.3** Understand a fraction *a/b* with *a* > 1 as a sum of fraction *1/b*. **Evidence Required:** 1. The student adds and subtracts fractions with like denominators by joining and separating parts referring to the same whole. **Tools:** None	**Prompt Features:** The student is prompted to solve an addition or subtraction fraction problem. **Stimulus Guidelines:** • The student is presented with either an addition or subtraction fraction equation with a box for an unknown number. • In addition problems, the unknown will be an addend. • In subtraction problems, the unknown will be the minuend or subtrahend. • Item difficulty can be adjusted via these example methods: o The location of the unknown in the equation ▪ In addition problems, the first or second addend ▪ In subtraction problems, the minuend or subtrahend o The location of the result in the equation (on the left or right side of the equal sign) o How "friendly" the numbers are to work with o Using mixed numbers that have to be broken into parts prior to doing an operation (e.g., $1\frac{2}{5}-\frac{4}{5}=\frac{5}{5}+\frac{2}{5}-\frac{4}{5}=\frac{7}{5}-\frac{4}{5}=\frac{3}{5}$) **TM1b** **Stimulus:** The student is presented with a fraction addition or subtraction equation with a box for an unknown number. **Example Stem 1:** Enter the unknown number that makes the equation true. $\frac{7}{5}-\square=\frac{4}{5}$ **Example Stem 2:** Enter the unknown number that makes the equation true. $\frac{4}{5}=\square+\frac{2}{5}$ **Rubric:** (1 point) The student shows understanding of addition and subtraction of fractions by entering the correct sum or difference of two fractions with like denominators (e.g., $\frac{3}{5}$; $\frac{2}{5}$). **Response Type:** Equation/Numeric

Task Model 2a

Response Type: Matching Tables

DOK Level 2

4.NF.B.3
Understand a fraction *a*/*b* with *a* > 1 as a sum of fraction *1*/*b*.

Evidence Required:
2. The student expresses an equivalent form of a fraction or mixed number by considering each as a sum of fractions with the same denominator.

Tools: None

Prompt Feature: The student is prompted to select an expression that represents a decomposition of a fraction into a sum of fractions with the same denominator.

Stimulus Guidelines:

- The table will contain addition expressions with two or more fractions each that have the same denominator.
- Item difficulty can be adjusted via these example methods:
  - Presenting a proper fraction, improper fraction, or mixed number as the given fraction
  - Decomposing the given fraction into a greater or lesser number of terms
  - Ordering the addends in the expression by value or not (e.g., putting it as a middle or end term instead of the initial term in the expression)

TM2a
Stimulus: The student is presented with a fraction and three fraction addition expressions. The number 1 may be used in place of a fraction with like denominator.

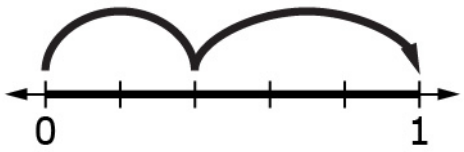
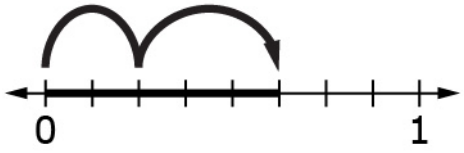
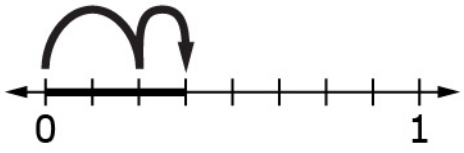
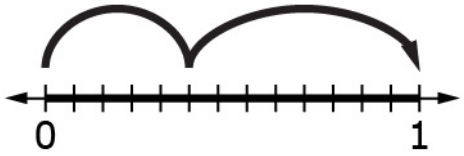
Example Stem: Decide whether each expression is equal to $1\frac{5}{8}$. Click in the table to respond.

	Equal to $1\frac{5}{8}$	**Not Equal to $1\frac{5}{8}$**
$1+\frac{5}{8}$		
$\frac{8}{8}+\frac{3}{8}+\frac{2}{8}$		
$1+\frac{3}{8}+\frac{1}{8}+\frac{2}{8}$		

Rubric: (1 point) The student correctly identifies all three expressions as either equal or not equal to the given fraction (e.g., Equal, Equal, Not Equal).

Response Type: Matching Tables

Task Model 2b **Response Type:** **Drag and Drop** **DOK Level 2** **4.NF.B.3** Understand a fraction *a*/*b* with *a* > 1 as a sum of fraction *1*/*b*. **Evidence Required:** 2. The student expresses an equivalent form of a fraction or mixed number by considering each as a sum of fractions with the same denominator. **Tools:** None **Accessibility Note:** Drag and Drop items are not currently able to be Brailled. Minimize the number of items developed to this TM.	**Prompt Features:** The student is prompted to enter two different ways to decompose a fraction into a sum of fractions with the same denominator. **Stimulus Guidelines:** • Item difficulty can be adjusted via these example methods: o The number of addends that the given fraction is decomposed into o The number of numerators the student must provide **TM2b** **Stimulus:** The student is presented with two equations representing the decomposition of a fraction or mixed number. **Example Stem:** Drag numbers to the numerators of the fractions to show **two different** correct equations. $\frac{7}{8} = \frac{\square}{8} + \frac{\square}{8} + \frac{\square}{8}$ $\frac{7}{8} = \frac{\square}{8} + \frac{\square}{8} + \frac{\square}{8}$ **Rubric:** (1 point) The student correctly completes the equations provided (e.g., 2, 1, 4 and 4, 3, 0). **Response Type:** Drag and Drop

Task Model 2c **Response Type: Multiple Choice, single correct response** **DOK Level 2** **4.NF.B.3** Understand a fraction *a/b* with *a* > 1 as a sum of fraction *1/b*. **Evidence Required:** 2. The student expresses an equivalent form of a fraction or mixed number by considering each as a sum of fractions with the same denominator. **Tools:** None	**Prompt Features:** The student is prompted to identify the decomposition of a fraction represented by a visual fraction model. **Stimulus Guidelines:** • The answer choices should be in the form of one of these types of visual fraction models: o Parts of a whole o Parts of a set o Intervals on a number line diagram • Item difficulty can be adjusted via this example method: o Number of addends/jumps used in each equation/model **TM2c** **Stimulus:** The student is presented with an addition equation representing the decomposition of a fraction. **Example Stem:** Select the model that matches this equation. $\frac{5}{8} = \frac{2}{8} + \frac{3}{8}$ A.  B.  C.  D.  **Rubric:** (1 point) The student selects the correct visual representation of the decomposition of a fraction (e.g., B). **Response Type:** Multiple Choice, single correct response

Task Model 2d **Response Type: Equation / Numeric** **DOK Level 2** **4.NF.B.3** Understand a fraction *a*/*b* with *a* > 1 as a sum of fraction *1*/*b*. **Evidence Required:** 2. The student expresses an equivalent form of a fraction or mixed number by considering each as a sum of fractions with the same denominator. **Tools:** None	**Prompt Features:** The student is prompted to express the sum of unit fractions with the same denominator as a fraction or mixed number. **Stimulus Guidelines:** • All addends in the expression are unit fractions. • Item difficulty can be adjusted via this example method: o Number of addends in the expression **TM2d** **Stimulus:** The student is presented with an expression representing the decomposition of a fraction into unit fractions. **Example Stem:** Enter the fraction that is equivalent to the expression: $\frac{1}{8}+\frac{1}{8}+\frac{1}{8}$. **Rubric:** (1 point) The student enters the correct fraction (e.g., $\frac{3}{8}$). **Response Type:** Equation/Numeric

Task Model 3a **Response Type: Equation / Numeric** **DOK Level 1** **4.NF.B.3** Understand a fraction *a/b* with *a* > 1 as a sum of fraction *1/b*. **Evidence Required:** 3. The student solves contextual problems involving addition and subtraction of fractions referring to the same whole and having like denominators by using visual fraction models and equations to represent the problem. **Tools:** None	**Prompt Features:** The student is prompted to solve a contextual problem involving the addition and subtraction of fractions. **Stimulus Guidelines:** • The wording of the item provides a clue to the type of operation needed to solve the problem (e.g., item may use words like "combine," "separate," "altogether," "more than," "less than," etc.). • Items may reflect Add To/Take From, Put Together/Take Apart, or Compare situations (refer to Operations and Algebraic Thinking Progression document, Table 1, pg. 7). • Item difficulty can be adjusted via this example method: o Using mixed numbers that have to be broken into parts prior to doing an operation (e.g., $1\frac{2}{5}-\frac{4}{5}=\frac{5}{5}+\frac{2}{5}-\frac{4}{5}=\frac{7}{5}-\frac{4}{5}=\frac{3}{5}$) **TM3a** **Stimulus:** The student is presented with a contextual problem involving the addition or subtraction of fractions referring to the same whole and having like denominators. **Example Stem 1:** John has $\frac{5}{6}$ of a liter of juice. Jill has $\frac{3}{6}$ of a liter of juice. How many liters of juice do John and Jill have together? Enter the number. **Example Stem 2:** Eric has $\frac{7}{8}$ of a pound of nuts. Jill has $\frac{2}{8}$ of a pound of nuts. How many more pounds of nuts does Eric have than Jill? Enter the number. **Rubric:** (1 point) The student enters the correct fraction (e.g, $\frac{8}{6}$ or $\frac{4}{3}$ or $1\frac{2}{6}$ or $1\frac{1}{3}$; $\frac{5}{8}$). **Response Type:** Equation/Numeric

Task Model 3b **Response Type: Equation / Numeric** **DOK Level 2** **4.NF.B.3** Understand a fraction *a/b* with *a* > 1 as a sum of fraction *1/b*. **Evidence Required:** 3. The student solves contextual problems involving addition and subtraction of fractions referring to the same whole and having like denominators by using visual fraction models and equations to represent the problem. **Tools:** None	**Prompt Features:** The student is prompted to solve a contextual problem involving the addition and subtraction of fractions. **Stimulus Guidelines:** • The wording of the item does not provide a clue to the type of operation needed to solve the problem (e.g., item does not use words such as "combines," "altogether," etc.). • Items may reflect Add To/Take From, Put Together/Take Apart, or Compare situations (refer to Operations and Algebraic Thinking Progression document, Table 1, pg. 7). • Item difficulty can be adjusted via this example method: o using mixed numbers that have to be broken into parts prior to doing an operation (e.g., $1\frac{2}{5}-\frac{4}{5}=\frac{5}{5}+\frac{2}{5}-\frac{4}{5}=\frac{7}{5}-\frac{4}{5}=\frac{3}{5}$). **TM3b** **Stimulus:** The student is presented with a contextual problem involving the addition or subtraction of fractions referring to the same whole and having like denominators. **Example Stem 1:** Jack has $2\frac{3}{4}$ feet of rope. Together, Jack and Diane have $4\frac{1}{4}$ feet of rope. How many feet of rope does Diane have? Enter your answer in the response box. **Example Stem 2:** A baker has $3\frac{3}{4}$ cups of sugar. She has $2\frac{1}{4}$ more cups of sugar than cups of flour. How many cups of flour does she have? Enter your answer in the response box. **Rubric:** (1 point) The student enters the correct fraction (e.g, $1\frac{2}{4}$, $1\frac{2}{4}$). **Response Type:** Equation/Numeric

Task Model 3c

Response Type:
Hot Spot

DOK Level 1

4.NF.B.3
Understand a fraction *a/b* with *a* > 1 as a sum of fraction *1/b*.

Evidence Required:
3. The student solves contextual problems involving addition and subtraction of fractions referring to the same whole and having like denominators by using visual fraction models and equations to represent the problem.

Tools: None

Accessibility Note: Hot Spot items are not currently able to be Brailled. Minimize the number of items developed to this TM.

Prompt Features: The student is prompted to manipulate a model representing the addition or subtraction of fractions.

Stimulus Guidelines:

- Items may reflect Add To/Take From, Put Together/Take Apart, or Compare situations (refer to Operations and Algebraic Thinking Progression document, Table 1, pg. 7).
- Item difficulty can be adjusted via this example method:
  - Using mixed numbers that have to be broken into parts prior to doing an operation (e.g., $1\frac{2}{5} - \frac{4}{5} = \frac{5}{5} + \frac{2}{5} - \frac{4}{5} = \frac{7}{5} - \frac{4}{5} = \frac{3}{5}$)

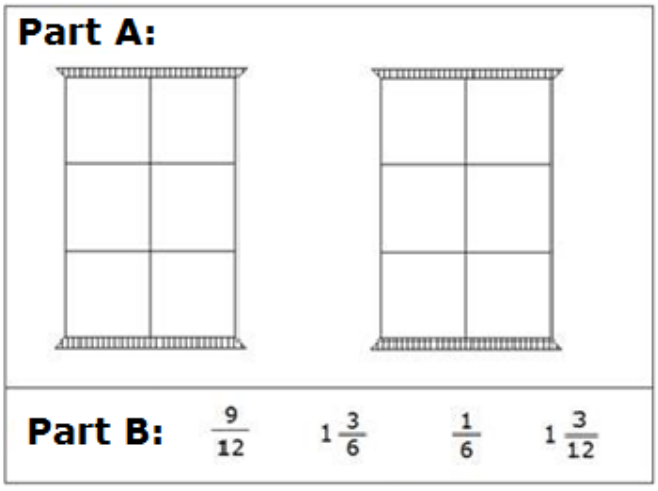
TM3c
Stimulus: The student is presented with a contextual problem involving the addition or subtraction of fractions.

Example Stem 1: Michael eats $\frac{4}{6}$ of a bar of chocolate. Erin eats $\frac{5}{6}$ of a bar of chocolate.

 represents one bar of chocolate

Part A: Shade the model to show how many bars of chocolate Michael and Erin eat together.

Part B: Click on the total number of bars of chocolate Michael and Erin eat together.



Task Model 3c

Response Type:
Hot Spot

DOK Level 1

4.NF.3
Understand a fraction *a/b* with *a* > 1 as a sum of fraction *1/b*.

Evidence Required:
3. The student solves contextual problems involving addition and subtraction of fractions referring to the same whole and having like denominators by using visual fraction models and equations to represent the problem.

Tools: None

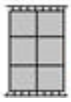
Accessibility Note: Hot Spot items are not currently able to be Brailled. Minimize the number of items developed to this TM.

TM3c (continued)

Rubric:
Part A: (1 point) The student builds a model that correctly represents a fraction addition or subtraction problem (e.g., $1\frac{3}{6}$).
Part B: (1 point) The student selects the correct number (e.g., $1\frac{3}{6}$).

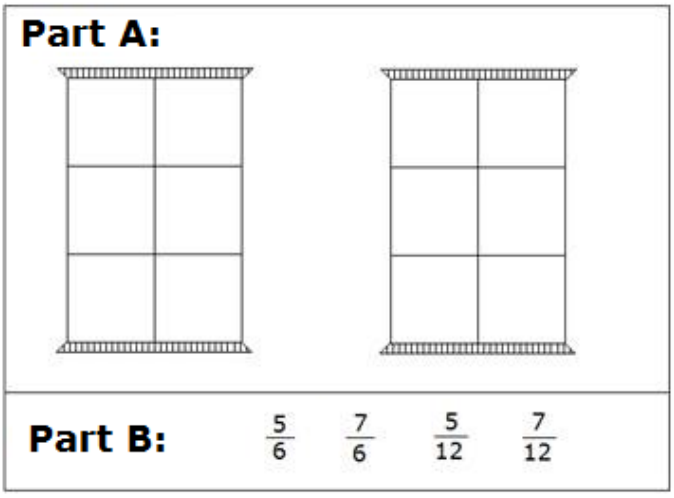
Response Type: Hot Spot

Example Stem 2: Michael and Erin have 2 bars of chocolate. Together they eat $1\frac{1}{6}$ bars of chocolate.

 represents one bar of chocolate

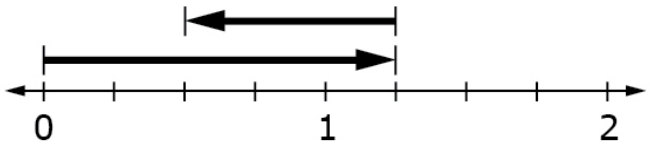
Part A: Shade the model to show the amount of chocolate they did **not** eat.

Part B: Click on the fraction that shows the amount of chocolate they did **not** eat.



Rubric:
Part A: (1 point) The student builds a model that correctly represents a fraction addition or subtraction problem (e.g., $\frac{5}{6}$).
Part B: (1 point) The student selects the correct number (e.g., $\frac{5}{6}$).

Response Type: Hot Spot

Task Model 3d **Response Type: Equation / Numeric** **DOK Level 1** **4.NF.B.3** Understand a fraction *a/b* with *a* > 1 as a sum of fraction *1/b*. **Evidence Required:** 3. The student solves contextual problems involving addition and subtraction of fractions referring to the same whole and having like denominators by using visual fraction models and equations to represent the problem. **Tools:** None **Version 3 Update:** Added new TM3d.	**Prompt Features:** The student is prompted to solve a contextual problem involving the addition and subtraction of fractions. **Stimulus Guidelines:** same as TM3a,b,c **TM3d** **Stimulus:** The student is presented with a model of a contextual problem involving the addition or subtraction of fractions. **Example stem:** José has 1 $\frac{1}{4}$ cups of a sports drink. He gives $\frac{3}{4}$ cup of his drink to his sister. How much sports drink, in cups, does José has left?  **Cups of Sports Drink** **Rubric:** (1 point) The student enters the correct amount (e.g., $\frac{2}{4}$ or $\frac{1}{2}$ or equivalent). **Response Type:** Equation/Numeric

Task Model 4a **Response Type: Equation / Numeric** **DOK Level 1** **4.NF.B.4** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. **Evidence Required:** 4. The student understands that a fraction *a*/*b* is a multiple of 1/*b*. **Tools:** None	**Prompt Features:** The student is prompted to enter the value of an unknown number in a fraction multiplication equation. **Stimulus Guidelines:** • Item difficulty can be adjusted via this example method: o The product is a whole number or a fraction. **TM4a** **Stimulus:** The student is presented with a multiplication equation of the form $\square = a \times \frac{1}{b}$. **Example Stem:** Enter the unknown number that makes the equation true. $\square = 4 \times \frac{1}{12}$ **Rubric:** (1 point) The student identifies the equivalent fraction or whole number which will make the equation true (e.g., $\frac{4}{12}$). **Response Type:** Equation/Numeric

Task Model 4b **Response Type: Equation / Numeric** **DOK Level 2** **4.NF.B.4** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. **Evidence Required:** 4. The student understands that a fraction *a*/*b* is a multiple of 1/*b*. **Tools:** None	**Prompt Features:** The student is prompted to enter the value of an unknown number in a fraction multiplication equation. **Stimulus Guidelines:** • The unknown number is one of the factors. • Item difficulty can be adjusted via these example methods: o The product is a whole number or a fraction. o The whole number factor (*a*) is replaced with a box (□). o The fractional factor ($\frac{1}{b}$) is replaced with a box (□). **TM4b** **Stimulus:** The student is presented with a multiplication equation of the form $\frac{a}{b} = a \times \frac{1}{b}$ with an unknown value. **Example Stem:** Enter the unknown number that makes the equation true. $\frac{4}{12} = \square \times \frac{1}{12}$ **Rubric:** (1 point) The student identifies the equivalent fraction or whole number which will make the equation true (e.g., 4). **Response Type:** Equation/Numeric

Task Model 4c

Response Types: Matching Tables

DOK Level 2

4.NF.4
Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

Evidence Required:
4. The student understands that a fraction *a*/*b* is a multiple of 1/*b*.

Tools: None

Prompt Features: The student is prompted to identify expressions that are equivalent to an expression of the form $c \times \frac{a}{b}$.

Stimulus Guidelines:

- Fractions presented in stem should have a denominator of 2, 3, 4, 5, 6, 8, 10, 12, or 100.
- Item difficulty may be adjusted via these example methods:
  - Use of fractions with denominators that are multiples of 2, 3, 4, 5, 6, 8, 10, 12, or 100
  - Use of an expression in the numerator or denominator

TM4c
Stimulus: The student is presented with a fraction multiplication expression of the form $c \times \frac{a}{b}$.

Example Stem 1: Decide whether each expression is equal to $5 \times \frac{2}{4}$. Click in the table to respond.

	Equal to $5 \times \frac{2}{4}$	**Not Equal to** $5 \times \frac{2}{4}$
$2 \times \frac{1}{20}$		
$4 \times \frac{2}{5}$		
$10 \times \frac{1}{4}$		

Example Stem 2: Decide whether each expression is equal to $5 \times \frac{2}{4}$. Click in the table to respond.

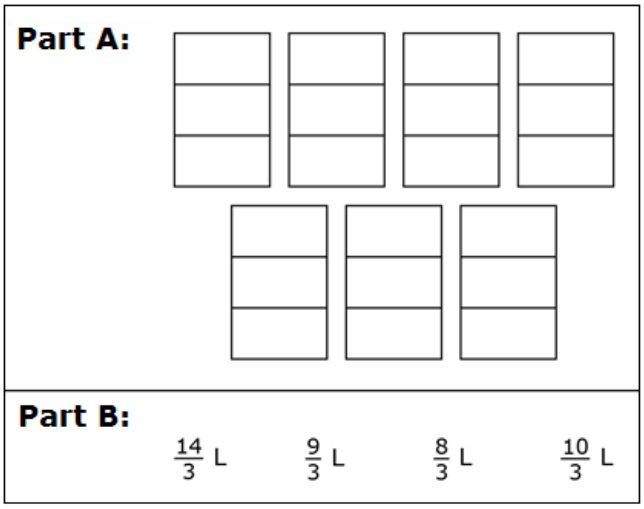
	Equal to $5 \times \frac{2}{4}$	**Not Equal to** $5 \times \frac{2}{4}$
$2 \times \frac{1}{20}$		
$2 \times \frac{5}{4}$		
$\frac{5 \times 2}{10}$		

Rubric: (1 point) The student correctly identifies the expressions as Equal or Not Equal (e.g., Not Equal, Not Equal, Equal; Not Equal, Equal, Not Equal).

Response Type: Matching Tables

Task Model 5 **Response Type: Equation / Numeric** **DOK Level 1** **4.NF.B.4** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. **Evidence Required:** 5. The student multiplies a fraction by a whole number. **Tools:** None	**Prompt Features:** The student is prompted to write the correct product of a fraction and a whole number. **Stimulus Guidelines:** • All items have an unknown product ($c \times \frac{a}{b} = \square$). • Items may present proper or improper fractions, but not mixed numbers. • Item difficulty may be adjusted via these example methods: o Unit fraction times a whole number, product is a whole number o Unit fraction times a whole number, product is not a whole number o Non-unit fraction times a whole number, product is a whole number o Non-unit fraction times a whole number, product is not a whole number **TM5** **Stimulus:** The student is presented with a fraction multiplication equation with an unknown product. **Example Stem:** Enter the unknown number that makes the equation true. $6 \times \frac{5}{8} = \square$ **Rubric:** (1 point) The student multiplies a fraction and a whole number and enters the correct product (e.g., $\frac{30}{8}$ or $3\frac{6}{8}$ or $3\ \frac{3}{4}$ or equivalent). **Response Type:** Equation/Numeric

Task Model 6a **Response Type: Equation / Numeric** **DOK Level 2** **4.NF.B.4** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. **Evidence Required:** 6. The student solves contextual problems involving the multiplication of a fraction by a whole number by using visual fraction models and equations to represent the problem. **Tools:** None	**Prompt Features:** The student is prompted to solve contextual problems involving the multiplication of a fraction by a whole number. **Stimulus Guidelines:** • All items have an unknown product $(c \times \frac{a}{b} = \square)$. • Items may present proper or improper fractions, but not mixed numbers. • Item difficulty can be adjusted via these example methods: o Unit fraction times a whole number, product is a whole number o Unit fraction times a whole number, product is not a whole number o Non-unit fraction times a whole number, product is a whole number o Non-unit fraction times a whole number, product is not a whole number **TM6a** **Stimulus:** The student is presented with a contextual problem involving the multiplication of a fraction by a whole number. **Example Stem:** A bottle holds $\frac{3}{5}$ liter of water. Sam needs 8 bottles of water to fill his fish tank. How many liters of water does Sam need to fill the fish tank? Enter the number of liters. **Rubric:** (1 point) The student enters the correct product (e.g., $\frac{24}{5}$ or $4\frac{4}{5}$). **Response Type:** Equation/Numeric

Task Model 6b **Response Type:** **Hot Spot** **DOK Level 2** **4.NF.B.4** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. **Evidence Required:** 6. The student solves contextual problems involving the multiplication of a fraction by a whole number by using visual fraction models and equations to represent the problem. **Tools:** None **Accessibility Note:** Hot Spot items are not currently able to be Brailled. Minimize the number of items developed to this TM.	**Prompt Features:** The student is prompted to solve contextual problems involving the multiplication of a fraction by a whole number using visual fraction models to solve the problem. **Stimulus guidelines:** Same as for TM6a. **TM6b** **Stimulus:** The student is presented with a contextual problem involving the multiplication of a fraction by a whole number. **Example Stem:** There are 7 people at a picnic. Each person drinks $\frac{2}{3}$ of a liter of lemonade. **Part A:** Each pitcher holds 1 liter. Click on the pitchers to shade the amount of lemonade needed for the picnic. Use the fewest number of pitchers possible. **Part B:** Click the total amount of lemonade that is needed.  **Rubric:** **Part A:** (1 point) The student correctly shades the model to represent the product (e.g., $4\frac{2}{3}$). **Part B:** (1 point) The student selects the correct product (e.g., $\frac{14}{3}$). **Response Type:** Hot Spot

Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.	
Content Domain: **Number and Operations—Fractions**	
Target H [m]: Understand decimal notation for fractions, and compare decimal fractions. (DOK 1, 2) Tasks for this target will ask students to express a fraction with denominator 10 as an equivalent fraction with denominator 100 and express fractions with either denominator as decimals. Some tasks will ask students to add fractions with unlike denominators (limited to 10 and 100). Other tasks will ask students to compare decimals to hundredths, using symbols (<, =, or >) or by location on a number line. Tasks written for Claim 2 or 4 will contextualize the concepts in this target using measurement conversion and displaying data as described in 4.MD Targets I and J. Problems for Claim 3 may explicitly connect addition of decimals to reasoning about fractions with denominators 10 and 100, using flawed reasoning or justification.	
Standards: 4.NF.C, 4.NF.C.5, 4.NF.C.6, 4.NF.C.7	**4.NF.C Understand decimal notation for fractions, and compare decimal fractions.** **4.NF.C.5** Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.* **4.NF.C.6** Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.* **4.NF.C.7** Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.
Related Below-Grade and Above-Grade Standards for Purposes of Planning for Vertical Scaling: 3.NF.A, 3.NF.A.3, 3.NF.A.3a, 3.NF.A.3b, 3.NF.A.3c, 3.NF.A.3d 5.NBT.A, 5.NBT.A.3 5.NF.A, 5.NF.A.1	**Related Grade 3 Standards** **3.NF.A Develop understanding of fractions as numbers.** **3.NF.A.3** Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. **a.** Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. **b.** Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model. **c.** Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*

	d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. **Related Grade 5 Standards** **5.NBT.A Understand the place value system.** **5.NBT.A.3** Read, write, and compare decimals to thousandths. **a.** Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000). **b.** Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. **5.NF.A Use equivalent fractions as a strategy to add and subtract fractions.** **5.NF.A.1** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*
DOK Levels:	1, 2
Achievement Level Descriptors:	
RANGE Achievement Level Descriptor (Range ALD) Target H: Understand decimal notation for fractions, and compare decimal fractions.	**Level 1** No Descriptor
	Level 2 Students should be able to express a fraction with denominator 10 as an equivalent fraction with denominator 100 and express those fractions as decimals.
	Level 3 Students should be able to add two fractions with respective denominators 10 and 100 by first converting to two fractions with like denominators; compare two decimals to the hundredths using >, <, =, or on a number line; and compare decimals by reasoning about their size.
	Level 4 Students should be able to compare two decimals to the hundredths using <, >, and = and justify the conclusions by using visual models.

Evidence Required:	1. The student expresses a fraction with denominator 10 as an equivalent fraction with denominator 100. 2. The student adds two fractions with respective denominators 10 and 100. 3. The student uses decimal notation to represent fractions with denominators 10 or 100. 4. The student locates decimal numbers to the hundredths place on a number line. 5. The student compares two decimals to the hundredths place by reasoning about their size, using the symbols <, >, or =.
Allowable Response Types:	Matching Tables; Hot Spot; Equation/Numeric; Graphing
Allowable Stimulus Materials:	<, >, and = symbols, fractions, decimals to the hundredths, decimal models, number lines, fraction addition problems
Construct-Relevant Vocabulary:	equivalent, equal, decimal, kilometers, meters, centimeters, kilograms, grams, liters, milliliters, length, mass, volume, number line, fraction, denominator, equation, expression
Allowable Tools:	None
Target-Specific Attributes:	• Denominators are limited to 10 and 100. Decimals are limited to tenths and hundredths. • Unless otherwise specified, improper fractions and mixed numbers do not receive special treatment.
Non-Targeted Constructs:	None
Accessibility Guidance:	Item writers should consider the following Language and Visual Element/Design guidelines[1] when developing items. Language Key Considerations: • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level • Avoid ambiguous or obscure words, idioms, jargon, unusual names and references Visual Elements/Design Key Considerations: • Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context • Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary • Avoid crowding of details and graphics

[1] For more information, refer to the General Accessibility Guidelines at: http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf

	Items are selected for a student's test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.[2]
Development Notes:	Much of 4.NF.C.7 will be measured in Claims 2, 3, and 4.

[2] For more information about student accessibility resources and policies, refer to http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

Task Model 1a **Response Type: Equation / Numeric** **DOK Level 1** **4.NF.C.5** Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.* **Evidence Required:** 1. The student expresses a fraction with denominator 10 as an equivalent fraction with denominator 100. **Tools:** None	**Prompt Features:** The student is prompted to find equivalent fractions with denominators 10 or 100. **Stimulus Guidelines:** • Item difficulty can be adjusted via these example methods: o Location of the unknown o Use of fractions greater than 1 **TM1a** **Stimulus:** The student is presented with an equation with an unknown numerator that sets a fraction with denominator 10 equal to a fraction with denominator 100. **Example Stem 1:** Enter the unknown number that makes this equation true. $\frac{\square}{10} = \frac{40}{100}$ **Example Stem 2:** Enter the unknown number that makes this equation true. $\frac{4}{10} = \frac{\square}{100}$ **Rubric:** (1 point) The student determines an equivalent fraction and enters the correct number (e.g., 4; 40). **Response Type:** Equation/Numeric

Task Model 1b

Response Type: Matching Tables

DOK Level 1

4.NF.C.5
Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.*

Evidence Required:
1. The student expresses a fraction with denominator 10 as an equivalent fraction with denominator 100.

Tools: None

Prompt Features: The student is prompted to find equivalent fractions with denominators 10 or 100.

Stimulus Guidelines:

- Each equation shows a fraction with denominator 10 equal to a fraction with denominator 100.
- Item difficulty can be adjusted via this example method:
  - Use of fractions greater than 1

TM1b
Stimulus: The student is presented with an equation that sets a fraction with denominator 10 equal to a fraction with denominator 100.

Example Stem: Determine if each equation is true or false. Select True or False for each equation.

	True	False
$\frac{4}{10} = \frac{40}{100}$		
$\frac{5}{10} = \frac{50}{10}$		
$\frac{11}{10} = \frac{110}{100}$		

Rubric: (1 point) The student correctly identifies all three equations as true or false, showing understanding of equivalent fractions with denominators 10 or 100 (e.g., T, F, T).

Response Type: Matching Tables

Task Model 2a

Response Type: Matching Tables

DOK Level 2

4.NF.C.5
Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.*

Evidence Required:
2. The student adds two fractions with respective denominators 10 and 100.

Tools: None

Prompt Features: The student is prompted to identify correctly solved fraction addition problems.

Stimulus Guidelines:

- In each item, fraction addition equations/expressions **must** include exactly
  - one addend with the denominator 10; and
  - one addend with the denominator 100.
- Item difficulty can be adjusted via this example method:
  - Use of fractions greater than 1

TM2a
Stimulus: The student is presented with three fraction addition equations in the answer choices.

Example Stem: Determine if each equation is true or false. Select True or False for each equation.

	True	False
$\frac{5}{10}+\frac{18}{100}=\frac{68}{100}$		
$\frac{11}{10}+\frac{13}{100}=\frac{24}{100}$		
$\frac{10}{10}+\frac{45}{100}=\frac{145}{100}$		

Rubric: (1 point) The student shows the ability to add fractions with denominators 10 and 100 by correctly identifying all three equations as true or false (e.g., T, F, T).

Response Type: Matching Tables

Task Model 2b **Response Type: Equation / Numeric** **DOK Level 2** **4.NF.C.5** Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.* **Evidence Required:** 2. The student adds two fractions with respective denominators 10 and 100. **Tools:** None	**Prompt Features:** The student solves a fraction addition problem involving fractions with denominators 10 and 100. **Stimulus Guidelines:** • In each item, fraction addition equations/expressions **must** include exactly o one addend with the denominator 10; and o one addend with the denominator 100. • The unknown number in the equation is either the sum or the numerator of the sum. • Item difficulty can be adjusted via this example method: o Use of fractions greater than 1 **TM2b** **Stimulus:** The student is presented with a fraction addition equation with an unknown number. **Example Stem 1:** Enter the unknown numerator that makes this equation true. $\frac{6}{10} + \frac{3}{100} = \frac{\square}{100}$ **Example Stem 2:** Enter the unknown number that makes this equation true. $\frac{3}{10} + \frac{15}{100} = \square$ **Rubric:** (1 point) The student finds the sum of fractions with denominators 10 or 100 and correctly enters the value of the unknown number (e.g., 63; $\frac{45}{100}$). The student may also give a correct decimal equivalent to an unknown fraction (e.g., not possible for Example Stem 1 since the unknown is a numerator only; 0.45). **Response Type:** Equation/Numeric

Task Model 2c
 	
Response Type: Equation / Numeric
 	
DOK Level 2
 	
4.NF.A.5
Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.*
 	
Evidence Required:
2. The student adds two fractions with respective denominators 10 and 100.
 	
Tools: None
	Prompt Features: The student solves a fraction addition problem involving fractions with denominators 10 and 100.
	
	Stimulus Guidelines:
	• In each item, fraction addition equations/expressions **must** include exactly
	  o one addend with the denominator 10; and
	  o one addend with the denominator 100.
	• The unknown number in the equation is either an addend or the numerator of an addend.
	• Item difficulty can be adjusted via this example method:
	  o Use of fractions greater than 1
	
	TM2c
	Stimulus: The student is presented with a fraction addition equation with an unknown number.
	
	Example Stem 1: Enter the unknown numerator that makes this equation true.
	
	$\frac{\square}{10} + \frac{15}{100} = \frac{65}{100}$
	
	Example Stem 2: Enter the unknown number that makes this equation true.
	
	$\frac{3}{10} + \square = \frac{65}{100}$
	
	Rubric: (1 point) The student finds the sum of fractions with denominators 10 or 100 and correctly enters the value of the unknown number (e.g., 5; $\frac{35}{100}$). The student may also give a correct decimal equivalent to an unknown fraction (e.g., not possible for Example Stem 1 since the unknown is a numerator only; 0.35).
	
	Response Type: Equation/Numeric

Task Model 3a **Response Type: Equation / Numeric** **DOK Level 1** **4.NF.C.6** Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.* **Evidence Required:** 3. The student uses decimal notation to represent fractions with denominators 10 or 100. **Tools:** None	**Prompt Features:** The student uses decimal notation to represent fractions with denominators 10 or 100. **Stimulus Guidelines:** • Item difficulty can be adjusted via this example method: o Use of fractions greater than 1 **TM3a** **Stimulus:** The student is presented with a fraction with denominator 10 or 100. **Example Stem:** Enter a decimal that is equivalent to $\frac{3}{10}$. **Rubric:** (1 point) The student determines an equivalent decimal representation of the given fraction and enters the correct decimal (e.g., 0.3). **Response Type:** Equation/Numeric

Task Model 3b

Response Type: Matching Tables

DOK Level 1

4.NF.C.6
Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*

Evidence Required:
3. The student uses decimal notation to represent fractions with denominators 10 or 100.

Tools: None

Prompt Features: The student selects equivalent representations of decimals and fractions with the denominators 10 or 100.

Stimulus Guidelines:

- Answer choices should be in the form of equations.
- Each answer choice should reflect a different fraction and decimal.
- Item difficulty can be adjusted via this example method:
  - Use of fractions/decimals greater than 1

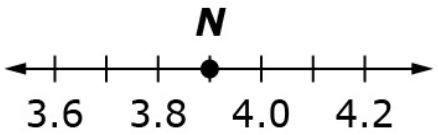
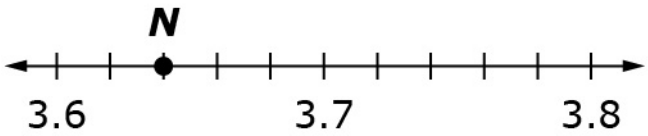
TM3b
Stimulus: The student is presented with three equations that set fractions with denominators of 10 or 100 equal to decimals, in the answer choices.

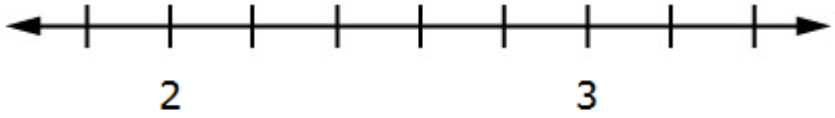
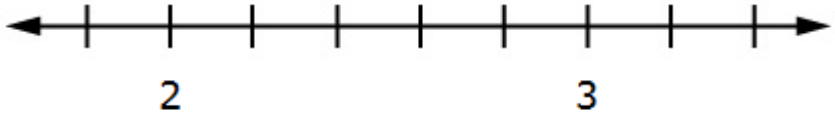
Example Stem: Determine if each equation is true or false. Select True or False for each equation.

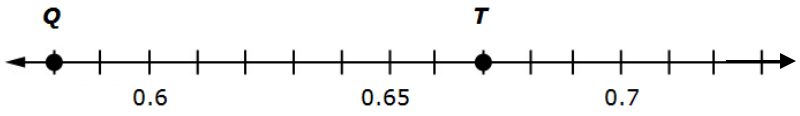
	True	False
$\frac{85}{100} = 85.100$		
$\frac{20}{100} = 0.2$		
$\frac{14}{100} = 0.014$		

Rubric: (1 point) The student correctly identifies all three equations as true or false, showing the ability to translate between decimal and fraction representations (e.g., F, T, F).

Response Type: Matching Tables

Task Model 4a **Response Type: Equation / Numeric** **DOK Level 1** **4.NF.C.6** Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.* **Evidence Required:** 4. The student locates decimal numbers to the hundredths place on a number line. **Tools:** None	**Prompt Features:** The student is prompted to identify the decimal value of a point on a number line. **Stimulus Guidelines:** • Each item presents a decimal number line. • Items must not require students to select/identify decimals that go beyond the hundredths place. **TM4a** **Stimulus:** The student is presented with a number line with a point marked on a tick mark or between two tick marks and labeled with a variable. **Example Stem 1:** Enter the decimal value of the unknown number located at point *N*.  **Example Stem 2:** Enter the decimal value of the unknown number located at point *N*.  **Rubric:** (1 point) The student locates a decimal number on a number line and enters the correct value of the variable (e.g., 3.9; 3.64). **Response Type:** Equation/Numeric

Task Model 4b **Response Type: Graphing** **DOK Level 2** **4.NF.C.6** Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.* **Evidence Required:** 4. The student locates decimal numbers to the hundredths place on a number line. **Tools:** None **Accessibility Note:** Graphing items are not currently able to be Brailled. Minimize the number of items developed to this TM.	**Prompt Features**: The student is prompted to locate a point on a decimal number line. **Stimulus Guidelines:** • Each item presents a number line. • Items must not require students to select/identify decimals that go beyond the hundredths place. **TM4b** **Stimulus:** The student is presented with a number line and a number whose value is located at a tick mark or between two tick marks on the number line. **Example Stem 1:** Use the Add Point tool to put a point on the number line to show the location of 2.2.  **Rubric:** (1 point) The student locates a decimal number on a number line and places the point on the correct tick mark (e.g., student places the point at 2.2). **Response Type:** Graphing **Example Stem 2:** Use the Add Point tool to put a point on the number line to show the location of 2.32.  **Rubric:** (1 point) The student locates a decimal number on a number line and places the point within a range equal to 10% of the interval above or below the correct spot, without placing the point on or beyond the nearest tick mark (e.g., student places the point in the range of 2.30 – 2.34). **Response Type:** Graphing

Task Model 4c **Response Type: Equation / Numeric** **DOK Level 1** **4.NF.C.6** Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.* **Evidence Required:** 4. The student locates decimal numbers to the hundredths place on a number line. **Tools:** None	**Prompt Features:** The student is prompted to select the correct location of decimal numbers to the hundredths place on a number line. **Stimulus Guidelines:** • Each item presents a decimal number line with two labeled points above different tick marks. • Items must not require students to select/identify decimals that go beyond the hundredths place. **TM4c** **Stimulus:** The student is presented with a number line which includes two labeled points. **Example Stem:** Use this number line to identify the numbers that each letter represents.  Enter the numbers represented by *Q* and *T* in the response boxes. **Rubric:** (1 point) The student shows an understanding of decimal number lines by correctly identifying the value of both letters on the number line (e.g., 0.58 and 0.67). **Response Type:** Equation/Numeric (2 labeled response boxes)

Task Model 5

Response Type: Matching Table

DOK Level 2

4.NF.C.7
Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

Evidence Required:
5. The student compares two decimals to the hundredths place by reasoning about their size, using the symbols <, >, or =.

Tools: None

Version 3 Update:
Changed TM5 from an equation/numeric response type to a matching table response type. Updated the stimulus and stem to match the new format.

Prompt Features: The student identifies the correct symbol (<, >, or =) to compare two decimals.

Stimulus Guidelines:

- Item difficulty can be adjusted via these example methods:
  - Both decimals have the same number of places represented before and after the decimal point.
  - Decimals have a different number of places represented before the decimal point, but the same number after the decimal point.
  - Decimals have a different number of places represented after the decimal point.

TM5
Stimulus: The student is presented with two pairs of decimal numbers, up to the hundredths place and directed to compare them using (<, >, or =).

Example Stem:
Select the symbol (<, >, or =) that correctly compares each pair of numbers.

	<	>	=
0.09 □ 0.7			
1.2 □ 0.37			

Rubric: (1 point) The student identifies the correct symbol to compare pairs of decimals (e.g., <, >).

Response Type: Matching Table

<table>
<tr><td colspan="2">Claim 1: Concepts and Procedures
Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.</td></tr>
<tr><td colspan="2">Content Domain: Measurement and Data</td></tr>
<tr><td colspan="2">Target I [s]: Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. (DOK 1, 2)

Tasks for this target generally require students to solve straightforward one-step contextual word problems using the four operations in a situation involving one or more of the following: measurement conversion within a single system (including decimal representations, such as expressing 62 centimeters as 0.62 meters), distances, time intervals, liquid volume in liters, mass, money, area and perimeter of rectangles.

Tasks written for Claims 2 and 4 will connect the concepts from this target to the operations described in 4.OA Target A and 4.NF Targets G and H.</td></tr>
<tr><td>Standards:
4.MD.A, 4.MD.A.1, 4.MD.A.2, 4.MD.A.3</td><td>4.MD.A Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

4.MD.A.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz; L, mL; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...

4.MD.A.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

4.MD.A.3 Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.</td></tr>
<tr><td>Related Below-Grade and Above-Grade Standards for Purposes of Planning for Vertical Scaling:

3.MD.A, 3.MD.A.1, 3.MD.A.2, 3.MD.C, 3.MD.C.7, 3.MD.C.7a, 3.MD.C.7b, 3.MD.C.7c,</td><td>Related Grade 3 Standards

3.MD.A Solve problems involving measurement and estimation of intervals of time, liquid volumes and masses of objects.

3.MD.A.1 Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

3.MD.A.2 Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (L). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same</td></tr>
</table>

3.MD.C.7d, 3.MD.D, 3.MD.D.8 5.MD.A, 5.MD.A.1	units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. **3.MD.C Geometric measurement: understand concepts of area and relate area to multiplication and to addition.** **3.MD.C.7** Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. b. Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area model to represent the distributive property in mathematical reasoning. d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems. **3.MD.D Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.** **3.MD.D.8** Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters. **Related Grade 5 Standards** **5.MD.A Convert like measurement units within a given measurement.** **5.MD.A.1** Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real-world problems.
DOK Levels:	1, 2
Achievement Level Descriptors:	
RANGE Achievement Level Descriptor (Range ALD) Target I: Solve problems involving measurement and conversion of measurements from	**Level 1** Students should be able to know relative sizes of measurement units within one system of units, including in, ft, yd; km, m, cm; kg, g; lb, oz.; L, mL; and hr, min, sec.
	Level 2 Students should be able to express measurements in a larger unit in terms of a smaller unit within a single system of measurement, record measurement equivalents in a two-column table, and apply the perimeter formula to rectangles in mathematical problems.
	Level 3 Students should be able to use the four operations to solve

a larger unit to a smaller unit.	problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit; represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale; and apply the area formula to rectangles in mathematical problems. **Level 4** Students should be able to apply the perimeter and area formulas to rectangles in word problems.
Evidence Required:	1. The student converts measurements from larger units to smaller units **within** a single system of units. 2. The student records measurement equivalents in a two-column table. 3. The student identifies measurement quantities from diagrams, such as number line diagrams that feature a measurement scale, and uses the information to solve word problems. 4. The student applies the area and perimeter formulas for rectangles to solve mathematical and real-world problems.
Allowable Response Types:	Equation/Numeric; Graphing; Matching Tables; Fill-in Tables
Allowable Stimulus Materials:	Number lines featuring measurement scales, diagrams, tables, graphics of rectangles, equations, whole numbers, decimals (to the hundredths), fractions (limited to denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100), measurements (in units of km, m, cm; kg, g; lb, oz; L, mL; hr, min, sec)
Construct-Relevant Vocabulary:	equivalent, mass, volume, interval, area, perimeter, square units
Allowable Tools:	None
Target-Specific Attributes:	• Conversion factors will not be given to students. • All conversions are within one system of measurement, from a larger unit to a smaller unit. • Multiplication items are limited to four-digit by one-digit or two-digit by two-digit. Items may not include multiplication with decimals, but may include a whole number times a fraction. • Division items are limited to four-digit by one-digit division of whole numbers. No division with decimals or fractions. • Operations on fractions may include addition or subtraction of fractions with like denominators, including mixed numbers. • Required conversions may include converting from fractions (including improper fractions and mixed numbers), decimals (to the hundredths), or whole numbers to fractions, decimals, or whole numbers.
Non-Targeted Constructs:	None

Accessibility Guidance:	Item writers should consider the following Language and Visual Element/Design guidelines[1] when developing items. Language Key Considerations: • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level • Avoid ambiguous or obscure words, idioms, jargon, unusual names and references Visual Elements/Design Key Considerations: • Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context • Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary • Avoid crowding of details and graphics Items are selected for a student's test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.[2]
Development Notes:	Tasks written for Claims 2 and 4 will connect the concepts from this target to the operations described in 4.OA Target A and 4.NF Targets G and H. Items for which the student uses the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money will be presented in Claim 2. Items that involve making conversions using compound measurements, e.g., 5 ft 2 in, will be presented in Claim 2. Items requiring the student to read a measurement off a diagram and use that data to solve a word problem will be presented in Claim 2. Solving problems that involve finding the area/perimeter of a rectangle given one dimension and the other of area/perimeter will be assessed in Claim 2. Solving multi-step problems that involve comparing areas or perimeters of different rectangles will be assessed in Claim 2.

[1] For more information, refer to the General Accessibility Guidelines at: http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf

[2] For more information about student accessibility resources and policies, refer to http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

Task Model 1a **Response Type: Equation / Numeric** **DOK Level 1** **4.MD.A.1** Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz; L, mL; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), …* **Evidence Required:** 1. The student converts measurements from larger units to smaller units within a single system of units. **Tools:** None	**Prompt Features:** The student converts measurements within one system of units. **Stimulus Guidelines:** • Conversion must be from larger unit to smaller unit within the same system of measurement. • Items will involve one of these: o Distances (km, m, cm; in, ft, yd) o Intervals of time (hr, min, sec) o Liquid volumes (L, mL) o Masses of objects (kg, g; lb, oz) • Item difficulty can be adjusted via these example methods: o Conversions involving one or two levels of separation within the same system of measurement (e.g., feet to inches would be one level of separation; yards to inches would be two levels of separation) o Use of fractions and decimals **TM1a** **Stimulus:** The student is presented with a measurement. **Example Stem 1:** Enter the unknown number that makes the statement true. 3.5 centimeters = □ millimeters **Example Stem 2:** Enter the unknown number that makes the statement true. 6 feet = □ inches **Rubric:** (1 point) The student enters the correct value (e.g., 35; 72). **Response Type:** Equation/Numeric

Task Model 1b

Response Type: Matching Tables

DOK Level 1

4.MD.A.1
Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz; L, mL; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), …*

Evidence Required:
1. The student converts measurements from larger units to smaller units within a single system of units.

Tools: None

Prompt Features: The student converts measurements within one system of units.

Stimulus Guidelines:

- Measurements in the table must be in smaller units than the measurement in the stem.
- Answer choices should be in the form of whole number, decimal (only if dealing with money), or fractional units from the same system.
- Items will involve one of these:
  - Distances (km, m, cm; in, ft, yd)
  - Intervals of time (hr, min, sec)
  - Liquid volumes (L, mL)
  - Masses of objects (kg, g; lb, oz)
- Item difficulty can be adjusted via this example method:
  - Conversions involving one or two levels of separation within the same system of measurement (e.g., feet to inches would be one level of separation; yards to inches would be two levels of separation)

TM1b
Stimulus: The student is presented with one measurement in the stem.

Example Stem: Decide whether each measurement is equal to 5 yards. Select Yes or No for each measurement.

	Yes	No
180 inches		
27 inches		
15 feet		

Rubric: (1 point) Student correctly selects yes or no for each of the given equivalencies (e.g., Y, N, Y).

Response Type: Matching Tables

<table>
<tr>
<td>Task Model 2

Response Type:
Fill-in Table

DOK Level 2

4.MD.A.1
Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz; L, mL; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), …

Evidence Required:
2. The student records measurement equivalents in a two-column table.

Tools: None</td>
<td>Prompt Features: The student is prompted to complete a table of equivalent measurements.

Stimulus Guidelines:
• The left column represents larger units and the right is for smaller units.
• The right column should be empty.
• Items will involve one of these:
o Distances (km, m, cm; in, ft, yd)
o Intervals of time (hr, min, sec)
o Liquid volumes (L, mL)
o Masses of objects (kg, g; lb, oz)
• Item difficulty can be adjusted via these example methods:
o Conversions involving one or two levels of separation within the same system of measurement (e.g., feet to inches would be one level of separation; yards to inches would be two levels of separation)
o The use of fractions or decimals
o The number of conversions the student must make (from 2 to 4 conversions per item)

TM2
Stimulus: The student is presented with a two-column table.

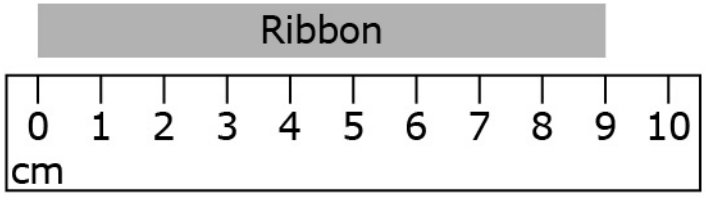
Example Stem: Enter the unknown numbers to complete the table of equal measurements.

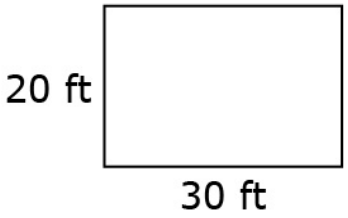
<table>
<tr><th>Feet</th><th>Inches</th></tr>
<tr><td>3</td><td></td></tr>
<tr><td>5</td><td></td></tr>
</table>

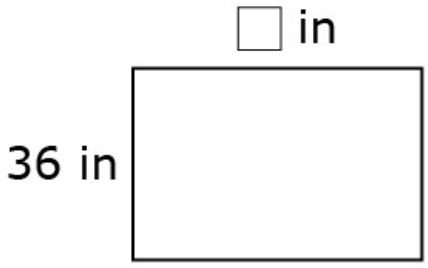
Interaction: The student enters the number of inches into the response boxes.

Rubric: (1 point) The student enters the correct equivalencies for the given measurements (e.g., 36, 60).

Response Type: Fill-in Table</td>
</tr>
</table>

Task Model 3a **Response Type: Equation / Numeric** **DOK Level 2** **4.MD.A.2** Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. **Evidence Required:** 3. The student identifies measurement quantities from diagrams, such as number line diagrams that feature a measurement scale, and uses the information to solve word problems. **Tools:** None **Version 3 update:** Revised the stem in TM3a to clarify the change in units. Retired TM3b.	**Prompt Features:** The student uses a diagram, such as a number line that features a measurement scale, to solve word problems. **Stimulus Guidelines:** • Items will involve one of these: o Distances (km, m, cm; in, ft, yd) o Intervals of time (hr, min, sec) o Liquid volumes (L, mL) o Masses of objects (kg, g; lb, oz) **TM3a** **Stimulus:** The student is presented with an object or quantity that can be measured using a number line diagram with a measurement scale. **Example Stem:** A ribbon is shown above a centimeter ruler. Enter the length, in **millimeters**, of the ribbon.  **Rubric:** (1 point) The student enters the correct measurement shown in the diagram (e.g., 90). Note: Depending on the diagram, a range of responses may need to be accepted. It is reasonable to allow 89-91 mm for the example shown above. **Response Type:** Equation/Numeric

Task Model 4a **Response Type: Equation / Numeric** **DOK Level 1** **4.MD.A.3** Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.* **Evidence Required:** 4. The student uses the area and perimeter formulas for rectangles to solve problems in mathematical and real-world contexts. **Tools:** None	**Prompt Features:** The student uses the area formula and/or perimeter formula to solve a problem in a mathematical or real-world context. **Stimulus Guidelines:** • Items may describe rectangles (in pure math context) or rectangular shapes (in a real-world context). The shapes presented in real-world contextual items must be described as "rectangular" (e.g., a rectangular garden, a rectangular kitchen, etc.). • The dimensions should be whole numbers with units listed. • Item difficulty can be adjusted via these example methods: o How "friendly" the numbers are to work with o Including a visual diagram with labeled sides **TM4a** **Stimulus:** The student is presented with the dimensions of a rectangle. **Example Stem:** Use the diagram of the rectangular garden to solve the problem.  Enter the area, in square feet, of the garden. **Rubric:** (1 point) The student enters the correct number (e.g., 600). **Response Type:** Equation/Numeric

Task Model 4b **Response Type: Equation / Numeric** **DOK Level 2** **4.MD.A.3** Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.* **Evidence Required:** 4. The student uses the area and perimeter formulas for rectangles to solve problems in mathematical and real-world contexts. **Tools:** None	**Prompt Features:** The student uses the area formula and/or perimeter formula to solve a problem in a mathematical or real-world context. **Stimulus Guidelines:** • Items may describe rectangles (in pure math context) or rectangular shapes (in a real-world context). The shapes presented in real-world contextual items must be described as "rectangular" (e.g., a rectangular garden, a rectangular kitchen, etc.). • The dimensions, areas, and perimeters should be whole numbers with units listed. • Item difficulty can be adjusted via these example methods: o How "friendly" the numbers are to work with o Including a visual diagram with labeled sides **TM4b** **Stimulus:** The student is presented with one dimension and either the area or perimeter of a rectangle and must find the unknown side length. **Example Stem:** Use the diagram of the rectangle to solve the problem.  The perimeter of the rectangle is 192 inches. What is the length, in inches, of the unknown side? **Rubric:** (1 point) The student enters the correct number (e.g., 60). **Response Type:** Equation/Numeric

Task Model 4c

Response Type: Matching Tables

DOK Level 2

4.MD.A.3
Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*

Evidence Required:
4. The student uses the area and perimeter formulas for rectangles to solve problems in mathematical and real-world contexts.

Tools: None

Prompt Features: The student is prompted to identify possible dimensions of a rectangle.

Stimulus Guidelines:

- Items may describe rectangles (in pure math context) or rectangular shapes (in a real-world context). The shapes presented in real-world contextual items must be described as "rectangular" (e.g., a rectangular garden, a rectangular kitchen, etc.).
- The dimensions, areas, and perimeters should be whole numbers with units listed.
- Item difficulty can be adjusted via this example method:
  - How "friendly" the numbers are to work with

TM4c
Stimulus: The student is presented with the area or perimeter of a rectangle.

Example Stem 1: The dimensions for three rectangles are shown. Decide whether each rectangle has an area equal to 100 square feet. Select Yes or No for each rectangle.

	Yes	No
Rectangle 1: • Length = 5 ft • Width = 20 ft		
Rectangle 2: • Length =10 ft • Width = 10 ft		
Rectangle 3: • Length = 4 ft • Width = 25 ft		

Example Stem 2: The dimensions for three rectangular gardens are shown. Decide whether each garden has a perimeter equal to 100 meters. Select Yes or No for each garden.

	Yes	No
Garden 1: • Length = 5 m • Width = 45 m		
Garden 2: • Length = 50 m • Width = 50 m		
Garden 3: • Length = 4 m • Width = 25 m		

Rubric: (1 point) The student selects all of the correct dimensions for the rectangle (e.g., Y, Y, Y; Y, N, N).

Response Type: Matching Tables

Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.	
Content Domain: **Measurement and Data**	
Target J [s]: Represent and interpret data. (DOK 1, 2) Tasks for this target will ask students to create or use a line plot and provide context for 4.NF Target G (specifically, addition and subtraction of fractions with like denominators).	
Standards: 4.MD.B, 4.MD.B.4	**4.MD.B Represent and interpret data.** **4.MD.B.4** Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*
Related Below-Grade and Above-Grade Standards for Purposes of Planning for Vertical Scaling: 3.MD.B, 3.MD.B.4 5.MD.B, 5.MD.B.2	**Related Grade 3 Standards** **3.MD.B Represent and interpret data.** **3.MD.B.4** Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters. **Related Grade 5 Standards** **5.MD.B Represent and interpret data.** **5.MD.B.2** Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*
DOK Levels:	1, 2
Achievement Level Descriptors:	
RANGE Achievement Level Descriptor (Range ALD) Target J: Represent and interpret data.	**Level 1** Students should be able to identify data from a given line plot using whole numbers.
	Level 2 Students should be able to use data from a given line plot using fractions 1/2, 1/4, and 1/8 to solve one-step problems.
	Level 3 Students should be able to create a line plot to represent a data set using fractions 1/2, 1/4, and 1/8, and interpret data from a line plot to solve problems involving addition and subtraction of fractions with like denominators.
	Level 4 No Descriptor
Evidence Required:	1. The student completes a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). 2. The student solves problems involving addition and subtraction of fractions with like denominators by using information presented in line plots.
Allowable Response Types:	Matching Tables; Hot Spot; Equation/Numeric

Allowable Stimulus Materials:	line plots, tables, fractions of a unit (1/2, 1/4, 1/8)
Construct-Relevant Vocabulary:	line plot, data set, interval, fractions, unit fractions, numerator, denominator, sum, difference, add, subtract
Allowable Tools:	None
Target-Specific Attributes:	Fractions of a unit are limited to denominators 1/2, 1/4, and 1/8. All contextual items should refer to objects that can be measured in fractions of a unit.
Non-Targeted Constructs:	None
Accessibility Guidance:	Item writers should consider the following Language and Visual Element/Design guidelines[1] when developing items. Language Key Considerations: • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level • Avoid ambiguous or obscure words, idioms, jargon, unusual names and references Visual Elements/Design Key Considerations: • Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context • Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary • Avoid crowding of details and graphics Items are selected for a student's test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.[2]
Development Notes:	Creating a line plot from scratch (where the student must partition the number line, choose an appropriate scale, and label the scale accordingly) will be assessed in Claim 4. Solving two-step addition and subtraction problems for 4.MD.B.4 will be assessed in Claim 2. Interpreting data that is presented in a line plot will be assessed in Claim 4.

[1] For more information, refer to the General Accessibility Guidelines at: http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf

[2] For more information about student accessibility resources and policies, refer to http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

Task Model 1

Response Type:
Hot Spot

DOK Level 1

4.MD.B.4
Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*

Evidence Required:
1. The student creates a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8).

Tools: None

Accessibility Note:
Hot spot items are not currently able to be Brailled. Minimize the number of items developed to this TM.

Prompt Features: The student completes a line plot that displays a given data set of measurements in fractional units.

Stimulus Guidelines:

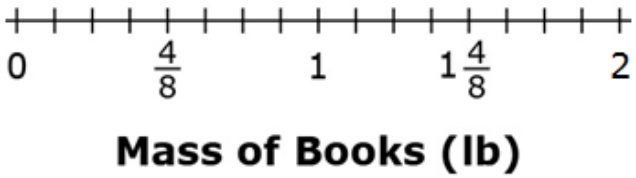
- At least two whole number endpoints must be labeled on the scale of the line plot.
- Measurement data may reflect classroom contexts or scientific contexts (appropriate to 4th grade), and are limited to these attributes and units:
  - distances (km, m, cm; in, ft, yd)
  - intervals of time (hr, min, sec)
  - liquid volumes (L, mL)
  - masses of objects (kg, g; lb, oz)
- Item difficulty can be adjusted via these example methods:
  - How many tick marks are pre-labeled or how many the student is prompted to label
  - The number of data points listed in the data set
  - Whether the data points are listed in order or given in a random sequence
  - The interval spanned by the data points—both its size and the actual endpoints
  - The form of fractions allowed as data points (e.g., proper fractions, improper fractions, mixed numbers, whole numbers)

TM1
Stimulus: The student is presented with a data set of measurements in list or table format and a number line.

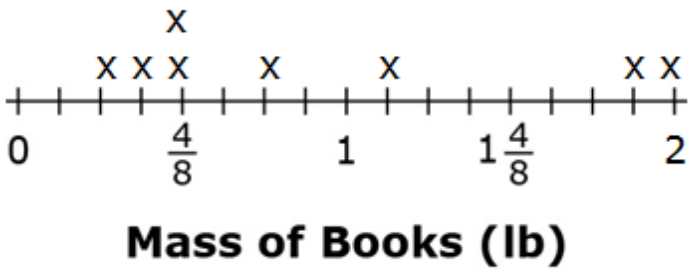
Example Stem: Michelle measures the mass of the books in her desk. The list shows the mass of each book in pounds.

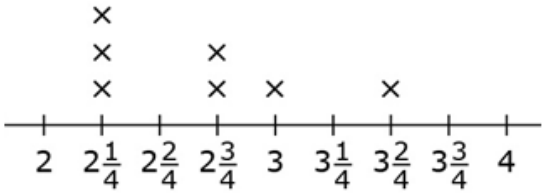
$$\frac{4}{8}, \frac{2}{8}, \frac{3}{8}, \frac{4}{8}, \frac{9}{8}, \frac{6}{8}, 1\frac{7}{8}, 2$$

Click above a tick mark to complete the line plot that displays the data.



Mass of Books (lb)

Task Model 1 **Response Type:** **Hot Spot** **DOK Level 1** **4.MD.B.4** Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.* **Evidence Required:** 1. The student creates a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). **Tools:** None **Accessibility Note:** Hot spot items are not currently able to be Brailled. Minimize the number of items developed to this TM.	**TM1 (continued)** **Rubric:** (1 point) The student places all of the correct data points to complete the line plot with no incorrect or missing points (e.g., as shown below).  **Mass of Books (lb)** **Response Type:** Hot Spot

Task Model 2 **Response Type: Equation / Numeric** **DOK Level 2** **4.MD.B.4** Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.* **Evidence Required:** 2. The student solves problems involving addition and subtraction of fractions with like denominators by using information presented in line plots. **Tools:** None	**Prompt Features:** The student solves problems involving addition and subtraction of fractions with like denominators by using information presented in line plots. **Stimulus Guidelines:** • Measurement data may reflect classroom contexts or scientific contexts (appropriate to 4th grade), and are limited to these attributes and units: o Distances (km, m, cm; in, ft, yd) o Intervals of time (hr, min, sec) o Liquid volumes (L, mL) o Masses of objects (kg, g; lb, oz) • Item difficulty can be adjusted via these example methods: o The form that the fractions take (e.g., proper fraction, improper fraction, mixed number, whole number) o The number of data points plotted in the line plot o What each X represents (e.g., does it stand for one measurement or multiple measurements?) o The interval spanned by the data points—both its size and the actual endpoints o How many of the tick marks are labeled on the line plot scale (labels must be evenly spaced) o Adding/subtracting data points that come before or after one particular point **TM2** **Stimulus:** The student is presented with a line plot that presents measurement data and a one-step question about that data. **Example Stem:** A student measured how much rain fell each week. This line plot shows the amount of rain, in inches, that fell each week.  **Amount of Rain That Fell Each Week (in)** How much more rain, in inches, was there during the week with the greatest amount of rain than during the week with the least amount of rain? Enter your answer in the response box. **Rubric:** (1 point) The student enters the correct response to solve addition or subtraction problems involving fractions based on the use of information from the line plot (e.g., $1\frac{1}{4}$). **Response Type:** Equation/Numeric

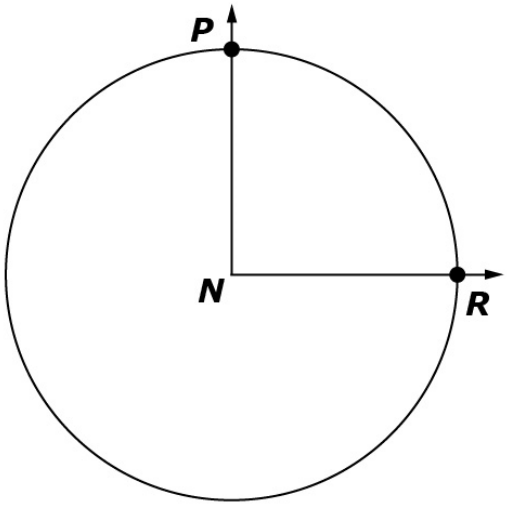
Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.	
Content Domain: **Measurement and Data**	
Target K [a]: Geometric measurement: understand concepts of angle and measure angles. (DOK 1, 2) Tasks for this target will ask students to construct and measure angles using a protractor; to provide multiple ways to decompose a larger angle into two or more smaller angles that have the same sum as the original angle; and to determine an unknown angle measure in a diagram. Some tasks will connect the angle measure back to the number of adjacent one-degree angles that comprise the whole.	
Standards: 4.MD.C, 4.MD.C.5, 4.MD.C.5a, 4.MD.C.5b, 4.MD.C.6, 4.MD.C.7	**4.MD.C Geometric measurement: understand concepts of angle and measure angles.** **4.MD.C.5** Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: **a.** An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. **b.** An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees. **4.MD.C.6** Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. **4.MD.C.7** Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.
Related Below-Grade and Above-Grade Standards for Purposes of Planning for Vertical Scaling: None	**Related Grade 3 Standards** None **Related Grade 5 Standards** None
DOK Levels:	1, 2

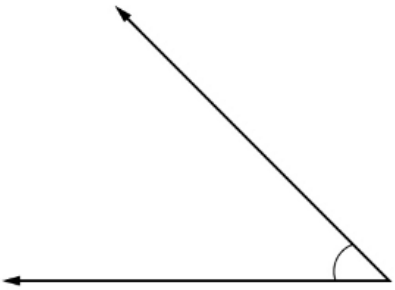
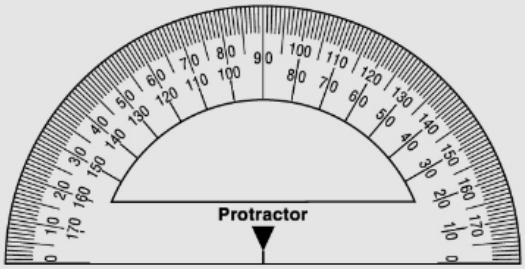
Achievement Level Descriptors:	
RANGE Achievement Level Descriptor (Range ALD) Target K: Geometric measurement: understand concepts of angles and measure angles.	**Level 1** No Descriptor
	Level 2 Students should be able to recognize whole-number degrees on a protractor and measure angles in whole-number degrees using a protractor.
	Level 3 Students should be able to construct angles in whole-number degrees using a protractor, use understanding of angle concepts to decompose a larger angle with two or more smaller angles that have the same sum as the original, and determine an unknown angle measure in a diagram.
	Level 4 Students should be able to solve addition and subtraction problems to find unknown angles on a diagram in problems by using an equation with a symbol for the unknown angle measure.
Evidence Required:	1. The student relates the concept of an angle to the fraction of a circular arc between two points on a circle. 2. The student uses a protractor to measure angles (composed of one-degree angles) and construct angles to whole-number degrees. 3. The student decomposes an angle into smaller non-overlapping parts and adds the measures of these smaller parts to find the measure of the whole angle. 4. The student solves addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems.
Allowable Response Types:	Equation/Numeric; Graphing; Drag and Drop
Allowable Stimulus Materials:	graphics of angles, turns, and rotations; protractors
Construct-Relevant Vocabulary:	protractor, angle, ray, intersect, one-degree angle, vertex, ray
Allowable Tools:	protractor
Target-Specific Attributes:	Benchmark angles are 30°, 45°, 60°, 90°, 180°
Non-Targeted Constructs:	None
Accessibility Guidance:	Item writers should consider the following Language and Visual Element/Design guidelines[1] when developing items. Language Key Considerations: • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the

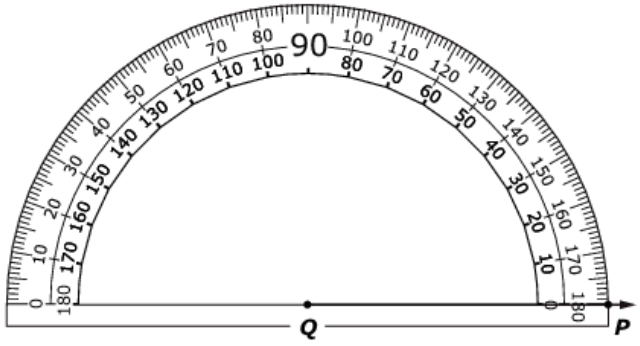
[1] For more information, refer to the General Accessibility Guidelines at: http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf

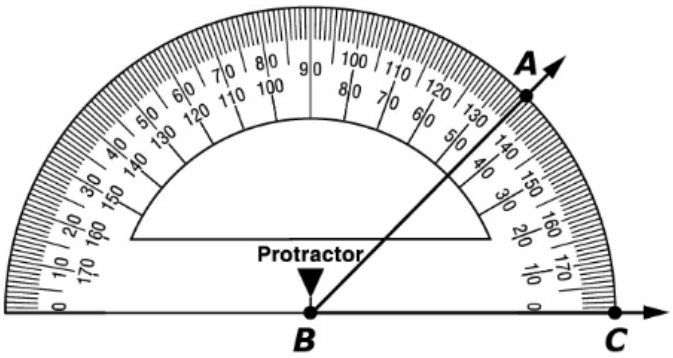
	understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level • Avoid ambiguous or obscure words, idioms, jargon, unusual names and references Visual Elements/Design Key Considerations: • Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context • Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary • Avoid crowding of details and graphics Items are selected for a student's test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.[2]
Development Notes:	Representing the addition or subtraction of angle measures with an equation will be assessed in Claim 4. Identifying angles of specified measures will be assessed at Claim 2.

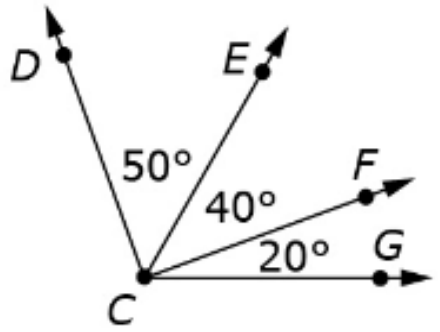
[2] For more information about student accessibility resources and policies, refer to http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

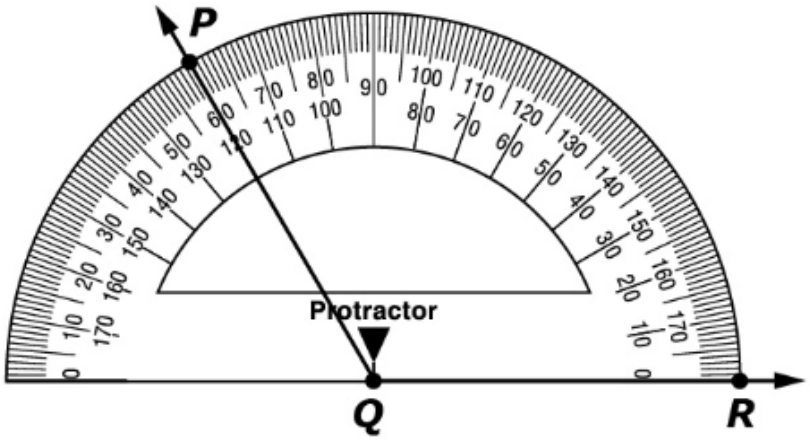
Task Model 1 **Response Type: Equation / Numeric** **DOK Level 2** **4.MD.C.5** Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: **a.** An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. **b.** An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees. **Evidence Required:** 1. The student relates the concept of an angle to the fraction of a circular arc between two points on a circle. **Tools:** None	**Prompt Features:** The student is prompted to write the measure of an angle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where two rays intersect the circle. **Stimulus Guidelines:** • Item difficulty can be adjusted via these example methods: o Whether the angle presented represents a benchmark arc size (e.g., 1/4 of a circle is easier than 1/5 of a circle) o Whether the item presents a real-world context or mathematical context **TM1** **Stimulus:** The student is presented with an angle superimposed on a circle with its vertex at the center of the circle and the fraction of a circular arc that it represents. **Example Stem:** The vertex of $\angle PNR$ is at the center of the circle. The circular arc between Point *P* and Point *R* is $\frac{1}{4}$ of the circle.  Enter the measure, in degrees, of $\angle PNR$. **Rubric:** The student enters the correct number of degrees (e.g., 90). **Response Type:** Equation/Numeric

Task Model 2a **Response Type: Drag and Drop** **DOK Level 1** **4.MD.C.6** Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. **Evidence Required:** 2. The student uses a protractor to measure angles (composed of one-degree angles) and construct angles to whole-number degrees. **Tools:** Protractor **Accessibility Note:** Drag and Drop items are not currently able to be Brailled. Minimize the number of items developed to this TM.	**Prompt Features:** The student is prompted to measure an angle using a protractor. **Stimulus Guidelines:** • A protractor must be present in the palette. • Item difficulty can be adjusted via these example methods: o The orientation of the angle presented (e.g., with a horizontal base and opening up to the left) o Whether the angle presented is a benchmark (i.e., 30°, 45°, 90°, or 180°) or non-benchmark angle **TM2a** **Stimulus:** The student is presented with an image of an angle. **Example Stem:** • Use the protractor to measure the angle. • Then drag the numbers into the box to enter the measure of the angle, in degrees.    0 5 1 6 2 7 3 8 4 9 **Rubric:** (1 point) The student enters the correct number of degrees in the angle (e.g., 45). **Response Type:** Drag and Drop

Task Model 2b **Response Type: Graphing** **DOK Level 2** **4.MD.C.6** Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. **Evidence Required:** 2. The student uses a protractor to measure angles (composed of one-degree angles) and construct angles to whole-number degrees. **Tools:** Protractor **Accessibility Note:** Graphing items are not currently able to be Brailled. Minimize the number of items developed to this TM.	**Prompt Features:** The student uses a protractor to construct an angle of a given measure. **Stimulus Guidelines:** • Item difficulty can be adjusted via these example methods: o The orientation of the ray that is presented (e.g., horizontal and pointing to the left) o Whether the student is prompted to draw a benchmark (i.e., 30°, 45°, 90°, or 180°) or non-benchmark angle **TM2b** **Stimulus:** The student is presented with an angle measure and instructions to generate the angle. **Example Stem:** Use the Add Arrow tool to draw a 45° angle that has ray *QP* as one of its sides.  **Rubric:** (1 point) The student creates an angle of given measure with the given ray as one of its sides (e.g., 45°). **Response Type:** Graphing

Task Model 2c **Response Type: Equation / Numeric** **DOK Level 2** **4.MD.C.6** Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. **Evidence Required:** 2. The student uses a protractor to measure angles (composed of one-degree angles) and construct angles to whole-number degrees. **Tools:** None	**Prompt Features:** The student uses a protractor to find the measure of a given angle. **Stimulus Guidelines:** • Item difficulty can be adjusted via these example methods: o The orientation of the rays that are presented (e.g., horizontal, pointing to the left and diagonal, pointing to the right) o Whether the student is presented with a benchmark (i.e., 30°, 45°, 90°, or 180°) or non-benchmark angle **TM2c** **Stimulus:** The student is presented with an angle imposed on a protractor and given instructions to find the measure of the angle. **Example Stem:** Enter the measure, in degrees, of ∠*ABC*.  **Rubric:** (1 point) The student enters the correct angle measure, in degrees (e.g., 45). **Response Type:** Equation/Numeric

Task Model 3a **Response Type: Equation / Numeric** **DOK Level 2** **4.MD.C.7** Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure. **Evidence Required:** 3. The student decomposes an angle into smaller non-overlapping parts and adds the measures of these smaller parts to find the measure of the whole angle. **Tools:** None	**Prompt Features:** The student enters the measure of a decomposed angle. **Stimulus Guidelines:** • All angle measures are given and student is asked to find the sum. • Angle measures can be listed or a part of the drawing. • Item difficulty can be adjusted via these example methods: o How many non-overlapping parts the angle is decomposed into o The orientation of the angle o The presence of a horizontal or vertical ray o How "friendly" the numbers are to work with **TM3a** **Stimulus:** The student is presented with an angle that is decomposed into non-overlapping parts. **Example Stem:** Use the diagram to solve the problem. • The measure of ∠*DCE* = 50°. • The measure of ∠*ECF* = 40°. • The measure of ∠*FCG* = 20°.  Enter the measure, in degrees, of ∠*DCG*. **Rubric:** (1 point) The student enters the measure of the whole angle, in degrees (e.g., 110). **Response Type:** Equation/Numeric

Task Model 3b **Response Type: Graphing** **DOK Level 2** **4.MD.C.7** Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure. **Evidence Required:** 3. The student decomposes an angle into smaller non-overlapping parts and adds the measures of these smaller parts to find the measure of the whole angle. **Tools:** None **Accessibility Note:** Graphing items are not currently able to be Brailled. Minimize the number of items developed to this TM.	**Prompt Features:** The student decomposes an angle. **Stimulus Guidelines:** • Item difficulty can be adjusted via these example methods: o How many non-overlapping parts the student decomposes the angle into o The orientation of the angle o The presence of a horizontal or vertical ray o How "friendly" the numbers are to work with **TM3b** **Stimulus:** The student is presented with a protractor showing an angle. **Example Stem:** The protractor shows the measure of ∠*PQR*. Use the Add Arrow tool to divide ∠*PQR* into two equal angles.  **Rubric:** (1 point) The student draws a ray that correctly divides the angle into two equal angles (e.g., two 60° angles). **Response Type:** Graphing

Task Model 4

Response Type: Equation / Numeric

DOK Level 2

4.MD.C.7
Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

Evidence Required:
4. The student solves addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems.

Tools: None

Prompt Features: The student solves for the unknown angle measure in a problem involving addition or subtraction of angle measures.

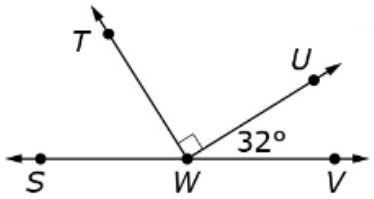
Stimulus Guidelines:

- Items that involve using the measure of a straight angle should specify the measure is 180°.
- Item difficulty can be adjusted via these example methods:
  - How many non-overlapping parts the student decomposes the angle into
  - The orientation of the angle
  - The presence of a horizontal or vertical ray
  - Whether the item refers to a real-world context or a mathematical context
  - How "friendly" the numbers are to work with

TM4
Stimulus: The student is presented with problems in real-world or mathematical contexts involving the use of angle measures of decomposed angles.

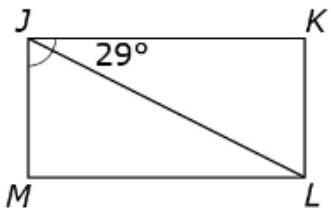
Example Stem 1: Use the figure to answer the question.

- The measure of $\angle UWV = 32°$.
- $\angle TWU$ is a right angle.
- The measure of $\angle SWV = 180°$.



Enter the measure, in degrees, of $\angle SWT$.

Example Stem 2: In the figure shown, *JKLM* is a rectangle and $\angle KJL = 29°$.



Enter the measure, in degrees, of $\angle MJL$.

Source: Illustrative Mathematics
http://www.illustrativemathematics.org/illustrations/1168

Task Model 4

Response Type: Equation / Numeric

DOK Level 2

4.MD.C.7
Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

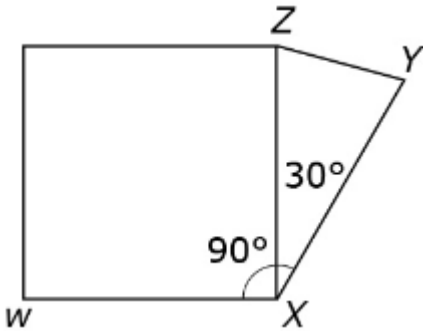
Evidence Required:
4. The student solves addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems.

Tools: None

TM4 (continued)

Example Stem 3: A student made the design shown with shapes.

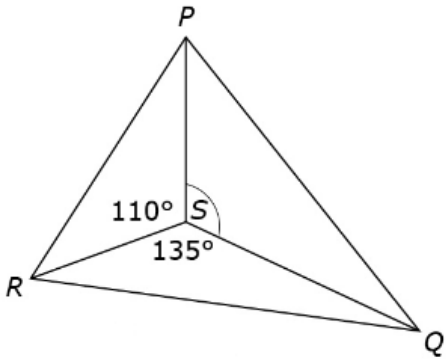
- The measure of $\angle WXZ = 90°$.
- The measure of $\angle YXZ = 30°$.



Enter the measure, in degrees, of $\angle WXY$.

Example Stem 4: A student made the design shown with shapes.

- The measure of $\angle PSR = 110°$.
- The measure of $\angle RSQ = 135°$.



Enter the measure, in degrees, of $\angle PSQ$.

Rubric: (1 point) The student enters the correct angle measure in degrees (e.g., 58; 61; 120; 115).

Response Type: Equation/Numeric

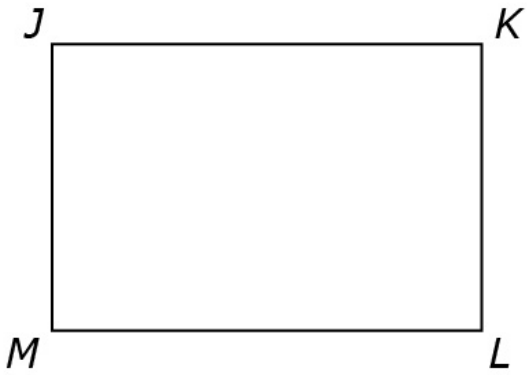
Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.	
Content Domain: **Geometry**	
Target L [a]: Draw and identify lines and angles, and classify shapes by properties of their lines and angles. (DOK 1, 2) Tasks for this target will ask students to draw or identify points, lines, line segments, rays, and parallel and perpendicular lines; to classify angles as right, acute, or obtuse (often paired with 4.MD Target K); to classify two-dimensional figures based on angles and parallel or perpendicular lines; and to draw or identify lines of symmetry in two-dimensional figures. More difficult items for this target may use symmetry as the basis for classification of two-dimensional figures (e.g., What lines of symmetry does a rectangle have to have for it to be considered a square?).	
Standards: 4.G.A, 4.G.A.1, 4.G.A.2, 4.G.A.3	**4.G.A Draw and identify lines and angles, and classify shapes by properties of their lines and angles.** **4.G.A.1** Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. **4.G.A.2** Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. **4.G.A.3** Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.
Related Below-Grade and Above-Grade Standards for Purposes of Planning for Vertical Scaling: 3.G.A, 3.G.A.1 5.G.B, 5.G.B.3, 5.G.B.4	**Related Grade 3 Standards** **3.G.A Reason with shapes and their attributes.** **3.G.A.1** Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. **Related Grade 5 Standards** **5.G.B Classify two-dimensional figures into categories based on their properties.** **5.G.B.3** Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.* **5.G.B.4** Classify two-dimensional figures in a hierarchy based on properties.
DOK Levels:	1, 2

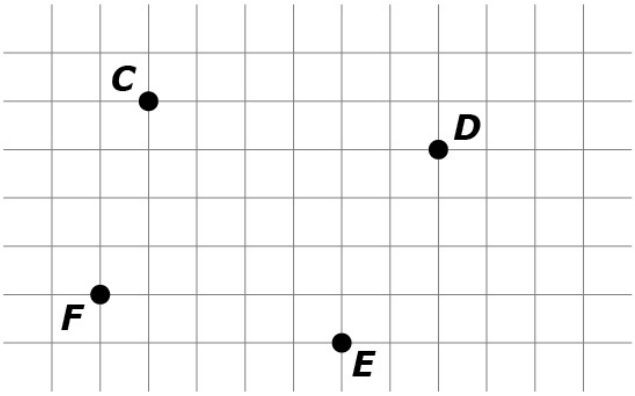
Achievement Level Descriptors:	
RANGE Achievement Level Descriptor (Range ALD) Target L: Draw and identify lines and angles, and classify shapes by properties of their lines and angles.	**Level 1** Students should be able to draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines; recognize a line of symmetry for a familiar two-dimensional figure; and identify right triangles.
	Level 2 Students should be able to identify points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines in two-dimensional figures and recognize all lines of symmetry in unfamiliar two-dimensional figures.
	Level 3 Students should be able to draw lines of symmetry for two-dimensional figures, classify two-dimensional figures based on parallel or perpendicular lines or angles of specified lines, and recognize right triangles as a category.
	Level 4 No Descriptor
Evidence Required:	1. The student draws points, lines, line segments, rays, and angles and identifies these in two-dimensional figures. 2. The student classifies two-dimensional figures based on the presence or absence of parallel/perpendicular line segments and angles of a specified size, including identifying right triangles. 3. The student identifies and draws lines of symmetry in line-symmetric figures, and distinguishes line-symmetric figures from line-asymmetric figures.
Allowable Response Types:	Matching Tables; Graphing; Hot Spot
Allowable Stimulus Materials:	drawings of two-dimensional figures, points, lines, line segments, rays, angles
Construct-Relevant Vocabulary:	point, ray, angle, line, line segment, parallel, perpendicular, right, obtuse, acute, sides, polygon, triangle, quadrilateral, pentagon, hexagon, octagon, right triangle, line of symmetry, greater than, less than, equal to
Allowable Tools:	None
Target-Specific Attributes:	None
Non-Targeted Constructs:	None
Accessibility Guidance:	Item writers should consider the following Language and Visual Element/Design guidelines[1] when developing items. Language Key Considerations: • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level • Avoid ambiguous or obscure words, idioms, jargon, unusual names and references

[1] For more information, refer to the General Accessibility Guidelines at: http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf

	Visual Elements/Design Key Considerations: • Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context • Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary • Avoid crowding of details and graphics Items are selected for a student's test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.[2]
Development Notes:	More difficult items for this target may use symmetry as the basis for classification of two-dimensional figures (e.g., What lines of symmetry does a rectangle have to have for it to be considered a square?).

[2] For more information about student accessibility resources and policies, refer to http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

Task Model 1a **Response Type:** **Hot Spot** **DOK Level 1** **4.G.A.1** Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. **Evidence Required:** 1. The student draws points, lines, line segments, rays, and angles and identifies these in two-dimensional figures. **Tools:** None **Accessibility Note:** Hot spot items are not currently able to be Brailled. Minimize the number of items developed to this TM.	**Prompt Features:** The student is prompted to identify a point, line, line segment, or ray. **Stimulus Guidelines:** • Item difficulty can be adjusted via these example methods: o The complexity of the geometric figure o The indicated element **TM1a** **Stimulus:** The student is presented with a two-dimensional geometric figure. **Example Stem:** Click on line segment *ML*.  **Rubric:** (1 point) The student selects the correct element (e.g., line segment *ML*). **Response Type:** Hot Spot

Task Model 1b **Response Type: Graphing** **DOK Level 1** **4.G.A.1** Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. **Evidence Required:** 1. The student draws points, lines, line segments, rays, and angles and identifies these in two-dimensional figures. **Tools:** None **Accessibility Note:** Graphing items are not currently able to be Brailled. Minimize the number of items developed to this TM.	**Prompt Features:** The student is prompted to draw a point, line, line segment, ray, or angle. **Stimulus Guidelines:** • Points are labeled with letter names. • Item difficulty can be adjusted via this example method: o The complexity of the indicated element • Scoring is based on whether student draws a specific point, line, line segment, ray, or angle, as identified in the stem, as opposed to drawing *any* point, line, line segment, ray, or angle. **TM1b** **Stimulus:** The student is presented with three to five points on a grid. **Example Stem:** Use the Connect Line tool to draw line segment *CD*.  **Rubric:** (1 point) The student draws the correct line segment (e.g., line segment *CD*). **Response Type:** Graphing

Task Model 2a

Response Type: Matching Tables

DOK Level 2

4.G.A.2
Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

Evidence Required:
2. The student classifies two-dimensional figures based on the presence or absence of parallel/perpendicular line segments and angles of a specified size, including identifying right triangles.

Tools: None

Prompt Features: The student is prompted to match figures to a description based on the presence or absence of angles of a specified size (right, acute, or obtuse) and/or the presence or absence of parallel or perpendicular sides.

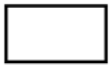
Stimulus Guidelines:

- Item difficulty can be adjusted via these example methods:
  - Convex vs. concave shapes
  - Shapes in a standard orientation vs. a non-standard orientation
  - Whether the name of the shape is given
  - Whether the shape is drawn on a grid

TM2a

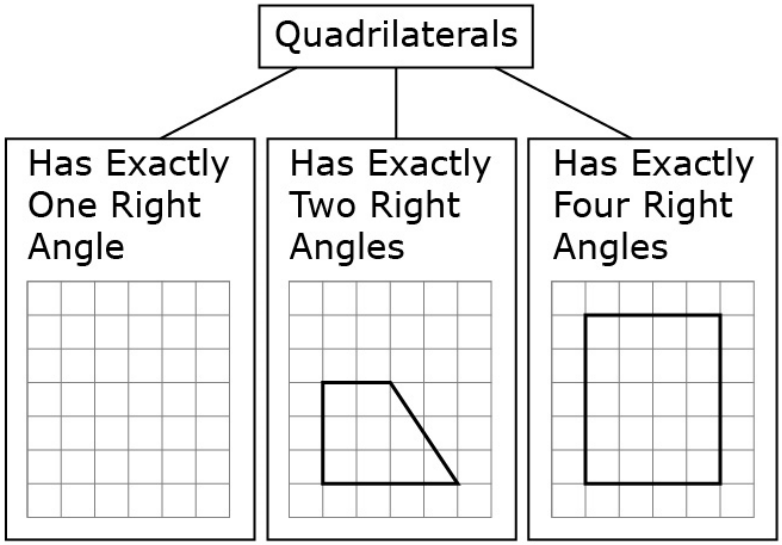
Stimulus: The student is presented with drawings of two-dimensional geometric figures and three categories based on the presence or absence of angles of a specified size (right, acute, or obtuse) and/or the presence or absence of parallel or perpendicular sides.

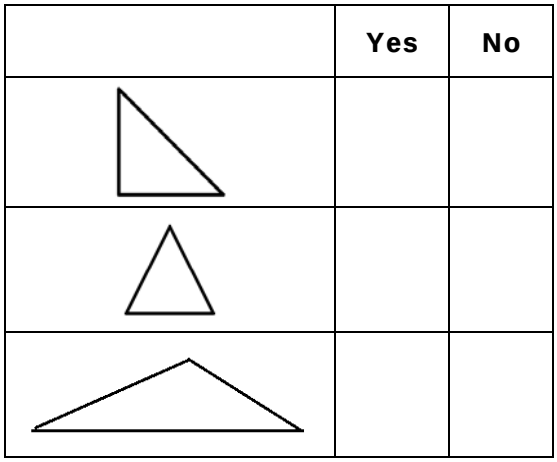
Example Stem: Click in the box that matches each figure with its description. Each figure may be matched to more than one description.

	Has one or more right angles	Has one or more pairs of perpendicular sides	Has one or more pairs of parallel sides
 Rectangle			
 Rhombus			
 Parallelogram			

Rubric: (1 point) The student correctly classifies the given figures (e.g., Rectangle: Right, Perpendicular, Parallel; Rhombus: Parallel; Parallelogram: Parallel).

Response Type: Matching Tables

Task Model 2b **Response Type: Graphing** **DOK Level 2** **4.G.A.2** Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. **Evidence Required:** 2. The student classifies two-dimensional figures based on the presence or absence of parallel/perpendicular line segments and angles of a specified size, including identifying right triangles. **Tools:** None **Accessibility Note:** Graphing items are not currently able to be Brailled. Minimize the number of items developed to this TM.	**Prompt Features:** The student is prompted to generate a two-dimensional figure that meets the requirements of a particular classification schema involving the presence or absence of angles of a specified size (right, acute, or obtuse) and/or perpendicular or parallel sides. **Stimulus Guidelines:** • Item difficulty will be adjusted via these example methods: o Whether the shapes drawn have horizontal or non-horizontal bases o How many "normal" ways there are to draw a shape that matches the empty box description **TM2b** **Stimulus:** The student is presented with a classification schema involving the presence or absence of angles of a specified size (right, acute, or obtuse) and/or perpendicular or parallel sides. **Example Stem:** This chart shows one way to classify quadrilaterals. Use the Connect Line tool to draw a quadrilateral that belongs in the box labeled "Has Exactly One Right Angle."  **Rubric:** (1 point) The student constructs a shape that meets the requirements of a classification schema (e.g., a quadrilateral with exactly one right angle). **Response Type:** Graphing

Task Model 2c **Response Type: Matching Tables** **DOK Level 1** **4.G.A.2** Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. **Evidence Required:** 2. The student classifies two-dimensional figures based on the presence or absence of parallel/perpendicular line segments and angles of a specified size, including identifying right triangles. **Tools:** None	**Prompt Features:** The student is prompted to identify right triangles. **Stimulus Guidelines:** • Triangles that are considered "not right" cannot use angles within 80-100 degrees. • The correct answer(s) will show isosceles or scalene right triangles at any rotation. • Item difficulty can be adjusted via this example method: o The orientation of the triangles' legs/hypotenuse **TM2c** **Stimulus:** The student is presented with three triangles. **Example Stem:** Decide whether the shape appears to be a right triangle. Select Yes or No for each triangle.  **Rubric:** (1 point) The student correctly identifies three triangles as right triangles or not right triangles (e.g., Y, N, N). **Response Type:** Matching Tables

Task Model 3a

Response Type: Matching Tables

DOK Level 1

4.G.A.3
Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Evidence Required:
3. The student identifies and draws lines of symmetry in line-symmetric figures, and distinguishes line-symmetric figures from line-asymmetric figures.

Tools: None

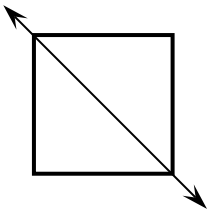
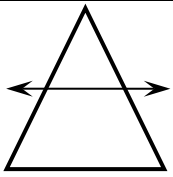
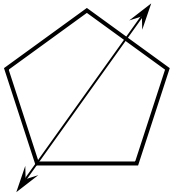
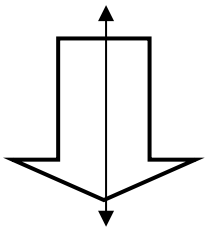
Prompt Features: The student is prompted to identify lines of symmetry in line-symmetric figures.

Stimulus Guidelines:

- Item difficulty can be adjusted via these example methods:
  - "Basic" vs. "non-basic" shapes
  - Convex vs. concave shapes
  - Shapes in a standard orientation vs. a non-standard orientation

TM3a
Stimulus: The student is presented with three shapes, each with a line drawn through it.

Example Stem: Decide whether the line appears to be a line of symmetry for the shape. Select Yes or No for each shape.

	Yes	No
		
		
		
		

Rubric: (1 point) The student correctly identifies three lines as being lines of symmetry or not (e.g., Y, N, Y, Y).

Response Type: Matching Tables

Task Model 3b

Response Types: Graphing and Hot Spot

DOK Level 2

4.G.A.3
Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Evidence Required:
3. The student identifies and draws lines of symmetry in line-symmetric figures, and distinguishes line-symmetric figures from line-asymmetric figures.

Tools: None

Accessibility Note:
Graphing and hot spot items are not currently able to be Brailled. Minimize the number of items developed to this TM.

Prompt Features: The student is prompted to generate lines of symmetry in line-symmetric figures.

Stimulus Guidelines:

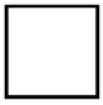
- Item difficulty can be adjusted via these example methods:
  - "Basic" vs. "non-basic" shapes
  - Convex vs. concave shapes
  - Shapes in a standard orientation vs. a non-standard orientation
  - The number of lines of symmetry the shape has (limited to no more than 4 lines of symmetry)

TM3b
Stimulus: The student is presented with a set of three line-symmetric, two-dimensional figures.

Example Stem 1: Use the Add Arrow tool to draw **all** the lines of symmetry for the shape. If there are no lines of symmetry, click None.

Example Stem 2: Use the Add Arrow tool to draw **all** the lines of symmetry for the shape. If there are no lines of symmetry, click None.

Example Stem 3: Use the Add Arrow tool to draw **all** the lines of symmetry for the shape. If there are no lines of symmetry, click None.



Rubric:
(1 point) The student correctly draws all lines of symmetry with no incorrect lines, or correctly selects None (e.g., as shown below).


;

(None is selected);


Response Type: Graphing and Hot Spot

Task Model 3c

Response Type: Matching Tables

DOK Level 2

4.G.A.3
Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Evidence Required:
3. The student identifies and draws lines of symmetry in line-symmetric figures, and distinguishes line-symmetric figures from line-asymmetric figures.

Tools: None

Prompt Features: The student is prompted to identify figures that have line-symmetry and figures that do not have line symmetry.

Stimulus Guidelines:

- Item difficulty can be adjusted via these example methods:
  - Convex vs. concave shapes
  - Shapes in a standard orientation vs. a non-standard orientation
  - Regular and irregular shapes

TM3c
Stimulus: The student is presented with three two-dimensional geometric figures.

Example Stem: Determine the number of lines of symmetry for each shape. Click in the box that matches the shape to the correct number of lines of symmetry.

	None	Exactly 1	Exactly 2	Exactly 3	More than 3
 Rectangle					
 Triangle					
 Circle					

Rubric: (1 point) The student correctly identifies the number of lines of symmetry in each shape (e.g., Exactly 2, None, More than 3).

Response Type: Matching Tables

Grade 3-5 Mathematics Item Specification Claim 2	
Problem solving, which of course builds on a foundation of knowledge and procedural proficiency, sits at the core of *doing* mathematics. Proficiency at problem solving requires students to choose to use concepts and procedures from across the content domains and check their work using alternative methods. As problem solving skills develop, student understanding of and access to mathematical concepts becomes more deeply established. (*Mathematics Content Specifications, p.56*)	
Primary Claim 2: Problem Solving Students can solve a range of well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.	
Secondary Claim(s): Items/tasks written primarily to assess Claim 2 will necessarily involve some Claim 1 content targets. Related Claim 1 targets should be listed below the Claim 2 targets in the item form. If Claim 3 or 4 targets are also directly related to the item/task, list those following the Claim 1 targets in order of prominence.	
Primary Content Domain: Each item/task should be classified as having a primary, or dominant, content focus. The content should draw upon the knowledge and skills articulated in the progression of standards leading up to and including the targeted grade within and across domains.	
Secondary Content Domain(s): While tasks developed to assess Claim 2 will have a primary content focus, components of these tasks will likely produce enough evidence for other content domains that a separate listing of these content domains needs to be included where appropriate. The standards in the NBT domain in grades 3-5 can be used to construct higher difficulty items for the adaptive pool. The integration of the OA, G, and MD domains with NBT allows for higher content limits within the grade level than might be allowed when staying within the primary content domain.	
DOK Levels	1, 2, 3
Allowable Response Types	**Response Types:** Multiple Choice, single correct response (MC); Multiple Choice, multiple correct response (MS); Equation/Numeric (EQ); Drag and Drop, Hot Spot, and Graphing (GI); Matching Tables (MA); Fill-in Table (TI) No more than five choices in MS and MA items. Short Text–Performance tasks only **Scoring:** Scoring rules and answer choices will focus on a student's ability to solve problems and/or to apply appropriate strategies to solve problems. For some problems, multiple correct responses and/or strategies are possible. • MC and MS items will be scored as correct/incorrect (1 point) • If MA items require two skills, they will be scored as:

	○ All correct choices (2 points); at least ½ but less than all correct choices (1 point) ○ Justification[1] for more than 1 point **must be** clear in the scoring rules ○ Where possible, include a "disqualifier" option that if selected would result in a score of 0 points, whether or not the student answered ½ correctly. • EQ, GI, and TI items will be scored as: ○ Single requirement items will be scored as correct/incorrect (1 point) ○ Multiple requirement items: All components correct (2 points); at least ½ but less than all correct (1 point) ○ Justification for more than 1 point **must be** clear in the scoring rules
Allowable Stimulus Materials	Effort must be made to minimize the reading load in problem situations. Use tables, diagrams with labels, and other strategies to lessen the reading load. Use simple subject-verb-object (SVO) sentences; use contexts that are familiar and relevant to students at the targeted grade level. Target-specific stimuli will be derived from the Claim 1 targets used in the problem situation. All real-world problem contexts will be relevant to the age of the students. Stimulus guidelines specific to task models are given below.
Construct Relevant Vocabulary	Refer to the Claim 1 specifications to determine Construct Relevant Vocabulary associated with specific content standards.
Allowable Tools	Any mathematical tools appropriate to the problem situation and the Claim 1 target(s). Some tools are identified in Standard for Mathematical Practice #5 and others can be found in the language of specific standards.
Target-Specific Attributes	CAT items should take from 2 to 5 minutes to solve; Claim 2 items that are part of a performance task may take 2 to 8 minutes to solve.
Accessibility Guidance:	Item writers should consider the following Language and Visual Element/Design guidelines[2] when developing items. Language Key Considerations: • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level • Avoid ambiguous or obscure words, idioms, jargon, unusual names and references Visual Elements/Design Key Considerations: • Include visual elements only if the graphic is needed to assess the construct or it aids in the

[1] For a CAT item to score multiple points, either distinct skills must be demonstrated that earn separate points or distinct levels of understanding of a complex skill must be tied directly to earning one or more points.

[2] For more information, refer to the General Accessibility Guidelines at: http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf

	understanding of the context • Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary • Avoid crowding of details and graphics Items are selected for a student's test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.[3]

[3] For more information about student accessibility resources and policies, refer to http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

Development Notes	Tasks generating evidence for Claim 2 in a given grade will draw upon knowledge and skills articulated in the progression of standards up through that grade, though more complex problem-solving tasks may draw upon knowledge and skills from lower grade levels. Claim 1 *Specifications* that cover the following standards should be used to help inform an item writer's understanding of the difference between how these standards are measured in Claim 1 versus Claim 2. Development notes have been added to many of the Claim 1 specifications that call out specific topics that should be assessed under Claim 2. There are some other useful distinctions between Claim 1 and Claim 2 in grades 3-5 that have supported the approach to alignment. The following points describe some attributes of items in Claim 2: • Multiple approaches are feasible or a range of responses is expected (e.g., if a student can solve a word problem by identifying a key word or words and selecting operations, then it is Claim 1). • The use of tools in Claim 2 is intended to support the problem solving process. In some cases, students may be asked to display their answer on the tool (e.g., by clicking the appropriate point or interval on a number line or ruler). • Assessing the reasonableness of answers to problems is a Claim 2 skill with items that align to Target C. In grades 3-5, Claim 2 tasks should be written to support two key themes: • Solving problems with fractions • Solving problems with the four operations As noted in the table below, the Measurement/Data and Geometry clusters should be used to support these two key themes. At least 80% of the items written to Claim 2 should primarily assess the standards and clusters listed in the table.

Grade 3	Grade 4	Grade 5
3.OA.A	4.OA.A	5.NBT.B
3.OA.D	4.NBT.B	5.NF.A
3.NBT.A*	4.NF.A	5.NF.B
3.MD.A	4.NF.B	5.MD.A*
3.MD.B*	4.NF.C	5.MD.C
3.MD.C	4.MD.A*	5.G.A*
3.MD.D*	4.MD.C*	

* Denotes additional and supporting clusters

Assessment Targets: Any given item/task should provide evidence for two or more Claim 2 assessment targets. Each of the following targets should not lead to a separate task: it is in *using* content from different areas, including work studied in earlier grades, that students demonstrate their problem solving proficiency. Multiple targets should be listed in order of prominence as related to the item/task.

Target A: Apply mathematics to solve well-posed problems in pure mathematics and arising in everyday life, society, and the workplace. (DOK 2, 3)
Under Claim 2, the problems should be completely formulated, and students should be asked to find a solution path from among their readily available tools.

Target B: Select and use appropriate tools strategically. (DOK 1, 2)
Tasks used to assess this target should allow students to find and choose tools; for example, using a "Search" feature to call up a formula (as opposed to including the formula in the item stem) or using a protractor in physical space.

Target C: Interpret results in the context of a situation. (DOK 2)
Tasks used to assess this target should ask students to link their answer(s) back to the problem's context. In early grades, this might include a judgment by the student of whether to express an answer to a division problem using a remainder or not based on the problem's context. In later grades, this might include a rationalization for the domain of a function being limited to positive integers based on a problem's context (e.g., understanding that the number of buses required for a given situation cannot be 32½, or that the negative values for the independent variable in a quadratic function modeling a basketball shot have no meaning in this context).

Target D: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas). (DOK 1, 2, 3)
For Claim 2 tasks, this may be a separate target of assessment explicitly asking students to use one or more potential mappings to understand the relationship between quantities. In some cases, item stems might suggest ways of mapping relationships to scaffold a problem for Claim 2 evidence.

What sufficient evidence looks like for Claim 2 (Problem-Solving)[4]:
"Although items and tasks designed to provide evidence for this claim must primarily assess the student's ability to identify the problem and to arrive at an acceptable solution, mathematical problems nevertheless require students to apply mathematical concepts and procedures."

Properties of items/tasks that assess Claim 2: The assessment of many relatively discrete and/or single-step problems can be accomplished using short constructed response items, or even computer-enhanced or selected response items.
More extensive constructed response items can effectively assess multi-stage problem solving and can also indicate unique and elegant strategies used by some students to solve a given problem, and can illuminate flaws in student's approach to solving a problem. These tasks could:

- Present non-routine[5] problems where a substantial part of the challenge is in deciding what to do, and which mathematical tools to use; and
- Involve chains of autonomous[6] reasoning, in which some tasks may take a successful student 2 to 5 minutes, depending on the age of student and complexity of the task.

"A distinctive feature of both single-step and multi-step items and tasks for Claim 2 is that they are "well-posed." That is, whether the problem deals with pure or applied contexts, the problem itself is completely formulated; the challenge is in identifying or using an appropriate solution path."

[4] Text excerpted from the Smarter Balanced Mathematics Content Specifications (p. 56-57).
[5] As noted earlier, by "non-routine" we mean that the student will not have been taught a closely similar problem, so will not be expected to *remember* a solution path but will have to *adapt* or *extend* their earlier knowledge to find one.
[6] By "autonomous" we mean that the student responds to a single prompt, without further guidance within the task.

Grade 3 Content Combinations:	**The following standards can be effectively used in various combinations in Grade 3 Claim 2 items:** **Primary emphasis for Claim 2 items: Operations and Algebraic Thinking** **Operations and Algebraic Thinking (OA)** **3.OA.A: Represent and solve problems involving multiplication and division.** **3.OA.A.1** Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.* **3.OA.A.2** Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.* **3.OA.A.3** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1] **3.OA.A.4** Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = □ ÷ 3, 6 × 6 = ?.* **3.OA.D: Solve problems involving the four operations, and identify and explain patterns in arithmetic.** **3.OA.D.8** Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.[3] **3.OA.D.9** Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.* **Standards to integrate with the focus on whole number operations:** **Numbers and Operations—Base Ten (NBT)** **3.NBT.A: Use place value understanding and properties of operations to perform multi-digit arithmetic.** **3.NBT.A.1** Use place value understanding to round whole numbers to the nearest 10 or 100. **3.NBT.A.2** Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. **3.NBT.A.3** Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

Measurement and Data (MD)

3.MD.A: Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

3.MD.A.1 Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

3.MD.A.2 Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).[6] Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.[7]

3.MD.B: Represent and interpret data.

3.MD.B.3 Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*

3.MD.B.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.

3.MD.C: Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.C.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.

a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.

b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.

3.MD.C.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

3.MD.C.7 Relate area to the operations of multiplication and addition.

a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

b. Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.

d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-

	overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems. **3.MD.D: Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.** **3.MD.D.8** Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Grade 4 Content Combinations:	**The following standards can be effectively used in various combinations in Grade 4 Claim 2 items:** **Primary emphasis for Claim 2 items at Grade 4: Operations and Algebraic Thinking, Number and Operations—Base Ten, and Number and Operations—Fractions** **Operations and Algebraic Thinking (OA)** **4.OA.A: Use the four operations with whole numbers to solve problems.** **4.OA.A.1** Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. **4.OA.A.2** Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.[1] **4.OA.A.3** Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. **Number and Operations—Fractions (NF)** **4.NF.A: Extend understanding of fraction equivalence and ordering.** **4.NF.A.1** Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. **4.NF.A.2** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. **4.NF.B: Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.**

4.NF.B.3 Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*.

- **a.** Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
- **b.** Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*
- **c.** Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
- **d.** Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

4.NF.B.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

- **a.** Understand a fraction *a*/*b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*
- **b.** Understand a multiple of *a*/*b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*
- **c.** Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

4.NF.C: Understand decimal notation for fractions, and compare decimal fractions.

4.NF.C.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.[4] *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.*

4.NF.C.6 Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*

4.NF.C.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

Number and Operations—Base Ten (NBT)

4.NBT.B: Use place value understanding and properties of operations to perform multi-digit arithmetic.

4.NBT.B.4 Fluently add and subtract multi-digit whole numbers using the standard algorithm.

4.NBT.B.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate

and explain the calculation by using equations, rectangular arrays, and/or area models.

4.NBT.B.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Standards to integrate with the focus on operations:

Measurement and Data (MD)

4.MD.A: Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

4.MD.A.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...*

4.MD.A.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

4.MD.A.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*

4.MD.C: Geometric measurement: understand concepts of angle and measure angles.

4.MD.C.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.

b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.

4.MD.C.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

4.MD.C.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

Grade 5 Content Combinations:	**The following standards can be effectively used in various combinations in Grade 5 Claim 2 items:** **Primary emphasis for Grade 5 Claim 2 items: Number and Operations—Base Ten and Number and Operations—Fractions** **Number and Operations—Base Ten (NBT)** **5.NBT.B: Perform operations with multi-digit whole numbers and with decimals to hundredths.** **5.NBT.B.5** Fluently multiply multi-digit whole numbers using the standard algorithm. **5.NBT.B.6** Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **5.NBT.B.7** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. **Number and Operations—Fractions (NF)** **5.NF.A: Use equivalent fractions as a strategy to add and subtract fractions.** **5.NF.A.1** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example,* *2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)* **5.NF.A.2** Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.* **5.NF.B: Apply and extend previous understandings of multiplication and division to multiply and divide fractions.** **5.NF.B.3** Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

5.NF.B.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

- **a.** Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show* $(2/3) \times 4 = 8/3$, *and create a story context for this equation. Do the same with* $(2/3) \times (4/5) = 8/15$. *(In general,* $(a/b) \times (c/d) = ac/bd$.*)*
- **b.** Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

5.NF.B.5 Interpret multiplication as scaling (resizing), by:

- **a.** Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
- **b.** Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.

5.NF.B.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

5.NF.B.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[1]

- **a.** Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for* $(1/3) \div 4$, *and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that* $(1/3) \div 4 = 1/12$ *because* $(1/12) \times 4 = 1/3$.
- **b.** Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for* $4 \div (1/5)$, *and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that* $4 \div (1/5) = 20$ *because* $20 \times (1/5) = 4$.
- **c.** Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*

Standards to integrate with the focus on operations:

Measurement and Data (MD)

5.MD.A: Convert like measurement units within a given measurement system.

5.MD.A.1 Convert among different-sized standard measurement units within a given measurement

	system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems. **5.MD.C: Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.** **5.MD.C.3** Recognize volume as an attribute of solid figures and understand concepts of volume measurement. **a.** A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. **b.** A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units. **5.MD.C.4** Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units. **5.MD.C.5** Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. **a.** Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. **b.** Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real world and mathematical problems. **c.** Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems. **Geometry (G)** **5.G.A: Graph points on the coordinate plane to solve real-world and mathematical problems.** **5.G.A.1** Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate). **5.G.A.2** Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Range ALDs – Claim 2 Grades 3-5	**Level 1** Students should be able to identify important quantities in the context of a familiar situation and translate words to equations or other mathematical formulation. When given the correct math tool(s), students should be able to apply the tool(s) to problems with a high degree of scaffolding.
	Level 2 Students should be able to identify important quantities in the context of an unfamiliar situation and to select tools to solve a familiar and moderately scaffolded problem or to solve a less familiar or a non-scaffolded problem with partial accuracy. Students should be able to provide solutions to familiar problems using an appropriate format (e.g., correct units, etc.). They should be able to interpret information and results in the context of a familiar situation.
	Level 3 Students should be able to map, display, and identify relationships, use appropriate tools strategically, and apply mathematics accurately in everyday life, society, and the workplace. They should be able to interpret information and results in the context of an unfamiliar situation.
	Level 4 Students should be able to analyze and interpret the context of an unfamiliar situation for problems of increasing complexity and solve problems with optimal solutions.

Target 2A: Apply mathematics to solve well-posed problems in pure mathematics and those arising in everyday life, society, and the workplace.

General Task Model Expectations for Target 2A

- The student is asked to solve a well-posed problem arising in a mathematical context or everyday life, society, or the workplace.
- Mathematical information from the context is presented in a table, graph, or diagram, or is extracted from a verbal description or pictorial representation of the context.
- Solving the problem requires one or more steps consisting of one of the four operations with whole numbers or fractions (division of fractions is limited to division of a whole number by a unit fraction or a unit fraction by a whole number).
- Understandings from geometry or measurement may be needed to determine the operations to be performed.
- The task does not indicate by key words or other scaffolding which operations are to be performed or in what order.
- Difficulty of the task may be adjusted by varying (a) the difficulty of extracting information from the context, (b) the number of steps, (c) the complexity of the numbers used, or (d) the complexity of the interpretation required.
- Tasks have DOK Level 2 or 3

Task Model 2A.1

Expectations:

- The student solves a multi-step problem with the four operations in a context involving measurement quantities.
- Items in this task model require the student to identify quantities of interest and map their relationships, often via diagrams or equations.
- Dimensions along which to vary the item include (a) varying the context, (b) varying the type of operations to be used, or (c) varying the magnitude or the types of numbers to be used.

Example Item 2A.1a (Grade 3):
Primary Target 2A (Content Domain MD), Secondary Target 1D (CCSS 3.OA.D), Tertiary Target 1G (CCSS 3.MD.A), Quaternary Target 2D

James gets home from school at 3:30 p.m. He completes 2 chores. Then he plays his computer game until 5:00 p.m.

Chore	Time to Complete
Walk dog	20 minutes
Clean room	40 minutes

Enter the **greatest** number of minutes that James can play his computer game.

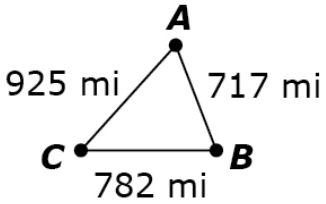
Rubric: (1 point) The student enters the correct number of minutes (30 or 30 min).

Response Type: Equation/Numeric

Commentary: This item requires the student to identify the relationship between given start and end times and the elapsed times presented in the table, and to identify the unknown quantity as the elapsed time remaining between the start and end times given. Seeing these different quantities and mapping their relationships draws on the skill set identified in Target 2D.

Example Item 2A.1b (Grade 4):
Primary Target 2A (Content Domain OA), Secondary Target 1A (CCSS 4.OA.A)

Pia's family drove from City A to City B, then City C, and back to City A. The map shows the distances.



How many miles did they drive all together? Enter your answer in the response box.

Rubric: (1 point) The student enters the correct total distance (2424 or 2424 mi).

Response Type: Equation/Numeric

Commentary: The level of difficulty for this item can be raised by changing the number of cities or having distances in the diagram that are not needed to answer the question, although adding in these extra levels of complexity moves the item closer to a Claim 4 task.

Grades 3-5, Claim 2

Example Item 2A.1c (Grade 5):

Primary Target 2A (Content Domain OA), Secondary Target 1A (CCSS 5.NF.B), Tertiary Target 2D

Luke buys a bicycle that is on sale for $\frac{1}{2}$ of the original price. The sale price is $80 less than the original price. What is the original price, in dollars, of the bicycle?

Enter your answer in the response box.

Rubric: (1 point) The student enters the correct original price (160 or $160).

Response Type: Equation/Numeric

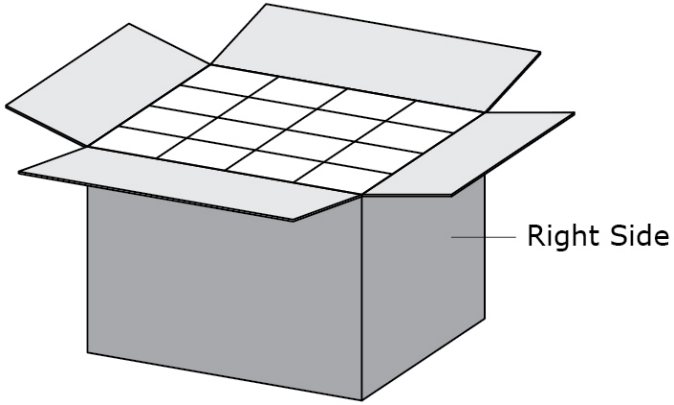
Commentary: This item requires the student to identify the sale price and the original price of a bicycle as the quantities of interest in this problem and to identify the relationship between them, and so draws on the skill set identified in Target 2D. Changing the fraction would change the difficulty level.

Grades 3-5, Claim 2

Example Item 2A.1d (Grade 5):

Primary Target 2A (Content Domain MD), Secondary Target 1I (CCSS 5.MD.C), Tertiary Target 1B (CCSS 4.OA.B), Quaternary Target 2D

A rectangular box is completely filled with 48 same-sized cubes arranged as shown. Julie opens the top of the box and sees 16 cubes.



Julie closes the top and then opens the right side of the box. How many cubes should she see?

Enter your answer in the response box.

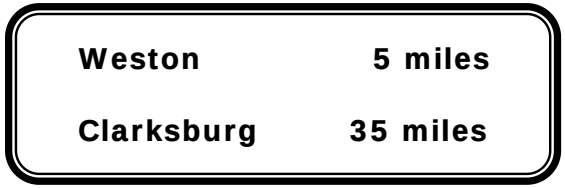
Rubric: (1 point) The student provides the correct number of cubes for the right side of the box (12).

Response Type: Equation/Numeric

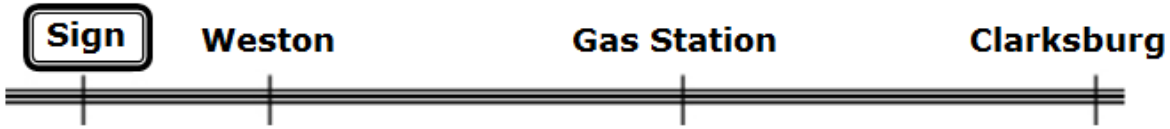
Commentary: This item requires the student to identify the volume and areas of the faces of the rectangular box as quantities of interest and to use the small cubes (and their faces) as units in order to relate the two quantities, and so draws on the skill set identified in Target 2D.

Example Item 2A.1e (Grade 5):
Primary Target 2A (Content Domain NF), Secondary Target 1F (CCSS 5.NF.B), Tertiary Target 2D

Mia is traveling along a road toward Clarksburg and sees the following sign.



Mia knows there is a gas station located halfway between Weston and Clarksburg, as shown on this diagram.



How many miles is it from Weston to Clarksburg?
Enter your answer in the first response box.

How many miles is it from the sign to the gas station?
Enter your answer in the second response box.

Rubric: (2 points) The student enters the correct distances for each question (30 or 30 mi; 20 or 20 mi).
(1 point) The student enters only one correct distance (e.g., 30 or 20).

Response Type: Equation/Numeric (2 response boxes)

Commentary: This item requires the student to identify the distances between the sign and the different cities as well as the distances between cities and understand the relationships between these quantities, and so draws on the skill set identified in Target 2D.

Task Model 2A.2

Expectations:

- The student solves a problem in a real-world or mathematical context that requires understanding of the base-ten number system.
- Items in this task model require the student to interpret base-ten numbers in terms of the context.
- Dimensions along which to vary the item include: (a) varying the context, (b) varying the type of operations to be used, or (c) varying the magnitude of the numbers to be used.

Example Item 2A.2a (Grade 3):
Primary Target 2A (Content Domain NBT), Secondary Target 1E (CCSS 3.NBT.A), Tertiary Target 2C

> Sabina has a jar full of dimes. A pack of cards costs 76 cents. How many dimes would she need to buy the cards if she uses no other coins?
>
> Enter your answer in the response box.

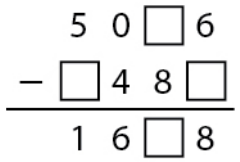
Rubric: (1 point) The student enters the correct number of dimes (8).

Response Type: Equation/Numeric

Commentary: This item requires the student to interpret the value of a collection of dimes as a multiple of ten, and so draws on the skill set identified in Claim 2C.

Example Item 2A.2b (Grade 4):
Primary Target 2A (Content Domain NBT), Secondary Target 1E (CCSS 4.NBT.B)

Drag one number into each box to complete the subtraction problem shown.



Interaction: The student drags digits 0-9 from the multi-use palette.

Rubric: (1 point) The student drags the correct digits to complete the subtraction problem (5096 – 3488 = 1608).

Response Type: Drag and Drop

Commentary: Small changes to this item change the complexity considerably. The reason that there is a unique solution is that the placement of the unknown digits and the value of the digits was highly engineered; just changing the 8 in the second number to a 5, for example, means that there will be four solutions instead of 1:

5096-3458=1638
5086-3458=1628
5076-3458=1618
5066-3458=1608

Allowing an unknown digit in the hundreds place instead of the ones place changes the complexity significantly.

Task Model 2A.3

Expectations:

- The student makes estimations about quantities in a context.
- Dimensions along which to vary the item include (a) varying the context, (b) requiring no operations (easier) or requiring computations with estimated quantities or estimating the result of computations with quantities (harder), (c) varying the magnitude of the numbers to be used.

Example Item 2A.3a (Grade 4):
Primary Target 2A (Content Domain NBT), Secondary Target 1E (CCSS 4.NBT.B), Tertiary Target 1I (CCSS 4.MD.A)

Select the response that correctly completes this statement:

41 inches is between _____.

A. 2 feet and 3 feet.
B. 3 feet and 4 feet.
C. 4 feet and 5 feet.
D. 5 feet and 6 feet.

Rubric: (1 point) The student selects the correct range (B).

Response Type: Multiple Choice, single correct response

Example Item 2A.3b (Grade 5):
Primary Target 2A (Content Domain NF), Secondary Target 1F (CCSS 5.NF.B)

Graciela walked her dog every day for the last 5 days. The time for each walk was between $\frac{1}{2}$ and $\frac{3}{4}$ of an hour. Make an estimate for the total number of minutes she walked her dog in the last 5 days.

Enter your estimate, in minutes, in the response box.

Rubric: (1 point) The student enters a value in the correct range (any number from 150 to 225, inclusive).

Response Type: Equation/numeric

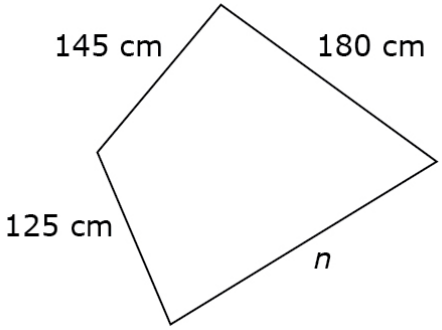
Task Model 2A.4

Expectations:

- The student solves a multi-step problem with the four operations involving whole-numbers and fractions in a purely mathematical context.
- Items in this task model require the student to identify quantities of interest and map their relationships, often via diagrams or equations.
- Dimensions along which to vary the item include (a) varying the context, (b) varying the type of operations to be used, or (c) varying the magnitude or the types of numbers to be used.

Example Item 2A.4a (Grade 3):
Primary Target 2A (Content Domain MD), Secondary Target 1D (CCSS, 3.OA.D), Tertiary Target 1J (CCSS 3.MD.D)

This quadrilateral has a perimeter of 680 centimeters.



Enter the length, in centimeters, of side *n*.

Rubric: (1 point) The student correctly enters the length of the unknown side (230 or 230 cm).

Response Type: Equation/Numeric

Example Item 2A.4b (Grade 4):
Primary Target 2A (Content Domain OA), Secondary Target 1A (CCSS 4.OA.A)

Tina and Marco play a number game. Tina gives Marco a number and he does three computations.

- He multiplies the number by 2.
- He adds 7 to the answer.
- Then, he subtracts 2 from that answer.

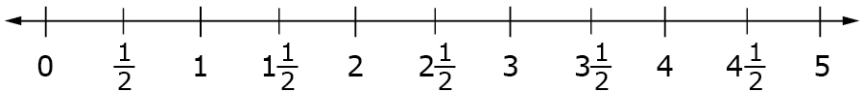
What number should Tina give Marco so that the final answer is 37 ?

Rubric: (1 point) The student enters the correct number (16).

Response Type: Equation/Numeric

Example Item 2A.4c (Grade 4):
Primary Target 2A (Content Domain NF), Secondary Target 1G (CCSS 4.NF.B), Tertiary Target 2B

Plot the value of $5 \times \frac{1}{2}$ on the number line shown.



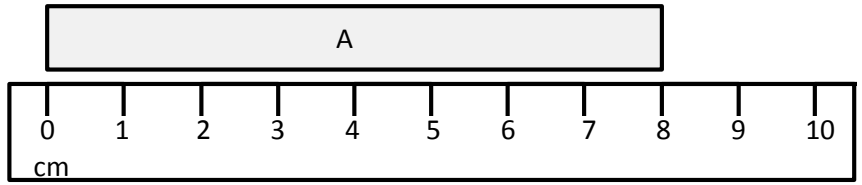
Rubric: (1 point) The student correctly plots a point at $2\frac{1}{2}$ (with a graphing tolerance of +/- $\frac{1}{16}$ or points snap to tick marks).

Response Type: Graphing (Interaction: The student is able to plot a single point somewhere on the line.)

Commentary: A variation on this item would show points on a number line and ask which one represents the product, or shows one point and asks which of four products it could be (MC). Asking for the approximate location on the number line for the results of computations would also be appropriate.

Example Item 2A.4d (Grade 5):
Primary Target 2A (Content Domain NF), Secondary Target 1F (CCSS 5.NF.B)

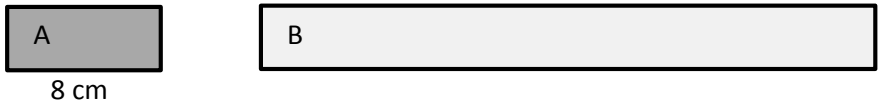
Rectangle A (shown) is $\frac{1}{4}$ as long as rectangle B (not shown). How long is rectangle B?



A. 2 cm
B. 6 cm
C. 8 cm
D. 32 cm

OR

Rectangle A is $\frac{1}{4}$ as long as rectangle B. How long is rectangle B?



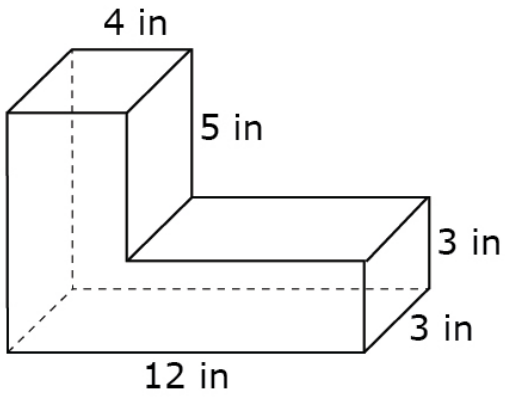
A. 2 cm
B. 6 cm
C. 8 cm
D. 32 cm

Rubric: (1 point) The student selects the correct option (D).

Response Type: Multiple Choice, single correct response

Example Item 2A.4f (Grade 5):
Primary Target 2A (Content Domain MD), Secondary Target 1I (CCSS 5.MD.C)

The figure shown was created by joining two rectangular prisms.



What is the total volume, in cubic centimeters, of the figure?

Enter your answer in the response box.

Rubric: (1 point) The student correctly enters the total volume of the figure in cubic centimeters (168 or 168 cm^3).

Response Type: Equation/Numeric

Target 2B: Select and use appropriate tools strategically.

General Task Model Expectations for Target 2B

- Mathematical information from the context is presented in a table, graph, or diagram, or is extracted from a verbal description or pictorial representation of the context.
- The student uses tools or makes strategic selection of tools.
- Tasks may require the student to use a familiar tool in a non-standard way, for example using a ruler from a non-standard starting point or using a number line to represent time.
- Difficulty of the task may be adjusted by varying (a) the difficulty of extracting information from the context (b) the number of steps (c) the complexity of the numbers used or (d) the complexity of the interpretation required.
- Task has DOK Level 1 or 2

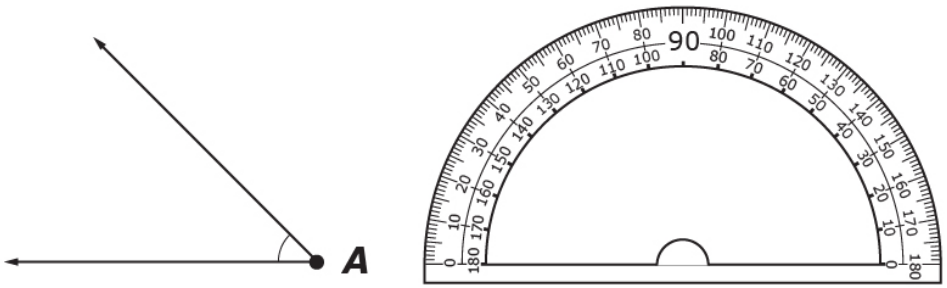
Task Model 2B.1

Expectations:

- The student demonstrates proficiency with a tool specifically identfied in the content standards.
- Tasks aligned to this task model focus on using tools (rather than selecting tools).
- Tools include measurement tools, such as rulers, protractors, and clocks, presented virtually, or number lines.
- Dimensions along which to vary the item include (a) varying the context, (b) varying the tool to be used, or (c) varying the complexity of the numbers to be used.

Example Item 2B.1a (Grade 4):
Primary Target 2B (Content Domain MD), Secondary Target 1G (CCSS 4.MD.C)

Use the protractor to find the measure of angle A.



Enter the measure of angle A, to the nearest whole degree, in the response box.

Interaction: The student can move the protractor to any point on the screen and rotate the protractor to align it with a side of the angle. See an example for how this could work here:
http://sandcastle.kasandbox.org/media/castles/Khan:master/exercises/measuring_angles.html

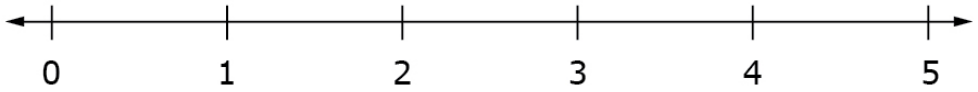
Rubric: (1 point) Student enters the correct angle measure in degrees (45+/-?).

Response Type: Equation/numeric

Commentary: Note that this technology is not currently available. An item that could assess the same construct with current technology would show a protractor with an angle whose vertex is aligned to the center point of the angle but whose rays are not aligned to the 0 or 180 marks on the protractor. This item type would fall under task model 2B.

Example Item 2B.1b (Grades 5):
Primary Target 2B (Content Domain NF), Secondary Target 1J (CCSS 5.NF.B)

Plot the value of $\frac{1}{3}$ x $\frac{5}{2}$ on the number line below. Add more tick marks and make sure the point is on a tick mark.



Interaction: The student sees a number line that has tick marks denoting the whole numbers. There is a slider or some other widget that allows the student to select the appropriate number of tick marks between whole numbers. See an example for how this could work here: https://www.youtube.com/watch?v=TEzH_PbHZIw

Rubric: (1 point) The student chooses a refinement of the number line that includes sixths and correctly plots a point at $\frac{5}{6}$ (with a graphing tolerance of +/- $\frac{1}{16}$ or there is a snap-to feature and points snap to tick marks).

Response Type: Graphing

Commentary: Note that this technology is not currently available. An item that could assess the same construct with current technology could show two or more number lines with different refinements and ask the student to use one to plot the product or to plot the product as close as possible to the correct location and have a tolerance around the location for scoring.

Grades 3-5, Claim 2

Task Model 2B.2

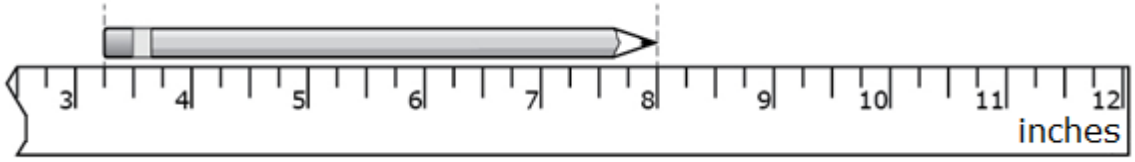
Expectations:

- The student uses a familiar tool in a non-standard way, in multi-step problem, or a problem that requires identifying quantities of interest and mapping the relationships between them.
- Dimensions along which to vary the item include (a) varying the context, (b) varying the type of operations to be used, or (c) varying the complexiy of the numbers to be used (d) varying the complexity of the interpretation required.

Example Item 2B.2a (Grade 4):

Primary Target 2B (Content Domain MD), Secondary Target 1G (CCSS 4.NF.B), Tertiary Target 1H (CCSS 3.MD.B)

What is the length, in inches, of the pencil shown?



Enter your answer in the response box.

Rubric: (1 point) The student enters the correct length in inches ($4\frac{3}{4}$).

Response Type: Equation/Numeric

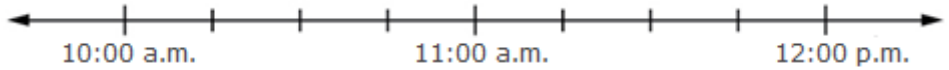
Grades 3-5, Claim 2

Example Item 2B.2b (Grade 3):

Primary Target 2B (Content Domain MD), Secondary Target 1G (CCSS 3.MD.A), Tertiary Target 2D

Math class begins at 10:45 a.m. and is 45 minutes long.

Use the Add Point tool to put a point on the number line that shows when math class ends.



Rubric: (1 point) Student places a point on the number line at the correct location (11:30 p.m.).

Response Type: Graphing

Scoring / Interaction: Scoring/interaction must allow for point to "snap to" tick marks or allow for a tolerance of +/- 5 minutes on the number line.

Commentary: This item requires the student to identify the start time, end time, and elapsed time as quantities of interest and map the relationship between them using the number line, and so draws on the skill set identified in Target 2D.

Grades 3-5, Claim 2
Example Item 2B.2c (Grade 3):

Primary Target 2B (Content Domain MD), Secondary Target 1G (CCSS 3.MD.A), Tertiary Target 2D

Mary started her homework 25 minutes before the time shown on the clock.

Fill in the table to show the time when Mary started her homework.

__ :__

Rubric: (1 point) The student shows the correct time (4:25).

Response Type: Fill-in-table

Commentary: This item requires the student to identify the start time, end time, and elapsed time as quantities of interest and map the relationship between them, and so draws on the skill set identified in Target 2D.

Task Model 2B.3

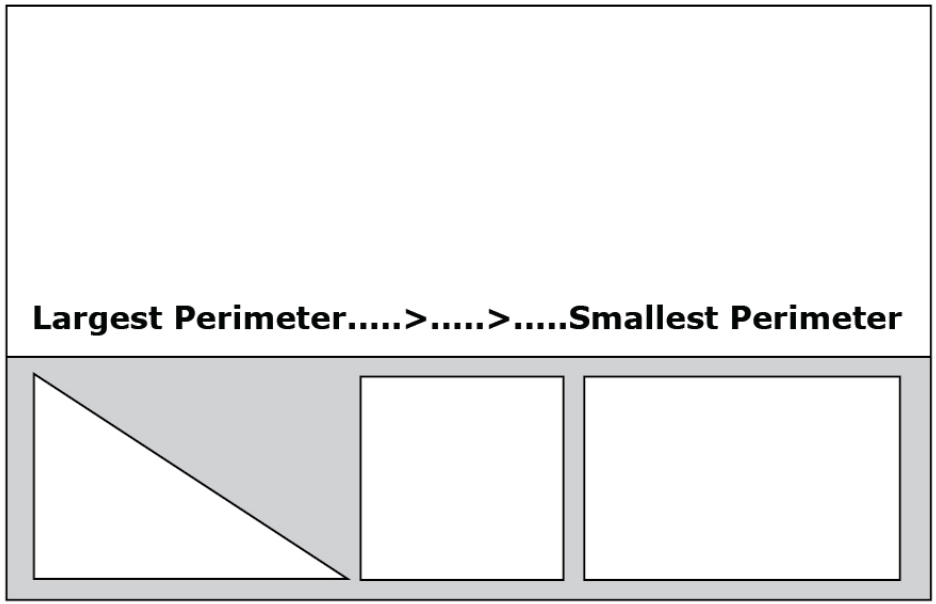
Expectations:

- The student makes strategic choices about using tools.
- The student has access to a tool that is more appropriate for some problems than others. The student may choose to use the tool or not.
- Dimensions along which to vary the item include: (a) varying the context, (b) varying the tool to be used, (c) varying the complexity of the numbers to be used.

Example Item 2B.3b (Grade 3):
Primary Target 2B (Content Domain MD), Secondary Target 1G (CCSS 3.MD.C)

Order all three figures so that the one on the left has the largest perimeter and the one on the right has the smallest perimeter.

Drag each figure into the space in order of its perimeter.



Rubric: (1 point) The student correctly orders the figures with the square first, the triangle second, and the rectangle third.

Response Type: Drag and drop.

Grades 3-5, Claim 2

Interaction: A GI background is given with active measuring and drawing tools. All three figures are presented in the bottom non-refreshable palette and the student must drag each figure into a correct arrangement, largest to smallest perimeter.

Commentary: The student has the choice of using the ruler in the Drawing and Measurement Tool or judging the perimeter without the use of tools. Strategic choices will make it easier for them to complete this item. It can be established that the rectangle has the largest perimeter by direct comparison, but it is harder to compare the perimeters of the square and the triangle without measuring the side-lengths.

Example Item 2B.3a (Grade 3):
Primary Target 2B (Content Domain MD), Secondary Target 1G (CCSS 3.MD.C)

What is the area of each figure?

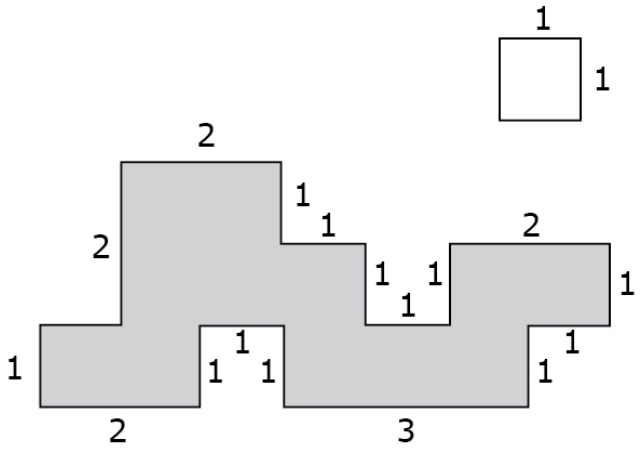


Figure A

The area of Figure A is ☐ square units.

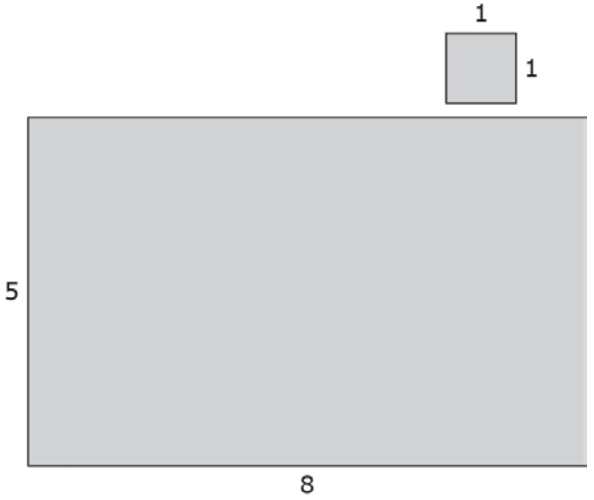


Figure B

The area of Figure B is ☐ square units.

See how the interface might work here: https://www.youtube.com/watch?v=EVoKzudbrE4

Rubric: (2 points) The student enters the correct area for each figure, 1 point for each (12 and 40).

Response Type: Equation/numeric with graphing and a combination of tiling and drag and drop as part of the unscored interaction.

Commentary: This item gives the student access to a tiling tool that can be used to cover a region with square units. The item has two parts, one where the tool can be profitably used to help the student keep track of the number of square units that are needed to cover the region without gaps or overlap, and one where knowing the relationship between the side-lengths and area of a rectangle is more efficient than using the tiling tool.

Target 2C: Interpret results in the context of a situation.

General Task Model Expectations for Target 2C

- The student provides a numeric answer to a problem where the context requires them to go beyond the result of a single computation.
- The student may be asked to choose a value that falls into a range of acceptable values limited by information given in a real-world context.
- The student may be asked to round up or round down based on the constraints of the context.
- The student may be asked to interpret the meaning of mathematical computations, for example, the different interpretations of arithmetic operations.
- The student may be asked to interpret the meaning of points on the number line or in the coordinate plane in a real-world context.
- The student may be asked to solve a problem that requires the integration of concepts and skills from multiple domains.
- Difficulty of the task may be adjusted by varying (a) the difficulty of extracting information from the context (b) the number of steps (c) the complexity of the numbers used or (d) the complexity of the interpretation required.
- Tasks have DOK Level 1 or 2.

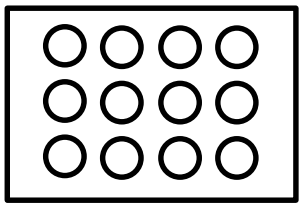
Task Model 2C.1

Expectations:

- The student chooses one value from a range of possible values that is determined by constraints in a context.
- Dimensions along which to vary the item include (a) varying the context, (b) varying the type of operations to be used, or (c) varying the type of numbers to be used.
- Tasks in this model have DOK Level 2-3.

Example Item 2C.1a (Grade 3):
Primary Target 2C (Content Domain OA), Secondary Target 1A (CCSS 3.OA.A)

Steven is baking cupcakes. A cupcake pan has 3 rows with a place to put 4 cupcakes in each row.
He filled two pans completely and part of another pan.

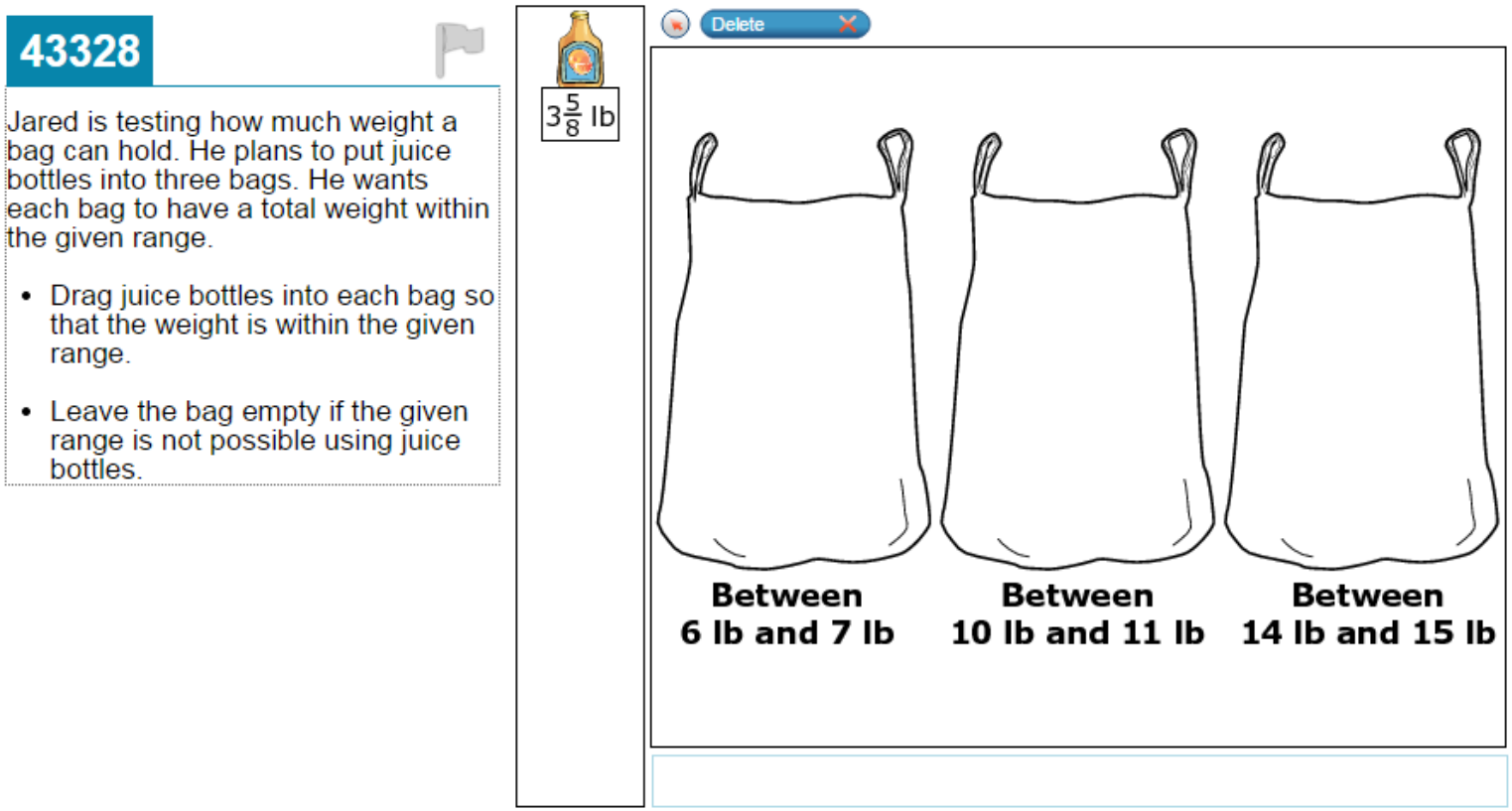


How many cupcakes could Steven have made? Enter your answer in the response box.

Rubric: (1 point) The student enters a whole number between 25 and 35, inclusive.

Response Type: Equation/Numeric

Example Item 2C.1b (Grade 4):
Primary Target 2C (Content Domain OA), Secondary Target 1G (CCSS 4.NF.B), Tertiary Target 2A (Content Domain NF).



Rubric: (2 point) The student enters the correct number of juice bottles for all three bags for 2 points or for two of the three bags for 1 point (no bottles, 3, 4).

Response Type: Drag and drop.

Example Item 2C.1c (Grade 5):
Primary Target 2C (Content Domain NF), Secondary Target 1E (CCSS 5.NF.A)

Janet has some money. She spends $\frac{1}{2}$ of her money on books. She spends some more money on videos.

Which number is a reasonable choice for the fraction of Janet's total money that she spends on books and videos?

A. $\frac{2}{7}$

B. $\frac{3}{5}$

C. $\frac{3}{2}$

D. $\frac{1}{2}$

Rubric: (1 point) The student enters the most reasonable choice (B).

Response Type: Multiple Choice, single correct response

Task Model 2C.2

Task Expectations:

- The student reports a number other than the direct result of the computations implied by the problem context because the context provides additional constraints on the allowable answers.
- Dimensions along which to vary the item include (a) varying the context, (b) varying the type of operations to be used, or (c) varying the type of numbers to be used.
- Tasks in this model have DOK Level 1 or 2.

Example Item 2C.2a (Grade 3)
Primary Target 2C (Content Domain OA), Secondary Target 1A (CCSS 3.OA.A)

Vera is making 6 picture frames. Each picture frame requires 8 craft sticks. Craft sticks are sold in packs of 10.

What is the **fewest** number of packs of craft sticks Vera can buy to get the total she needs?

Enter your answer in the response box.

Rubric: (1 point) The student enters the correct number of packs (5).

Response Type: Equation/Numeric

Grades 3-5, Claim 2

Example Item 2C.2b (Grade 4):

Primary Target 2C (Content Domain OA), Secondary Target 1A (CCSS 4.OA.A)

There are 70 students traveling to a soccer tournament. All of the vans can take 9 students each.

How many vans are needed to take all of the students to the tournament?

Enter your answer in the response box.

Rubric: (1 point) The student enters the correct number of vans needed (8).

Response Type: Equation/Numeric

Example Item 2C.2c (Grade 5):

Primary Target 2C (Content Domain NF), Secondary Target 1F (CCSS 5.NF.B)

Carl feeds his dog $2\frac{1}{2}$ cups of dog food every day. Each bag contains 64 cups of dog food.

What is the **maximum** number of days that Carl can feed his dog exactly $2\frac{1}{2}$ cups of dog food from one full bag?

Enter your answer in the response box.

Rubric: (1 point) The student is able to determine the total number of servings in one bag of food and interpret the remainder as not being enough for another whole serving (25).

Response Type: Equation/Numeric

Grades 3-5, Claim 2
Example Item 2C.2d (Grade 5):
Primary Target 2C (Content Domain NBT), Secondary Target 1D (CCSS 5.NBT.B)

Scott is buying water bottles and apples for his soccer team. The cost of buying packs of water bottles and bags of apples is shown in the table.

Item	Cost
One pack of 6 water bottles	$4.80
One bag of 5 apples	$3.20

What is the **least** amount of money that he can spend on whole packs of water bottles and bags of apple so that all 18 players on his team can have both a bottle of water and an apple?

Enter your answer, in dollars, in the response box.

Rubric: (1 point) The student enters the correct minimum cost (27.20).

Response Type: Equation/Numeric

Task Model 2C.3

Expectations:

- The student is asked to interpret the meaning of symbolic statements in a real-world context.
- Dimensions along which to vary the item include (a) varying the context, (b) varying the type of operations to be used, or (c) varying the type of the numbers to be used.
- Tasks in this model have DOK Level 2.

Example Item 2C.3a (Grade 3):
Primary Target 2C (Content Domain OA), Secondary Target 1A (CCSS 3.OA.A)

Billy has 9 full cans of juice. He has 9×8 ounces of juice all together. What could the 8 mean?

A. There are 8 ounces of juice in one full can.
B. There are 8 people who want juice.
C. He already drank 8 cans of juice.
D. He spilled 8 ounces of juice.

Rubric: (1 point) The student selects the correct option (A).

Response Type: Multiple choice, single correct response

Example Item 2C.3b (Grade 4):
Primary Target 2C (Content Domain OA), Secondary Target 1A (CCSS 4.OA.A)

Najoo is 10 years old. Her pet turtle is 40 years old. How do their ages compare?

A. Najoo is 4 years older than her turtle.
B. Her turtle is 4 years older than Najoo.
C. Najoo is 4 times as old as her turtle.
D. Her turtle is 4 times as old as Najoo.

Rubric: (1 point) The student selects the correct option (D).

Response Type: Multiple choice, single correct response

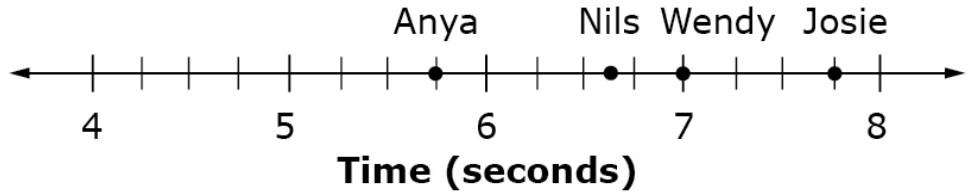
Task Model 2C.4

Task Expectations:

- The student is asked to interpret the meaning of points on a number line or in the coordinate plane in a real-world context.
- Dimensions along which to vary the item include (a) varying the context or (b) varying the type of the numbers to be used.
- Tasks in this model have DOK Level 1 or 2.

Example Item 2C.4a (Grade 3):
Primary Target 2C (Content Domain NF), Secondary Target 1F (CCSS 3.NF.A)

Three friends ran a race. The points on the number line represent the race times, in seconds, for each friend.



Who had the shortest time?

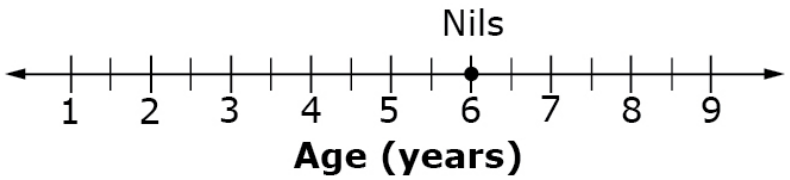
A. Anya
B. Nils
C. Wendy
D. Josie

Rubric: (1 point) The student selects the correct option (A).

Response Type: Multiple choice, single correct response

Example Item 2C.4b (Grade 3):
Primary Target 2C (Content Domain NF), Secondary Target 1F (CCSS 4.NF.C)

Hank is 8.5 years old. Nils' age in years is plotted on the number line shown.



How many years older is Hank than Nils?

Enter the number of years in the response box.

Rubric: (1 point) The student enters the correct age difference (2.5 or 2 ½).

Response Type: Equation/numeric

Grades 3-5, Claim 2
Example Item 2C.4c (Grade 5):
Primary Target 2C (Content Domain G), Secondary Target 1J (CCSS 5.G.A)

Six students ran a race. The graph shows the ages and times of the six students.

What was Petra's time in seconds?

Rubric: (1 point) The student correctly identifies Petra's time (e.g., 14.8).
Note: Accept a tolerance of +/- 0.2 seconds

Response Type: Equation/Numeric

Commentary: Variations on this item include comparing quantities that are represented by the coordinates of points on the graph or asking the student to plot a point that satisfies a given condition, for example, asking the student to plot a point for Wendy, who has a shorter race time than Petra.

Target 2D: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas).

Target 2D identifies a key step in the modeling cycle, and is thus frequently present in problems with real-world contexts. Note that Target 2D is rarely the primary target for an item, but is frequently a Secondary or Tertiary Target for an item with primary alignment to 2A, 2B, or 2C. See Items 1, 3, 4, and 5 in Task Model 1a, Item 1 in Task model 1d, and Items 2 and 3 in Task model 2a for examples that draw upon the skill set described in Target 2D.

General Task Model Expectations for Target 2D

- The student is presented with a mathematical problem in a real-world context where the quantities of interest are not named explicitly, are named but represented in different ways, or the relationship between the quantities is not immediately clear.
- The student is asked to solve a problem that may require the integration of concepts and skills from multiple domains.

Grades 3-5 Mathematics Item Specification Claim 3	
This claim refers to a recurring theme in the CCSSM content and practice standards: the ability to construct and present a clear, logical, convincing argument. For older students this may take the form of a rigorous deductive proof based on clearly stated axioms. For younger students this will involve more informal justifications. Assessment tasks that address this claim will typically present a claim or a proposed solution to a problem and will ask students to provide, for example, a justification, an explanation, or counter-example. (*Mathematics Content Specifications, p.63*) Communicating mathematical reasoning is not just a requirement of the Standards for Mathematical Practice—it is also a recurrent theme in the Standards for Mathematical Content. For example, many content standards call for students to explain, justify, or illustrate.	
Primary Claim 3: Communicating Reasoning: Students clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.	
Secondary Claim(s): Items/tasks written primarily to assess Claim 3 will necessarily involve some Claim 1 content targets. Related Claim 1 targets should be listed below the Claim 3 targets in the item form. If Claim 2 or Claim 4 targets are also directly related to the item/task, list those following the Claim 1 targets in order of prominence.	
Primary Content Domain: Each item/task should be classified as having a primary, or dominant, content focus. The content should draw upon the knowledge and skills articulated in the progression of standards leading up to and including the targeted grade within and across domains.	
Secondary Content Domain(s): While tasks developed to assess Claim 3 will have a primary content focus, components of these tasks will likely produce enough evidence for other content domains that a separate listing of these content domains needs to be included where appropriate.	
DOK Levels	1, 2, 3
Allowable Response Types	**Response Types:** Multiple Choice, single correct response (MC); Multiple Choice, multiple correct response (MS); Equation/Numeric (EQ); Drag and Drop, Hot Spot, and Graphing (GI); Matching Table (MA); Fill-in Table (TI) No more than five choices in MS and MA items. Short Text–Performance tasks only **Scoring:** Scoring rules and answer choices will focus on a student's ability to solve problems and/or to apply appropriate strategies to solve problems. For some problems, multiple correct responses and/or strategies are possible. • MC and MS items will be scored as correct/incorrect (1 point) • If MA items require two skills, they will be scored as: o All correct choices (2 points); at least ½ but less than all correct choices (1 point)

	○ Justification[1] for more than 1 point **must be** clear in the scoring rules ○ Where possible, include a "disqualifier" option that if selected would result in a score of 0 points, whether or not the student answered ½ correctly. • EQ, GI, and TI items will be scored as: ○ Single requirement items will be scored as correct/incorrect (1 point) ○ Multiple requirement items: All components correct (2 points); at least ½ but less than all correct (1 point) ○ Justification for more than 1 point **must be** clear in the scoring rules
Allowable Stimulus Materials	Effort must be made to minimize the reading load in problem situations. Use tables, diagrams with labels, and other strategies to lessen the reading load. Use simple subject-verb-object (SVO) sentences; use contexts that are familiar and relevant to students at the targeted grade level. Target-specific stimuli will be derived from the Claim 1 targets used in the problem situation. All real-world problem contexts will be relevant to the age of the students. Stimulus guidelines specific to task models are given below.
Construct Relevant Vocabulary	Refer to the Claim 1 specifications to determine Construct Relevant Vocabulary associated with specific content standards.
Allowable Tools	Any mathematical tools appropriate to the problem situation and the Claim 1 target(s). Some tools are identified in Standard for Mathematical Practice #5 and others can be found in the language of specific standards.
Target-Specific Attributes	CAT items should take from 2 to 5 minutes to solve; Claim 3 items that are part of a performance task may take 3 to 10 minutes to solve.
Accessibility Guidance:	Item writers should consider the following Language and Visual Element/Design guidelines[2] when developing items. Language Key Considerations: • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level • Avoid ambiguous or obscure words, idioms, jargon, unusual names and references Visual Elements/Design Key Considerations: • Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context • Use the simplest graphic possible with the greatest degree of contrast, and include clear,

[1] For a CAT item to score multiple points, either distinct skills must be demonstrated that earn separate points or distinct levels of understanding of a complex skill must be tied directly to earning one or more points.

[2] For more information, refer to the General Accessibility Guidelines at: http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf

	concise labels where necessary • Avoid crowding of details and graphics Items are selected for a student's test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.[3]
Development Notes	• Items and task assessing Claim 3 may involve application of more than one standard. The focus is on communicating reasoning rather than demonstrating mathematical concepts or simple applications of mathematical procedures. • Targeted content standards for Claim 3 should belong to the major work of the grade (reference table of standards shown below). • Claim 1 *Specifications* that cover the following standards should be used to help inform an item writer's understanding of the difference between how these standards are measured in Claim 1 versus Claim 3. Development notes have been added to many of the Claim 1 specifications that call out specific topics that should be assessed under Claim 3. • Claim 3 items that require any degree of hand scoring can only be developed for performance tasks for grades 3-5. At least 80% of the items written to Claim 3 should primarily assess the standards and clusters listed in the table that follows.

Grade 3	Grade 4	Grade 5
3.OA.B	4.OA.A.3	5.NBT.A.2
3.NF.A	4.NBT.A	5.NBT.B.6
3.NF.A.1	4.NBT.B.5	5.NBT.B.7
3.NF.A.2	4.NBT.B.6	5.NF.A.1
3.NF.A.3	4.NF.A	5.NF.A.2
3.MD.A	4.NF.A.1	5.NF.B
3.MD.C.7	4.NF.A.2	5.NF.B.3
	4.NF.B.3a	5.NF.B.4
	4.NF.B.3b	5.NF.B.7a
	4.NF.B.3c	5.NF.B.7b
	4.NF.B.4a	5.MD.C
	4.NF.B.4b	5.MD.C.5a
	4.NF.C	5.MD.C.5b
	4.NF.C.7	5.G.B*
		5.G.B.4*

*Denotes additional and supporting clusters

[3] For more information about student accessibility resources and policies, refer to http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

Assessment Targets: Any given item/task should provide evidence for several of the following assessment targets; each of the following targets should not lead to a separate task. Multiple targets should be listed in order of prominence as related to the item/task.

Target A: Test propositions or conjectures with specific examples. (DOK 2)
Tasks used to assess this target should ask for specific examples to support or refute a proposition or conjecture (e.g., An item stem might begin, "Provide 3 examples to show why/how…").

Target B: Construct, autonomously[4], chains of reasoning that will justify or refute propositions or conjectures[5]. (DOK 3, 4)
Tasks used to assess this target should ask students to develop a chain of reasoning to justify or refute a conjecture. Tasks for Target B might include the types of examples called for in Target A as part of this reasoning, but should do so with a lesser degree of scaffolding than tasks that assess Target A alone. Some tasks for this target will ask students to formulate and justify a conjecture.

Target C: State logical assumptions being used. (DOK 2, 3)
Tasks used to assess this target should ask students to use stated assumptions, definitions, and previously established results in developing their reasoning. In some cases, the task may require students to provide missing information by researching or providing a reasoned estimate.

Target D: Use the technique of breaking an argument into cases. (DOK 2, 3)
Tasks used to assess this target should ask students to determine under what conditions an argument is true, to determine under what conditions an argument is not true, or both.

Target E: Distinguish correct logic or reasoning from that which is flawed and—if there is a flaw in the argument—explain what it is. (DOK 2, 3, 4)
Tasks used to assess this target present students with one or more flawed arguments and ask students to choose which (if any) is correct, explain the flaws in reasoning, and/or correct flawed reasoning.

Target F: Base arguments on concrete referents such as objects, drawings, diagrams, and actions. (DOK 2, 3)
In earlier grades, the desired student response might be in the form of concrete referents. In later grades, concrete referents will often support generalizations as part of the justification rather than constituting the entire expected response.

[4] By "autonomous" we mean that the student responds to a single prompt, without further guidance within the task.

[5] At the secondary level, these chains may take a successful student 10 minutes to construct and explain. Times will be somewhat shorter for younger students, but still giving them time to think and explain. For a minority of these tasks, subtasks may be constructed to facilitate entry and assess student progress towards expertise. Even for such "apprentice tasks" part of the task will involve a chain of autonomous reasoning that takes at least 5 minutes.

Grade 3 standards that lend themselves to communicating reasoning	**The following standards can be effectively used in various combinations in Grade 3 Claim 3 items:** **Operations and Algebraic Thinking (OA)** **3.OA.B:** Understand properties of multiplication and the relationship between multiplication and division. **Number and Operations—Fractions (NF)** **3.NF.A:** Develop understanding of fractions as numbers. **3.NF.A.1** Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*. **3.NF.A.2** Understand a fraction as a number on the number line; represent fractions on a number line diagram. **3.NF.A.3** Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. **Measurement and Data (MD)** **3.MD.A:** Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects. **3.MD.C:** Geometric measurement: understand concepts of area and relate area to multiplication and to addition. **3.MD.C.7** Relate area to the operations of multiplication and addition.

Grade 4 standards that lend themselves to communicating reasoning	**The following standards can be effectively used in various combinations in Grade 4 Claim 3 items:** **Operations and Algebraic Thinking (OA)** **4.OA.A.3** Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. **Number and Operations in Base Ten (NBT)** **4.NBT.B:** Use place value understanding and properties of operations to perform multi-digit arithmetic **4.NBT.B5** Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **4.NBT.B.6** Find whole-number quotients and remainders with up to four-digit dividends and one-

Grade 4 standards that lend themselves to communicating reasoning	digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **Number and Operations—Fractions (NF)** **4.NF.A:** Extend understanding of fraction equivalence and ordering. **4.NF.A.1** Explain why a fraction *a*/*b* is equivalent to a fraction (*n* × *a*)/(*n* × *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. **4.NF.A.2** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. **4.NF.B:** Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. **4.NF.B.3** Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*. **a.** Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. **b.** Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.* **c.** Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. **4.NF.B.4** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. **a.** Understand a fraction *a*/*b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).* **b.** Understand a multiple of *a*/*b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)* **4.NF.C:** Understand decimal notation for fractions, and compare decimal fractions. **4.NF.C.7** Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

Grade 5 standards that lend themselves to communicating reasoning	**The following standards can be effectively used in various combinations in Grade 5 Claim 3 items:** **Number and Operations in Base Ten (NBT)** **5.NBT.A.2** Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. **5.NBT.B.6** Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **5.NTB.B.7** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. **Number and Operations—Fractions (NF)** **5.NF.A.1** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)* **5.NF.A.2** Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2*. **5.NF.B:** Apply and extend previous understandings of multiplication and division to multiply and divide fractions. **5.NF.B.3** Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?* **5.NF.B.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. **5.NF.B.7** **a.** Interpret division of a unit fraction by a non-zero whole number and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.*

Grade 5 standards that lend themselves to communicating reasoning	**b.** Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.* **Measurement and Data (MD)** **5.MD.C:** Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition. **5.MD.C.5** **a.** Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. **b.** Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real world and mathematical problems. **Standards to integrate with the focus on fractions and whole number operations:** **Geometry (G)** **5.G.B:** Classify two-dimensional figures into categories based on their properties. **5.G.B.4** Classify two-dimensional figures in a hierarchy based on properties.

Range ALDs – Claim 3 Grades 3-5	**Level 1** Students should be able to base arguments on concrete referents such as objects, drawings, diagrams, and actions and identify obvious flawed arguments in familiar contexts.
	Level 2 Students should be able to find and identify the flaw in an argument by using examples or particular cases. Students should be able to break a familiar argument given in a highly scaffolded situation into cases to determine when the argument does or does not hold.
	Level 3 Students should be able to use stated assumptions, definitions, and previously established results and examples to test and support their reasoning or to identify, explain, and repair the flaw in an argument. Students should be able to break an argument into cases to determine when the argument does or does not hold.
	Level 4 Students should be able to use stated assumptions, definitions, and previously established results to support their reasoning or repair and explain the flaw in an argument. They should be able to construct a chain of logic to justify or refute a proposition or conjecture and to determine the conditions under which an argument does or does not apply.

Target 3A: Test propositions or conjectures with specific examples.

General Task Model Expectations for Target 3A

- Items for this target should focus on the core mathematical work that students are doing around numbers and operations, with mathematical content from other domains playing a supporting role in setting up the reasoning contexts.
- Items in this task model should probe the key mathematical structures that students at that grade-level are studying, such as the structure of base-ten numbers, fractions, or the four operations and their properties.
- In response to a claim or conjecture, the student should:
    - Find a counterexample if the claim is false,
    - Find examples and non-examples if the claim is sometimes true, or
    - Provide supporting examples for a claim that is always true without concluding that the examples establish that truth, unless there are only a finite number of cases and all of them are established one-by-one. The main role for using specific examples in this case is for students to develop a hypothesis that the conjecture or claim is true, setting students up for work described in Claim 3B.
- False or partially true claims that students are asked to find counterexamples for should frequently draw upon commonly held mathematical misconceptions.
- Note: Use appropriate mathematical language in asking students for a single example. While a single example can be used to refute a conjecture, it cannot be used to prove one is always true unless that is the one and only case.

Task Model 3A.1

- The student is presented with a proposition or conjecture and asked to give
    - A counterexample if the claim is false,
    - Examples and non-examples if the claim is sometimes true, or
    - One or more supporting examples for a claim that is always true without concluding that the examples establish that truth.

Example Item 3A.1a (Grade 3)
Primary Target 3A (Content Domain OA), Secondary Target 1D (CCSS 3.OA.B), Tertiary Target 3F

Marquis said, "The more numbers you multiply, the greater the product." Then he wrote:

$2 \times 8 = 16$
$2 \times 5 \times 5 = 50$
$2 \times 3 \times 5 \times 2 = 60$

$60 > 50 > 16$

Give an example of a product of two numbers that is greater than $2 \times 5 \times 5$.

[ ] x [ ] > $(2 \times 5 \times 5)$

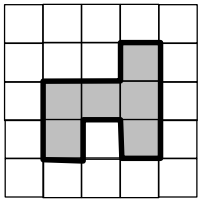
Enter the numbers in the two response boxes.

Rubric: (1 point) The student enters two numbers in the response boxes whose product is greater than 50. (e.g., 7 and 8).

Response Type: Equation/numeric

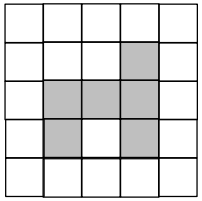
Example Item 3A.1b (Grade 4)
Primary Target 3A (Content Domain MD), Secondary Target 1I (CCSS 3.MD.D), Tertiary Target 3F

William shaded 6 squares in a grid to make the figure shown.

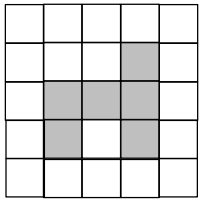


He claims that if he **adds 1 more** square to this figure in different places, the perimeter can be greater than, less than, or equal to the perimeter of the original figure.

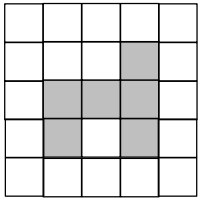
Part A. Click to shade one more square so the perimeter is greater than the original figure.



Part B. Click to shade one more square so the perimeter is less than the original figure.



Part C. Click to shade one more square so the perimeter is equal to the original figure.



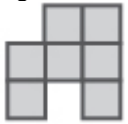
Rubric: (2 points) The student is able to provide an example that supports each conjecture.
(1 point) The student is able to provide two out of three correct examples.
(0 points) The student is unable to provide at least two correct examples.

Exemplar[6]:

For Part A, the perimeter has to be greater than 14 units.

For Part B, the perimeter of the figure has to be less than 14 units.

For Part C, the perimeter of the figure has to be equal to 14 units.



Response Type: Hot Spot

[6] An exemplar is just one example of a correct response. Other correct responses are possible.

Example Item 3A.1c (Grade 5)
Primary Target 3A (Content Domain NBT), Secondary Target 1D (CCSS 4.NBT.B), Tertiary Target 3F

Nina says, "If you multiply a 2-digit number and a 1-digit number, you get a 3-digit number."

Enter numbers in the table to give one example of when Nina's claim is true, and another example that shows her claim is **not** always true.

Example of when –	**2-digit number**	**1-digit number**	**3-digit product**
Nina's claim is true			
Nina's claim is **not** true			

Rubric: (2 points) The student gives an example where the product is a three-digit number (e.g., $90 \times 2=180$) and an example where it is not (e.g., $10 \times 2=20$).
(1 point) The student gives an example where the product is a three-digit number or an example where it is not.

Response Type: Fill-in Table

Task Model 3A.2

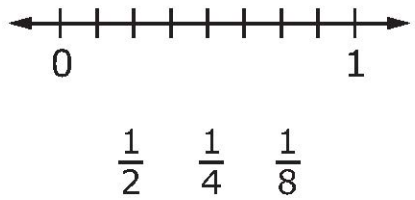
- The student is presented with one or more propositions or conjectures and several examples and asked implicitly or explicitly which examples support or refute each proposition.
- Items in this task model should cover all cases and not be unintentionally misleading about the truth status of a particular proposition or conjecture.

Example Item 3A.2a (Grade 3)
Primary Target 3A (Content Domain NF), Secondary Target 1F (CCSS 3.NF.3d), Tertiary Target 3F

Robert said, "When comparing two fractions with a numerator of 1, the fraction with the bigger denominator is always greater."

Part A
Drag each fraction to the correct location on the number line.

Part B
Is Robert's statement true? Click Yes or No.



Is Robert's statement true?
Click Yes or No.

Yes No

Interaction: The student drags fractions from the single-use palette to the number line and clicks on "Yes" or "No."

Rubric: (2 points) The student places all three fractions in the correct locations and answers "No."
(1 point) The student either places all the fractions in the correct locations and answers "Yes"; or places all fractions in the correct order but misses the correct location for one or more fractions and answers "No."

Response Type: Drag and Drop and Hot Spot

Example Item 3A.2b (Grade 4)
Primary Target 3A (Content Domain NBT), Secondary Target 1E (CCSS 4.NBT.B)

Click in the box that matches each division problem to the correct claim.

Claim	200 ÷ 5	777 ÷ 7	108 ÷ 9
When you divide a 3-digit number by a 1-digit number, the quotient can have **1 digit**.			
When you divide a 3-digit number by a 1-digit number, the quotient can have **2 digits**.			
When you divide a 3-digit number by a 1-digit number, the quotient can have **3 digits**.			

Rubric: (1 point) The student matches each quotient to the appropriate claim (e.g., Claim 2: 200 ÷ 5 and 108 ÷ 9. Claim 3: 777 ÷ 7.).

Response Type: Matching Table

Target 3B: Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures.

General Task Model Expectations for Target 3B

- Items for this target should focus on the core mathematical work that students are doing around numbers and operations, with mathematical content from other domains playing a supporting role in setting up the reasoning contexts.
- Items for this target can probe a key mathematical structure such as the structure of base-ten numbers, fractions, or the four operations and their properties.
- Items for this target can require students to solve a multi-step, well-posed problem involving the application of mathematics to a real-world context. The difference between items for Claim 2A and Claim 3B is that the focus in 3B is on communicating the reasoning process in addition to getting the correct answer.
- Note that in grades 3–5, items can provide more structure than items for later grades to help them understand the expectations for justifying or refuting a proposition or conjecture.

Task Model 3B.1

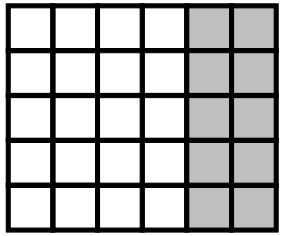
- The student is presented with a proposition or conjecture. The student is asked to identify or construct reasoning that justifies or refutes the proposition or conjecture.
- Items in this task model often address more generalized reasoning about a class of problems or reasoning that generalizes beyond the given problem context even when it is presented in a particular case.

Example Item 3B.1a (Grade 3)
Primary Target 3B (Content Domain OA), Secondary Target 1B (CCSS 3.OA.B), Tertiary Target 3F

Bev said, "I can find 5×6 by adding 5×4 and 5×2."

She wrote this equation and drew this picture to show her thinking.

$5 \times 6 = 5 \times 4 + 5 \times 2$

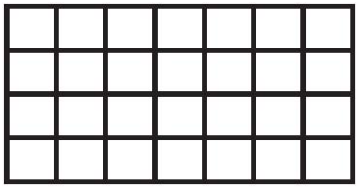


Mel wrote this equation: $4 \times 7 = 4 \times 3 + 4 \times 4$

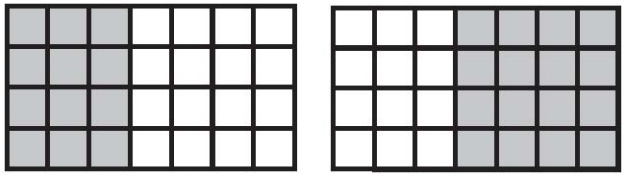
Is this equation true? Click on Yes or No.

Yes | No

Click on the squares to draw a picture that supports your answer.



Rubric: (1 point) The student identifies the equation as true and clicks to shade either a 4 x 3 rectangle or a 4 x 4 rectangle; see examples below.



Response Type: Hotspot

Example Item 3B.1b (Grade 4)
Primary Target 3B (Content Domain OA), Secondary Target 1B (CCSS 4.NBT.B), Tertiary Target 3F

Carter says, "8000 is 100 times as large as 80."

Choose three statements that support this claim.

Drag them into a logical order.

1.

2.

3.

So 8000 is 100 times as large as 80.

80 is 10 times as large as 8.
800 is 10 times as large as 80.
8000 is 10 times as large as 800.
$10 \times 10 = 100$
$10 \times 100 = 1000$
$80 \times 10 = 800$
$800 \times 10 = 8000$

Rubric: (1 point) The student selects three statements that complete an explanation for the claim and puts them in a logical order. In this particular example, the order doesn't matter.

Exemplars:

1. 800 is 10 times as big as 80.	1. $80 \times 10 = 800$
2. 8000 is 10 times as big as 800.	2. $800 \times 10 = 8000$
3. $10 \times 10 = 100$	3. $10 \times 10 = 100$

Response Type: Drag and Drop

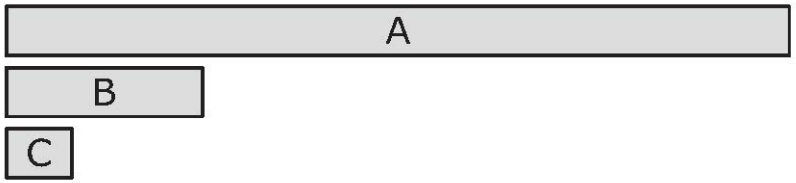
Task Model 3B.2

- The student is asked a mathematical question and is asked to identify or construct reasoning that justifies his or her answer.
- Items in this task model often address more generalized reasoning about a class of problems or reasoning that generalizes beyond the given problem context even when it is presented in a particular case.

Example Item 3B.2a (Grade 4)

Primary Target 3B (Content Domain OA), Secondary Target 1B (CCSS), Tertiary Target 3F

Rectangle A is 4 times as long as rectangle B.
Rectangle B is 3 times as long as rectangle C.



How many times greater is rectangle A than rectangle C?

☐ **times**

Choose three equations that, when taken together, support your claim. Drag them into a logical order.

1.	
2.	
3.	
$4 \times A = B$	$3 \times C = B$
$4 \times B = A$	$4 \times (3 \times C) = A$
$3 \times B = C$	$3 \times (4 \times C) = A$

Rubric: (2 point) The student enters the correct multiplicative factor in the response box (e.g., 12) and selects three statements that support the claim and puts them in a logical order.
(1 point) The student does one or the other.

Exemplars:

1. 4 x B = A
2. 3 x C = B
3. 4 x (3 x C) = C

1. 3 x C = B
2. 4 x B = A
3. 4 x (3 x B) = A

Response Type: Equation/Numeric and Drag and Drop

Note: Functionality to combine these items types doesn't currently exist. The item could be implemented as a 1 point item if the scale factor is given.

Example Item 3B.2b (Grade 5)
Primary Target 3B (Content Domain MD), Secondary Target 1I (CCSS 5.MD.5), Tertiary Target 3F

The dimensions of a right rectangular prism are:

- length = 9 centimeters
- width = 3 centimeters
- height = 5 centimeters

What will happen to the volume of the right rectangular prism if the length, the width, and the height are each doubled?

The new volume will be [drop-down choices: 2, 4, 6, 8] times the original volume because $(2 \times 9)(2 \times 3)(2 \times 5) =$ [drop-down choices: 2, 4, 6, 8] $\times (9 \times 3 \times 5)$.

Rubric: (1 point) The student selects the correct multiplier (e.g., 8) in both drop-down menus.

Response Type: Drop-down menu

Note: Functionality for this item doesn't currently exist, though we anticipate to be able to offer drop-down items by 2018. The item could be implemented as a multiple choice in the meantime.

Task Model 3B.3

- Items for this target require the student to solve a multi-step, well-posed problem involving the application of mathematics to a real-world context.
- The difference between Claim 2 task models and this task model is that the student needs to provide some evidence of his/her reasoning. The difference between Claim 4 task models and this task model is that the problem is completely well posed and no extraneous information is given.

Example Item 3B.3a (Grade 3)
Primary Target 3B (Content Domain OA), Secondary Target 1D (CCSS 3.OA.D)

A bird ate 400 grams of food in 3 days. The bird ate 120 grams of food on Day 1, 150 grams of food on Day 2, and g grams of food on Day 3.

Day	Grams of Food
1	120
2	150
3	g

How many grams of food did the bird eat on Day 3? Enter your answer in the first response box.

In the second response box, enter an equation that you could solve to find the amount of food the bird ate on Day 3.

Rubric: (2 points) The student enters the correct number of grams of food on Day 3 and enters a correct (e.g., 130; $400 - 120 - 150 = x$, $120 + 150 + x = 400$, or equivalent equation).
(1 point) The student enters the correct number of grams of food on Day 3 or enters a correct equation.

Response Type: Equation/Numeric (2 response boxes)

Example Item 3B.3b (Grade 4)
Primary Target 3B (Content Domain MD), Secondary Target 1G (CCSS 4.MD.A)

- There are 60 seconds in a minute.
- There are 60 minutes in an hour.
- There are 24 hours in a day.

What is the total number of minutes in 1 day? Enter your answer in the first response box.

Write an expression that shows how you found your answer. Enter your expression in the second response box.

Rubric: (2 points) The student enters the correct number of minutes in a day in the first response box (1440) and a correct equation in the second response box (e.g., 60×24, 144×10, or equivalent expressions).
(1 point) The student enters the correct number of minutes in a day in the first response box or a correct equation in the second response box.

Response Type: Equation/Numeric (2 response boxes)

Target 3C: State logical assumptions being used.

General Task Model Expectations for Target 3C

- Items for this target should focus on the core mathematical work that students are doing around numbers and operations, with mathematical content from other domains playing a supporting role in setting up the reasoning contexts.
- For some items, the student must explicitly identify assumptions that
    - Make a problem well-posed, or
    - Make a particular solution method viable.
- When possible, items in this target should focus on assumptions that are commonly made implicitly and can cause confusion when left implicit.
- For some items, the student will be given a definition and be asked to reason from that definition.

Task Model 3C.1

- The student is asked to identify an unstated assumption that would make the problem well-posed or allow them to solve a problem using a given method.

Example Item 3C.1a (Grade 3)
Primary Target 3C (Content Domain OA), Secondary Target 1B (CCSS 3.OA.B)

A 20 meter rope is cut into 4 pieces. Jenny says you can find the length of each piece by finding 20 ÷ 4.

What statement best describes Jenny's claim?

A. Jenny's claim is false. She should add 4 and 20 instead.
B. Jenny's claim is false. She should multiply 4 and 20 instead.
C. Jenny's claim is true if you assume that each piece is 4 meters long.
D. Jenny's claim is true if you assume that the pieces are all equal in length.

Rubric: (1 point) The student selects the correct statement (e.g., D).

Response Type: Multiple Choice, single correct response

Example Item 3C.1b (Grade 5)
Primary Target 3C (Content Domain OA), Secondary Target 1A (CCSS 4.OA.A)

Gil and Nina are comparing the numbers 3 and 12.

Gil says, “12 is 9 more than 3.”

Nina says, “12 is 4 times more than 3.”

What is true about Gil and Nina’s statements?

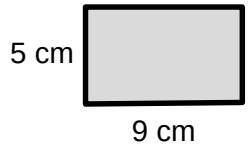
A. Nina is correct and Gil is not. You should multiply to compare the numbers.
B. Gil is correct and Nina is not. You should add to compare the numbers.
C. They are both correct. They just compared using different operations.
D. Neither one is correct. You have to compare like this: 12 > 3.

Rubric: (1 point) The student selects the correct statement (e.g., C).

Response Type: Multiple Choice, single correct response

Example Item 3C.1c (Grade 5)
Primary Target 3C (Content Domain G, MD), Secondary Target 1K (CCSS 5.G.B, 4.MD.A.3), Tertiary Target 3D

Carrie saw the figure below and said that its area is $5 \times 9 = 45$ square centimeters.



Which statement best supports Carrie’s claim?

A. It is true if the opposite sides have the same length.
B. It is true if the figure is a rectangle.
C. It is false if the opposite sides have the same length.
D. It is false if the figure is a rectangle.

Rubric: (1 point) The student selects the correct statement (e.g., B).

Response Type: Multiple Choice, single correct response

Example Item 3C.1d (Grade 5)
Primary Target 3C (Content Domain NF), Secondary Target 1F (CCSS 4.NF.A.2), Tertiary Target 3D

Flo ate $\frac{3}{4}$ of a sandwich and Arnie ate $\frac{2}{3}$ of a sandwich. If Arnie ate more, what must be true?

A. Flo's sandwich is bigger.
B. Arnie's sandwich is bigger.
C. The sandwiches are the same size.
D. It doesn't matter which sandwich is bigger.

Rubric: (1 point) The student selects the correct assumption (e.g., B).

Response Type: Multiple Choice, single correct response

Task Model 3C.2

- The student will be given one or more definitions or assumptions and be asked to reason from that set of definitions and assumptions.

Example Item 3C.2a (Grade 5)
Primary Target 3C (Content Domain G), Secondary Target 1K (CCSS 5.G.B)

Patrick is learning about quadrilaterals. He was given the following true statements.

- Opposite sides of all parallelograms have the same length.
- Opposite sides of all rectangles have the same length.
- All sides of a square have the same length.
- All rectangles are parallelograms.
- All rectangles have right angles.
- All squares have right angles.

Based on this information, Patrick assumes the following statements are always true. Which statement is **not** supported by the given information?

A. All squares are rectangles.
B. All squares are parallelograms.
C. All parallelograms are rectangles.
D. All parallelograms are quadrilaterals.

Rubric: (1 point) The student selects the correct response (e.g., C).

Response Type: Multiple choice, single correct response

Target 3D: Use the technique of breaking an argument into cases.

General Task Model Expectations for Target 3D

- Items for this target should focus on the core mathematical work that students are doing around numbers and operations, with mathematical content from other domains playing a supporting role in setting up the reasoning contexts.
- The student is given
  - A problem that has a finite number of possible solutions, some of which work and some of which don't, or
  - A proposition that is true in some cases but not others.
- Items for Claim 3 Target D should either present an exhaustive set of cases to consider or expect students to consider all possible cases in turn in order to distinguish it from items in other targets.
- In grades 3-5, the student will be given the cases to consider.

Task Model 3D.1

- The student is given a problem that has a finite number of possible solutions, some of which work and some of which don't.

Example Item 3D.1a (Grade 3)

Primary Target 3D (Content Domain OA), Secondary Target 1A (CCSS 3.OA.A)

Select **all** the ways can you divide 15 children into equal groups with none left over.

A. 2 groups
B. 3 groups
C. 4 groups
D. 5 groups

Rubric: (1 point) The student selects the possible number of groups (B and D).

Response Type: Multiple Choice, multiple select response

Example Item 3D.1b (Grade 4)
Primary Target 3D (Content Domain MD), Secondary Target 1K (CCSS 4.MD.C)

When you cut an obtuse angle into two smaller angles, what can be true? (Select **all** that apply.)

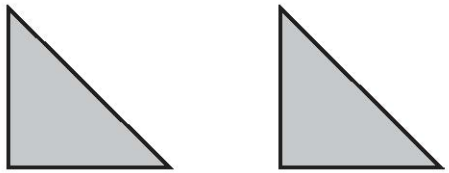
A. The two smaller angles can be less than 90 degrees.
B. At least one of the two smaller angles can be greater than 90 degrees.
C. Both of the two smaller angles can be greater than 90 degrees.

Rubric: (1 point) The student selects the possible cases (A and B).

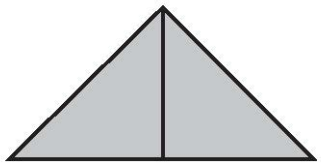
Response Type: Multiple Choice, multiple correct response

Example Item 3D.1c (Grade 5)
Primary Target 3D (Content Domain G), Secondary Target 1K (CCSS 5.G.B)

Nora has drawn two identical isosceles right triangles.



Here is a way to put them together so that they share a side and make another triangle.



Select **all** the quadrilaterals Nora can make with these triangles if she puts them together so that they share a side.

A. A square
B. A rectangle that is not a square
C. A rhombus that is not a square
D. A parallelogram that is not a rectangle

Rubric: (1 point) The student selects the possible cases (A and D).

Response Type: Multiple Choice, multiple select response

Task Model 3D.2

- The student is given a proposition and an exhaustive list of cases and asked to determine in which of those cases the proposition is true.

Example Item 3D.2a (Grade 3)
Primary Target 3D (Content Domain OA), Secondary Target 1B (CCSS 3.OA.B), Tertiary Target 3C

n is a whole number and $n \times 5 = 5$.

Identify which values of *n* make this equation true.

	True	False
When $n = 0$		
When $n = 1$		
When $n > 1$		
This is never true		

Rubric: (1 point) The student identifies the correct values of *n* (F, T, F, F)

Response Type: Matching Table

Example Item 3D.2b (Grade 4)
Primary Target 3D (Content Domain NF), Secondary Target 1G (CCSS 4.NF.A), Tertiary Target 3C

What must be true about *d* to make this inequality true?

$\frac{3}{d} \geq \frac{3}{10}$

Identify which values of *d* make this equation true.

	True	False
$d < 10$		
$d = 10$		
$d > 10$		

Rubric: (1 point) The student identifies the correct values of *d* (T, T, F)

Response Type: Matching Table

Example Item 3D.2c (Grade 5)
Primary Target 3D (Content Domain NF), Secondary Target 1? (CCSS 5.NF.B), Tertiary Target 3C

32×45 is greater than both 32 and 45. When is $a \times b$ between a and b?

Select **all** that apply.

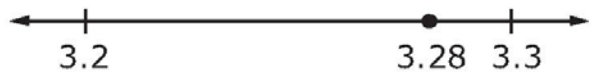
A. When $a > 1$ and $b > 1$
B. When $a < 1$ and $b > 1$
C. When $b < 1$ and $a > 1$
D. When $a < 1$ and $b < 1$

Rubric: (1 point) The student selects B and C.

Response Type: Multiple Choice, multiple correct response

Example Item 3D.2d (Grade 5)
Primary Target 3C (Content Domain NBT), Secondary Target 1C (CCSS 5.NBT.A), Tertiary Target 3F

Jenny says, "To round a decimal d between 3.2 and 3.3 to the nearest tenth, you just see which tenth it is closest to on the number line. For example, 3.28 is closer to 3.3 than 3.2, so it rounds to 3.3."



In which cases will Jenny's method work? (Select **all** that apply.)

A. Case 1: $3.25 < d \leq 3.3$
B. Case 2: $d = 3.25$
C. Case 3: $3.2 \leq d < 3.25$
D. Jenny's method doesn't usually doesn't work—it just worked for this example.

Rubric: (1 point) The student selects the correct cases (A and C).

Response Type: Multiple Choice, multiple correct response

Target 3E: Distinguish correct logic or reasoning from that which is flawed and—if there is a flaw in the argument—explain what it is.

General Task Model Expectations for Target 3E

- Items for this target should focus on the core mathematical work that students are doing around numbers and operations, with mathematical content from other domains playing a supporting role in setting up the reasoning contexts.
- The student is presented with valid or invalid reasoning and told it is flawed or asked to determine its validity. If the reasoning is flawed, the student identifies, explains, and/or corrects the error or flaw.
- The error should be more than just a computational error or an error in counting, and should reflect an actual error in reasoning.
- Analyzing faulty algorithms is acceptable so long as the algorithm is internally consistent and it isn't just a mechanical mistake executing a standard algorithm.

Task Model 3E.1

- Some flawed reasoning or student work is presented and the student identifies and/or corrects the error or flaw.
- The student is presented with valid or invalid reasoning and asked to determine its validity. If the reasoning is flawed, the student will explain or correct the flaw.

Example Item 3E.1a (Grade 3)
Primary Target 3E (Content Domain OA), Secondary Target 1A (CCSS 3.OA.A), Tertiary Target 3C

Tasha is solving this problem:

There 4 tanks with 10 fish in each tank. How many fish are there all together?

Tasha claims, "There are 4 + 10 = 14 fish all together."

Which statement best describes Tasha's claim?

A. Tasha correctly added to find the total.
B. Tasha should subtract instead.
C. Tasha should multiply instead.
D. Tasha should divide instead.

Rubric: (1 point) The student selects the correct statement (C).

Response Type: Multiple Choice, single correct response

Example Item 3E.1b (Grade 4)
Primary Target 3E (Content Domain NBT), Secondary Target 1E (CCSS 3.NBT.B)

Harvey was solving this problem:

There are 12 packets of gum each with a mass of 65 grams. What is the mass of all of the packets combined?

Harvey said, "I can multiply the tens places and the ones places and add them."

Then he wrote:
12 = 10 + 2
65 = 60 + 5
600 + 10 = 610
The total mass is 610 grams.

Which statement best describes Harvey's claim?

A. Harvey solved the problem correctly and got the right answer.
B. Harvey made a mistake in solving the problem but got the right answer anyway.
C. Harvey had a correct way of solving the problem but got the wrong answer.
D. Harvey's solution is not correct because he did not multiply the tens with the ones.

Rubric: (1 point) The student selects the correct statement (e.g., D).

Response Type: Multiple Choice, single correct response

Example Item 3E.1c (Grade 5)
Primary Target 3E (Content Domain NF), Secondary Target 1E (CCSS 5.NF.A)

Brian is adding $\frac{2}{3} + \frac{7}{5}$. He wrote: $\frac{2}{3} + \frac{7}{5} = \frac{2+7}{3+5} = \frac{9}{8}$

Brian's approach is **not** correct. Select **all** of the statements that could indicate mistakes with Brian's approach.

A. He added the denominators.
B. He didn't write $\frac{7}{5}$ as a mixed number.
C. He didn't write his answer as a mixed number.
D. He added the numerators when the denominators were different.

Rubric: (1 point) The student clicks on the mistakes in the algorithm (A and S).

Response Type: Multiple Choice, multiple correct response

Task Model 3E.2

- Two or more approaches or chains of reasoning are given and the student is asked to identify the correct method and justification OR identify the incorrect method/reasoning and the justification.

Example Item 3E.2a (Grade 4)
Primary Target 3E (Content Domain NBT), Secondary Target 1E (CCSS 4.NBT.A), Tertiary Target 3C, Quaternary Target 3F

Zach and Nate both rounded 6481, but used different methods.

Zach thought about it this way:	Nate thought about it this way:
6481 rounds to 6480 6480 rounds to 6500 6500 rounds to 7000 So 6481 rounds to 7000.	6481 is closer to 6000 than to 7000, so it rounds to 6000.

Which statement best describes these methods?

A. Zach's method is correct.
B. Nate's method is correct.
C. Both methods are correct.
D. Neither method is correct.

Rubric: (1 point) The student selects the correct method (B).

Response Type: Multiple Choice, single correct response

Example Item 3E.2a (Grade 5)
Primary Target 3E (Content Domain NBT), Secondary Target 1E (CCSS 4.NBT.A), Tertiary Target 3C

Mr. Spivak's class was finding the volume of a right rectangular prism with dimensions 20 cm, 45 cm, and 80 cm.

Brigit said, "I tried two ways of multiplying the dimensions and got different answers. I can't figure out what went wrong."

She explained her two ways to Mr. Spivak.

First method:	**Second method:**
Step 1: I distributed. $20 \times (45 \times 80) = (20 \times 45) + (20 \times 80)$	Step 1: I broke apart the numbers. $20 \times 45 \times 80 = (2 \times 10) \times (5 \times 9) \times (8 \times 10)$
Step 2: I multiplied 20 by 45 and 20 by 80. $= 900 + 1600$	Step 2: I rearranged the numbers.
Step 3: Then I added. $= 2500$	Step 3: Then I multiplied everything. $= 72 \times (10 \times 100) = 72{,}000$

Which method has an error? Which step has the first error in that method?

Brigit's [drop-down options: first, second] method has an error. She made the error in step [drop-down options: 1, 2, 3].

Rubric: (1 point) The student selects the incorrect method (first) and identifies the step in which the error occurred (1).

Response Type: Drop-down Menu[7]

[7] This response is not yet supported by the Smarter Balanced item authoring tool, but is expected as an enhancement by 2017.

Target 3F: Base arguments on concrete referents such as objects, drawings, diagrams, and actions

Task Model 3F.1

- The student uses concrete referents to help justify or refute an argument.
- Items in this task model should address content in standards that specifically call for number lines, diagrams, and contexts to be used as a basis for reasoning.

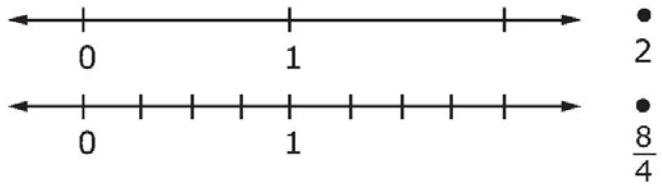
Example Item 3F.1a (Grade 3)

Primary Target 3F (Content Domain NBT), Secondary Target 1F (CCSS 3.NF.A), Tertiary Target 3B

Compare $\frac{8}{4}$ and 2.

Part A

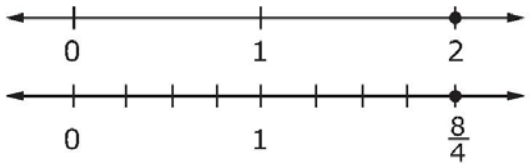
Plot each number on a number line.



Part B

$\frac{8}{4}$ [drop-down choices: <, =, >] 2

Rubric: (1 point) The student plots the points correctly (see below) and selects the correct comparison (=).



Response Type: Drop-down Menu, Graphing

Note: Functionality for this item type does not currently exist.

Example Item 3B.1b (Grade 3)
Primary Target 3F (Content Domain NF), Secondary Target 1F (CCSS 3.NF.A), Tertiary Target 3B

Part A

Which comparison between $\frac{1}{5}$ and $\frac{1}{8}$ is correct?

A. $\frac{1}{5} < \frac{1}{8}$

B. $\frac{1}{5} > \frac{1}{8}$

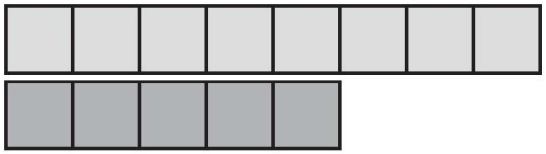
C. $\frac{1}{5} = \frac{1}{8}$

Part B

Choose a picture that supports your answer in *Part A*.

D.

E.



F.

Rubric: (1 point) The student selects the correct comparison and the correct picture (B, F).

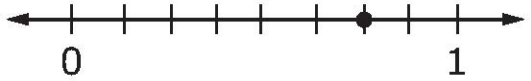
Response Type: Drop-down Menu and Multiple Choice, single correct response

Example Item 3F.1c (Grade 4)

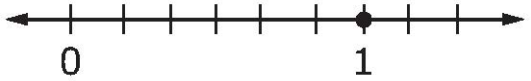
Primary Target 3F (Content Domain NBT), Secondary Target 1F (CCSS 4.NF.A), Tertiary Target 3B

Which number line shows that $\frac{3}{4} = \frac{6}{8}$?

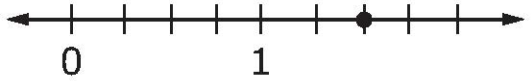
A.



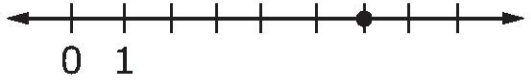
B.



C.



D.



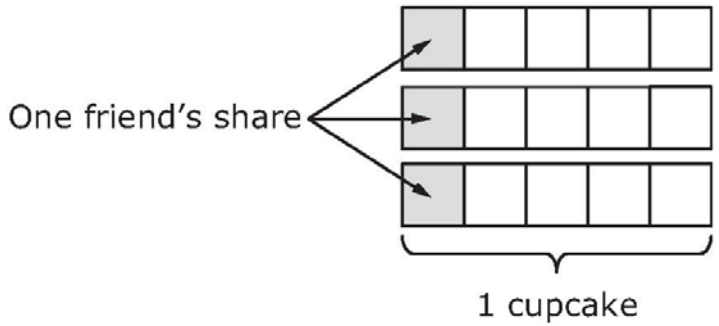
Rubric: (1 point) The student selects the correct number line (A).

Response Type: Multiple Choice, single correct response

Example Item 3F.1d (Grade 5)
Primary Target 3F (Content Domain NBT), Secondary Target 1F (CCSS 5.NF.B), Tertiary Target 3B

Complete the story about friends sharing cupcakes to show that $3 \div 5 = \frac{3}{5}$.

- 5 friends were sharing 3 cupcakes. They divided each cupcake into 5 equal pieces.
- Each piece is [drop-down menu choices: $\frac{1}{3}$, $\frac{1}{5}$, $\frac{3}{5}$] of a cupcake.
- Each friend got 1 piece of each cupcake.
- Each friend got [drop-down menu choices: $\frac{1}{3}$, $\frac{1}{5}$, $\frac{3}{5}$] of a cupcake in total.



Rubric: (1 point) The student selects the correct unit fraction ($\frac{1}{5}$) and the correct total amount each friend receives ($\frac{3}{5}$).

Response Type: Drop-down Menu

Grades 3–5 Mathematics Item Specification Claim 4	
"Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decision-making." (p.72, CCSSM)	
Primary Claim 4: Modeling and Data Analysis Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.	
Secondary Claim(s): Items/tasks written primarily to assess Claim 4 will necessarily involve some Claim 1 content targets. Related Claim 1 targets should be listed below the Claim 4 targets in the item form. If Claim 2 or Claim 3 targets are also directly related to the item/task, list those following the Claim 1 targets in order of prominence.	
Primary Content Domain: Each item/task should be classified as having a primary, or dominant, content focus. The content should draw upon the knowledge and skills articulated in the progression of standards leading up to and including the targeted grade with strong emphasis on the major work of previous grades.	
Secondary Content Domain(s): While tasks developed to assess Claim 4 will have a primary content focus, components of these tasks will likely produce enough evidence for other content domains that a separate listing of these content domains needs to be included where appropriate. The standards in the NBT domain in grades 3–5 can be used to construct higher difficulty items for the adaptive pool. The integration of the OA, G, and MD domains with NBT allows for higher content limits within the grade level than might be allowed when staying within the primary content domain.	
DOK Levels	1, 2, 3, 4
Allowable Response Types	**Response Types:** Multiple Choice, single correct response (MC); Multiple Choice, multiple correct response (MS); Equation/Numeric (EQ); Drag and Drop, Hot Spot, and Graphing (GI); Matching Table (MA); Fill-in Table (TI) No more than five choices in MS and MA items. Short Text – Performance tasks only **Scoring:** Scoring rules and answer choices will focus on a student's ability to use the appropriate reasoning. For some problems, multiple correct responses and/or strategies are possible. • MC and MS will be scored as correct/incorrect (1 point) • If MA items require two skills, they will be scored as: o All correct choices (2 points); at least ½ but less than all correct choices. (1 point) o Justification[1] for more than 1 point **must be** clear in the scoring rules.

[1] For a CAT item to score multiple points; either distinct skills must be demonstrated that earn separate points or distinct levels of understanding of a complex skill must be tied directly to earning one or more points.

	○ Where possible, include a "disqualifier" option that if selected would result in a score of 0 points, whether or not the student answered ½ correctly. • EQ, GI, and TI items will be scored as: ○ Single requirement items will be scored as correct/incorrect. (1 point) ○ Multiple requirement items: All components correct (2 points); at least ½ but less than all correct. (1 point) ○ Justification for more than 1 point **must be** clear in the scoring rules.
Allowable Stimulus Materials	Effort must be made to minimize the reading load in problem situations. Use tables, diagrams with labels, and other strategies to lessen the reading load. Use simple subject-verb-object (SVO) sentences; use contexts that are familiar and relevant to students at the targeted grade level. Target-specific stimuli will be derived from the Claim 1 targets used in the problem situation. All real-world problem contexts will be relevant to the age of the students. Stimulus guidelines specific to task models are given below.
Construct-Relevant Vocabulary	Refer to the Claim 1 specifications to determine Construct-Relevant Vocabulary associated with specific content standards.
Allowable Tools	Any mathematical tools appropriate to the problem situation and the Claim 1 target(s). Some tools are identified in Standard for Mathematical Practice 5 and others can be found in the language of specific standards.
Target-Specific Attributes	CAT Items should take from 3 to 6 minutes to solve. Claim 4 items that are part of a performance task may take 5 to 15 minutes to solve.
Accessibility Guidance	Item writers should consider the following Language and Visual Element/Design guidelines[2] when developing items. Language Key Considerations: • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level • Avoid ambiguous or obscure words, idioms, jargon, unusual names and references Visual Elements/Design Key Considerations: • Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context • Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary

[2] For more information, refer to the General Accessibility Guidelines at: http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf

	• Avoid crowding of details and graphics Items are selected for a student's test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.[3]
Development Notes	CAT items/tasks generating evidence for Claim 4 in a given grade will draw upon knowledge and skills articulated in the progression of standards up through that grade, though more complex problem-solving tasks may draw upon knowledge and skills from lower grade levels. Claim 1 *Specifications* that cover the following standards should be used to help inform an item writer's understanding of the difference between how these standards are measured in Claim 1 versus Claim 4. Development notes have been added to many of the Claim 1 specifications that call out specific topics that should be assessed under Claim 4. Distinguishing between Claim 4 and Claims 1 and 2: • In early grades when equations are still new to students, an important distinction between Claim 2 and Claim 4 is requiring a model that would lead to a problem's solution. • In Claim 2 problems are well posed, while in Claim 4 they may have extraneous or missing information. • In Claims 1 and 2, measurements of objects or figures can be accurately determined. In Claim 4, modeling is used to make approximations. • In Claim 1, data analysis is straightforward procedural. In Claim 4, the analysis should be tied to some useful purpose in the real-world. At least 80% of the items written to Claim 4 should primarily assess the standards and clusters listed in the table that follows.

[3] For more information about student accessibility resources and policies, refer to http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

Grades 3–5, Claim 4

Grade 3	Grade 4	Grade 5
3.OA.A	4.OA.A	5.NBT.B
3.OA.D	4.NF.B	5.NF.A
3.MD.A	4.MD.A*	5.NF.B
3.MD.C	4.MD.B*	5.MD.A*
3.MD.D*	4.MD.C*	5.MD.B*
		5.MD.C
		5.G.A*

* Denotes additional and supporting clusters

REMINDER: Claim 4 tasks may also ask students to apply content from prior grades in sophisticated applications.

Assessment Targets: Any given item/task should provide evidence for two or more Claim 4 assessment targets. Each of the following targets should not lead to a separate task. It is in *using* content from different areas, including work studied in earlier grades, that students demonstrate their problem-solving proficiency. Multiple targets should be listed in order of prominence as related to the item/task.

Target A: Apply mathematics to solve problems arising in everyday life, society, and the workplace. (DOK 2, 3)
Problems used to assess this target for Claim 4 should not be completely formulated (as they are for the same target in Claim 2), and require students to extract relevant information from within the problem and find missing information through research or the use of reasoned estimates.

Target B: Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem. (DOK 2, 3, 4)
Items that require the student to make decisions about the solution path needed to solve a problem are aligned with this target. Target B is not intended to be the primary target for an item, but should be a secondary, tertiary, or quaternary target for an item with primary alignment to other targets.

Target C: State logical assumptions being used. (DOK 1, 2)
Tasks used to assess this target ask students to use stated assumptions, definitions, and previously established results in developing their reasoning. In some cases, the task may require students to provide missing information by researching or providing a reasoned estimate.

Target D: Interpret results in the context of a situation. (DOK 2, 3)
Tasks used to assess this target should ask students to link their answer(s) back to the problem's context. (See Claim 2, Target C for further explication.)

Target E: Analyze the adequacy of and make improvements to an existing model or develop a mathematical model of a real phenomenon. (DOK 3, 4)
Tasks used to assess this target ask students to investigate the efficacy of existing models (e.g., develop a way to analyze the claim that a child's height at age 2 doubled equals his/her adult height) and suggest improvements using their own or provided data.

Other tasks for this target will ask students to develop a model for a particular phenomenon (e.g., analyze the rate of global ice melt over the past several decades and predict what this rate might be in the future).
Longer constructed-response items and extended performance tasks should be used to assess this target.

Target F: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas). (DOK 1, 2, 3)
Unlike Claim 2 where this target might appear as a separate target of assessment (see Claim 2, Target D), it will be

embedded in a larger context for items/tasks in Claim 4. The mapping of relationships should be part of the problem posing and solving related to Claim 4 Targets A, B, E, and G.

Target G*: Identify, analyze, and synthesize relevant external resources to pose or solve problems. (DOK 3, 4)
Especially in extended performance tasks, students should have access to external resources to support their work in posing and solving problems (e.g., finding or constructing a set of data or information to answer a particular question or looking up measurements of a structure to increase precision in an estimate for a scale drawing). Constructed-response items should incorporate "hyperlinked" information to provide additional detail (both relevant and extraneous) for solving problems in Claim 4.

*Measured in Performance Tasks only; functionality of linking to external resources is planned for future enhancements.

What sufficient evidence looks like for Claim 4[4]:

"A key feature of items and tasks in Claim 4 is that the student is confronted with a contextualized, or 'real world' situation and must decide which information is relevant and how to represent it. As some of the examples provided below illustrate, 'real world' situations do not necessarily mean questions that a student might really face; it means that mathematical problems are embedded in a practical application context. In this way, items and tasks in Claim 4 differ from those in Claim 2, because while the goal is clear, the problems themselves are not yet fully formulated (well-posed) in mathematical terms.

"Items/tasks in Claim 4 assess student expertise in choosing appropriate content and using it effectively in formulating models of the situations presented and making appropriate inferences from them. Claim 4 items and tasks should sample across the content domains, with many of these involving more than one domain. Items and tasks of this sort require students to apply mathematical concepts at a significantly deeper level of understanding of mathematical content than is expected by Claim 1. Because of the high strategic demand that substantial non-routine tasks present, the technical demand will be lower—normally met by content first taught in earlier grades, consistent with the emphases described under Claim 1. Although most situations faced by students will be embedded in longer performance tasks, within those tasks, some selected-response and short constructed-response items will be appropriate to use.

"Modeling and data analysis in the Common Core State Standards trace a visible arc of growing prominence across the grades, showing low prominence in grades K–5, higher prominence in grades 6–8 (which is when the Statistics and Probability domain first appears), and highest prominence in high school (which is when Modeling appears as a content category with the full modeling cycle). Therefore to align to the Standards, Claim 4 will be more important on the assessment in high school, less important in grades 6–8, and the least important in grades 3–5. Again, to align to the Standards, Claim 4 tasks will be most sophisticated and complete in high school (cf. the modeling cycle in CCSSM pp. 72, 73), less sophisticated/more tied to specific content in middle school, and least sophisticated/most tied to specific content in grades 3–5."

[4] Text excerpted from the Smarter Balanced Mathematics Content Specifications (p. 74-75).

Grade 3 Content Combinations:	**The following standards can be effectively used in various combinations in Grade 3 Claim 4 items:** **Primary emphases for Claim 4 Items at Grade 3: Operations and Algebraic Thinking and Measurement and Data** **Operations and Algebraic Thinking (OA)** **3.OA.A: Represent and solve problems involving multiplication and division.** **3.OA.A.1** Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.* **3.OA.A.2** Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.* **3.OA.A.3** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1] **3.OA.A.4** Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = ☐ ÷ 3, 6 × 6 = ?.* **3.OA.D: Solve problems involving the four operations, and identify and explain patterns in arithmetic.** **3.OA.D.8** Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.[3] **3.OA.D.9** Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.* **Measurement and Data (MD)** **3.MD.A: Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects**. **3.MD.A.1** Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by

	representing the problem on a number line diagram. **3.MD.A.2** Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).[6] Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.[7] **3.MD.C: Geometric measurement: understand concepts of area and relate area to multiplication and to addition.** **3.MD.C.5** Recognize area as an attribute of plane figures and understand concepts of area measurement. **a.** A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. **b.** A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units. **3.MD.C.6** Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units). **3.MD.C.7** Relate area to the operations of multiplication and addition. **a.** Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. **b.** Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. **c.** Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths *a* and *b* + *c* is the sum of *a* × *b* and *a* × *c*. Use area models to represent the distributive property in mathematical reasoning. **d.** Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems. **3.MD.D: Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.** **3.MD.D.8** Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Grade 4 Content Combinations:	**The following standards can be effectively used in various combinations in Grade 4 Claim 4 items:** **Primary emphases for Claim 4 Items at Grade 4: Operations and Algebraic Thinking, Number and Operations—Fractions, and Measurement and Data** **Operations and Algebraic Thinking (OA)** **4.OA.A: Use the four operations with whole numbers to solve problems.** **4.OA.A.1** Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. **4.OA.A.2** Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.[1] **4.OA.A.3** Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. **Number and Operations—Fractions (NF)** **4.NF.B: Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.** **4.NF.B.3** Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*. **a.** Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. **b.** Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.* **c.** Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. **d.** Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem. **4.NF.B.4** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

	a. Understand a fraction *a*/*b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).* **b.** Understand a multiple of *a*/*b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)* **c.** Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?* **Measurement and Data (MD)** **4.MD.A: Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.** **4.MD.A.1** Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...* **4.MD.A.2** Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. **4.MD.A.3** Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.* **4.MD.B: Represent and interpret data.** **4.MD.B.4** Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*

	4.MD.C: Geometric measurement: understand concepts of angle and measure angles. **4.MD.C.5** Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: **a.** An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. **b.** An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees. **4.MD.C.6** Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. **4.MD.C.7** Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

Grade 5 Content Combinations:	**The following standards can be effectively used in various combinations in Grade 5 Claim 4 items:** **Primary emphases for Grade 5 Claim 4 Items: Number and Operations—Base Ten, Number and Operations—Fractions, Measurement and Data, and Geometry** **Number and Operations—Base Ten (NBT)** **5.NBT.B: Perform operations with multi-digit whole numbers and with decimals to hundredths.** **5.NBT.B.5** Fluently multiply multi-digit whole numbers using the standard algorithm. **5.NBT.B.6** Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **5.NBT.B.7** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

	Number and Operations—Fractions (NF) **5.NF.A: Use equivalent fractions as a strategy to add and subtract fractions.** **5.NF.A.1** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)* **5.NF.A.2** Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.* **5.NF.B: Apply and extend previous understandings of multiplication and division to multiply and divide fractions.** **5.NF.B.3** Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?* **5.NF.B.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. **a.** Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)* **b.** Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas. **5.NF.B.5** Interpret multiplication as scaling (resizing), by: **a.** Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. **b.** Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a

	fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1. **5.NF.B.6** Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. **5.NF.B.7** Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[1] **a.** Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.* **b.** Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.* **c.** Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?* **Measurement and Data (MD)** **5.MD.A: Convert like measurement units within a given measurement system.** **5.MD.A.1** Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems. **5.MD.B: Represent and interpret data.** **5.MD.B.2** Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.* **5.MD.C: Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.** **5.MD.C.3** Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

	a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. **b.** A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units. **5.MD.C.4** Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units. **5.MD.C.5** Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. **a.** Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. **b.** Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real world and mathematical problems. **c.** Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems. **Geometry (G)** **5.G.A: Graph points on the coordinate plane to solve real-world and mathematical problems.** **5.G.A.1** Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate). **5.G.A.2** Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Range ALDs – Claim 4 Grades 3-5	**Level 1** Students should be able to identify important quantities in the context of a familiar situation and translate words to equations or other mathematical formulation. When given the correct math tool(s), students should be able to apply the tool(s) to problems with a high degree of scaffolding.
	Level 2 Students should be able to identify important quantities in the context of an unfamiliar situation and to select tools to solve a familiar and moderately scaffolded problem or to solve a less familiar or a non-scaffolded problem with partial accuracy. Students should be able to provide solutions to familiar problems using an appropriate format (e.g., correct units, etc.). They should be able to interpret information and results in the context of a familiar situation.
	Level 3 Students should be able to apply mathematics to solve unfamiliar problems arising in everyday life, society, and the workplace by identifying important quantities and mapping, displaying, explaining, or applying their relationship and by locating missing information from relevant external resources. They should be able to construct chains of reasoning to justify a model used, produce justification of interpretations, state logical assumptions, and compare and contrast multiple plausible solutions.
	Level 4 Students should be able to apply mathematics to solve unfamiliar problems by constructing chains of reasoning to analyze a model, producing and analyzing justification of interpretations, stating logical assumptions, and constructing and comparing/contrasting multiple plausible solutions and approaches.

Target 4A: Apply mathematics to solve problems arising in everyday life, society, and the workplace.

General Task Model Expectations for Target 4A

- The student is asked to solve a problem arising in everyday life, society, or the workplace.
- Information needed to solve the problem has a level of complexity that is not present in items within Claim 2 Target A. For example, the student must
    - distinguish between relevant and irrelevant information, or
    - identify information that is not given in the problem and request it, or
    - make a reasonable estimate for one or more quantities and use that estimate to solve the problem.
- The student must select a mathematical model independently and is not directly told what arithmetic operation or geometric structure to use to solve the problem.
- Tasks in this model often have secondary alignments to other Claim 4 targets, in particular Target 4B, constructing autonomous chains of reasoning, Target 4D, requiring the student to interpret results in the context of the problem, and Target 4F, requiring students to identify quantities and map relationships between them.
- The student is often required to draw upon knowledge from different domains, including knowledge from earlier grade-levels.
- Tasks have Depth of Knowledge Level 2 or 3.

Task Model 4A.1

Task Expectations

- The student solves a multi-step problem involving one or more of the four operations.
- The student identifies needed information and chooses which operations to perform. The student may
    - ignore irrelevant information,
    - request missing information, and/or
    - make an estimate for one or more quantities and use that estimate to solve the problem.
- Problems in this model may have a tertiary or quaternary alignment to 4B or 4D.
- Problems in this model may have more than one possible solution.

Example Item 4A.1a (Grade 3)
Primary Target 4A (Content Domain OA), Secondary Target 1D (CCSS 3.OA.D), Tertiary Target 4D, Quaternary Target 4F

Eva has 2 quarters, 4 dimes, and 6 nickels. She wants to buy a different gift for each of her 3 friends.

Click on the gifts in the table to show 3 gifts that Eva could buy.

Gift	Cost
Balloon	60 ¢
Eraser	35 ¢
Gumball	25 ¢
Kazoo	75 ¢
Mood ring	50 ¢
Pencil	35 ¢
Sticker	20 ¢

Rubric: (1 point) The student is able to identify three items whose total cost is less than $1 and 20¢. (e.g., Mood ring, pencil, and sticker).

Response Type: Hot Spot

Commentary: The item aligns to 4F because it requires that students identify the total amount of money that Eva has as a key quantity in solving the problem, and relate it to the prices of different items. Complexity of this item can be decreased by directly giving the total amount of money. If this is done, the alignment to 4F should be removed. The item can be varied by specifying that she wants to give the same gift to each of her friends, turning it into a multiplication problem. Complexity and grade level can be increased by increasing the amount of money she has, the prices of the objects, or the number of friends, so that 3-digit addition or multiplication is required. For larger numbers, other contexts might make more sense.

Example Item 4A.1b (Grade 4)
Primary Target 4A (Content Domain NBT), Secondary Target 1A (CCSS 4.OA.B), Tertiary Target 4B, Quaternary Target 4D

A bag of 5 apples at the grocery store has a mass of 825 grams. The largest apple has a mass of 185 grams.

What is a reasonable estimate for the mass, in grams, of the smallest apple in the bag? Select Yes for each reasonable mass and No for each mass that is **not** reasonable.

	Yes	No
50 grams		
100 grams		
150 grams		
200 grams		

Rubric: (1 point) The student selects numbers that are reasonable estimates for the mass of the smallest apple. The student could select just 150 since an argument can be made that if the apples are fairly similar in size, then 150 is the only reasonable estimate, but if they vary a lot, then 100 would be reasonable as well. 200 would not be possible as that is larger than the largest apple, and 50 is not possible because that would require at least one other apple to be 197 grams. (There are three correct response patterns: {100}, {150}, or {100, 150}).

Response Type: Matching Table

Example Item 4A.1c (Grade 5)
Primary Target 4A (Content Domain MD, NBT), Secondary Target 1E (CCSS 4.MD.A, 4.NBT.B), Tertiary Target 4B, Quaternary Target 4F

How many minutes are in 1 day? [Click here for more information if you need it]

Interaction: If the student clicks for more information, they get the following conversion data[5]:

- There are 60 seconds in 1 minute
- There are 60 minutes in 1 hour
- There are 24 hours in 1 day
- There are 7 days in 1 week
- There are 52 weeks in 1 year

Rubric: (1 point) The student enters the correct number of minutes (1440).

Response Type: Equation/Numeric (label the response box with minutes)

Commentary: This item requires students to recognize which quantities are of interest (minutes, hours, and days) and then identify the relationship between them. Identifying these different quantities and mapping their relationships draws on the skill set identified in Target 4F.

Example Item 4A.1d (Grade 5)
Primary Target 4A (Content Domain NBT), Secondary Target 1E (CCSS 5.NBT.B), Tertiary Target 4B, Quaternary Target 4D

A parking meter accepts nickels, dimes, and quarters. It holds up to 1500 coins. Estimate the value of the coins, in dollars, in the meter when it is full.

Rubric: (1 point) The student enters a reasonable estimate (a multiple of 5 between 75 and 375).

Response Type: Equation/Numeric

[5] The ability to pull up information interactively is not currently available, but part of the plan for enhancements to the item-authoring system in 2017.

Example Item 4A.1e (Grade 5)
Primary Target 4A (Content Domain NF), Secondary Target 1I (CCSS 5.MD.C), Tertiary Target 4F

Gina is making cookies. The last three steps used to make the cookies are shown.

> Step 5: Roll the dough into balls that are $\frac{1}{2}$-inch wide.
>
> Step 6: Place the balls on a baking tray 2 inches apart.
>
> Step 7: Bake for 12 minutes.
>
> **This recipe makes 18-24 cookies**

Gina plans to

- give cookies to 9 people;
- give each person 3 cookies; and
- have no extra cookies remaining.

Which action will help Gina get closest to the exact number of cookies she needs?

A. Place the cookies 3 inches apart.
B. Bake the cookies for only 10 minutes.
C. Roll the cookies slightly larger than $\frac{1}{2}$-inch wide.
D. Roll the cookies slightly smaller than $\frac{1}{2}$-inch wide.

Rubric: (1 point) The student correctly determines which action will help Gina get closest to the exact number of cookies (D).

Response Type: Multiple Choice, single correct response

Example Item 4A.1f (Grade 3)
Primary Target 4A (Content Domain OA), Secondary Target 1D (CCSS 3.OA.D), Tertiary Target 4F, Quaternary Target 1D (CCSS 3.MD.A)

Jenny went to the store to buy 15 bottles of water.

- The bags at the store can each hold 6 kilograms.
- The bottles of water each weigh 2 kilograms.
- Jenny bought 15 bottles of water.

What is the fewest number of bags that Jenny needs to hold all 15 water bottles?

Rubric: (1 point) The student enters the smallest number of bags needed (5).

Response Type: Equation/Numeric

Example Item 4F.1a (Grade 3)
Primary Target 4A (Content Domain MD), Secondary Target 1G (CCSS 3.MD.1), Tertiary Target 4F

The table shows the start and end times for runners in a race.

Racing Times		
Runner	**Start Time**	**End Time**
Mike	12:03 p.m.	12:26 p.m.
Ann	12:10 p.m.	12:17 p.m.
John	12:13 p.m.	12:19 p.m.
Patty	12:16 p.m.	12:25 p.m.

What is the difference, in minutes, between Patty's start time and Mike's start time?

Rubric: (1 point) The student enters the correct difference (13).

Response Type: Equation/Numeric

Target 4B: Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem.

Items that require the student to make decisions about the solution path needed to solve a problem are aligned with Target 4B. Note that Target 4B is never the primary target for an item, but is frequently a Tertiary or Quaternary Target for an item with primary alignment to other targets; see, for example, items in Task Models for 4A, 4C, and 4E.

General Task Model Expectations for Target 4B

- The student is presented with a multi-step problem with little or no scaffolding, or
- The student must make estimates or choose between different reasonable assumptions in order to solve the problem.

Target 4B is assessed in conjunction with Target 4A, 4C, and 4E.

Target 4C: State logical assumptions being used.

Task Model 4C.1

Task Expectations:

- The student is presented with a problem arising in everyday life, society, or the workplace. The student either
  - identifies information or assumptions needed to solve the problem or
  - provides a reasoned estimate of a quantity needed to solve the problem.

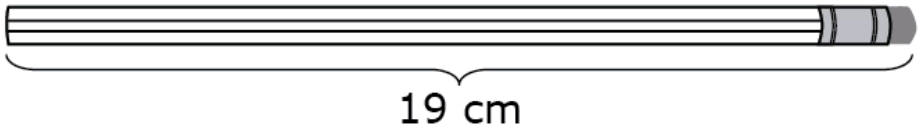
  It is not necessary that a student constructs a complete solution to the problem for this target.
- Tasks in this model generally have either more information than is needed solve the problem (and students must choose) or not enough information (and students must make a reasoned estimate).
- The student is often required to draw upon knowledge from different domains, including knowledge from earlier grade-levels.
- Tasks for this target may also assess Target 4B or 4D.
- Tasks have DOK Level 1 or 2

Example Item 4C.1a (Grade 3)
Primary Target 4C (Content Domain OA), Secondary Target 1D (CCSS 3.OA.D, 2.MD.A), Tertiary Target 4D, Quaternary Target 4E

Part A
Estimate the length of this unsharpened pencil, in centimeters. [ ]

Enter your estimate in the response box.

Part B
The length of the pencil is about 19 cm.



How much longer or shorter is your estimate than the real length? [ ]

Enter your answer in the response box.

Interaction: The student must enter an estimate for the length of the pencil before seeing the actual length and cannot change it once the actual length is shown. The student's estimate does not factor into the score he or she receives.

Rubric: (1 point) The student finds the difference between their estimate, *a*, and the actual length of the pencil (|19-*a*|).

Response Type: Equation/Numeric

Note: Functionality for this item type does not currently exist, but is planned for future enhancements.

Commentary: This item type is new and may be unfamiliar to item writers and is designed to activate a particular practice which is important in mathematical modeling. Students are often required to make an estimate as one of the logical assumptions on which they will base a mathematical model. In grades 3-5, students are learning how to make reasoned estimates by first developing the habit of making their best estimate (without penalty) and then reflecting on the accuracy of their estimate. The difference between items in this task model and Task Model 4E.3 is that the emphasis here is on making and reflecting on the accuracy of the estimate and the emphasis in Task Model 4E.3 is on making and revising the estimate.

Example Item 4C.1b (Grade 5)
Primary Target 4C (Content Domain NF), Secondary Target 1H (CCSS 4.NBT.A), Tertiary Target 4D, Quaternary Target 4E

Part A
A liter is more than a cup. Estimate the number of liters in a cup. You can use the picture to help you make an estimate.

Enter your estimate, in liters, in the response box. [ ]

Part B
There are about 0.24 liters in one cup. How much greater or less than your estimate is the real amount?

Enter the difference in the response box. [ ]

Interaction: The student must select an estimate for the number of liters in a cup before seeing the actual value and cannot change it once the actual value is shown. The students' estimate does not factor into the score he or she receives.

Rubric: (1 point) The student finds the difference between their estimate, *a*, and the actual number of liters ($|19-a|$).

Response Type: Equation/Numeric

Example Item 4C.1c (Grade 4)
Primary Target 4C (Content Domain OA), Secondary Target 1A (CCSS 3.OA.A), Tertiary Target 4B, Quaternary Target 4F

Sarah is helping her dad make cookies for her class using a recipe they found online. Her dad asks, "Do you think one batch of cookies will be enough?" Select **all** of the information they need to answer the question.

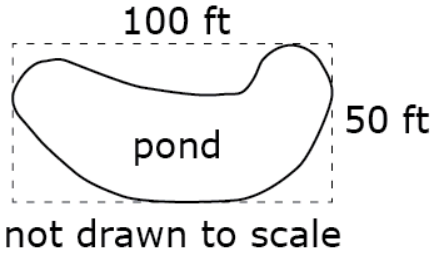
A. The amount of flour in the recipe.
B. The number of cookies in one batch.
C. The number of students in the class.
D. The temperature of the oven for baking the cookies.
E. The number of cookies you can fit onto a cookie sheet.

Rubric: (1 point) The student selects the correct pieces of information (B and C).

Response Type: Multiple Choice, multiple correct response

Example Item 4C.1d (Grade 4)
Primary Target 4C (Content Domain MD), Secondary Target 1I (CCSS 4.MD.3), Tertiary Target 4B, Quaternary Target 4D

Liam uses string to form a rectangle with length 100 feet and width 50 feet to estimate the area of a small pond.



Enter an estimate for the area of the pond in square feet in the response box. [ ]

Select a statement that supports your estimate:

A. The area of the rectangle is bigger than the area of the pond.
B. The area of the rectangle is smaller than the area of the pond.
C. The distance around the rectangle is bigger than the distance around the pond.
D. The distance around the rectangle is smaller than the distance around the pond.

Rubric: (1 point) The student enters a reasonable estimate and selects the supporting reason (a number between 2500 and 5000; A).

Response Type: Equation/Numeric; Multiple Choice, single correct response[6]

Note: Currently can be formatted as a Drag and Drop and Hot Spot.

[6] This combination of item types is currently not supported, but is planned for future enhancements to the item-authoring tool.

Target 4D: Interpret results in the context of a situation.

Target 4D identifies a key step in the modeling cycle, and is thus present in the majority of modeling problems that require students to find a numerical answer. Note that in Grades 3-5, Target 4D is never the primary target for an item, but is frequently a Tertiary or Quaternary Target for an item with primary alignment to other targets; see, for example, items in Task Models for 4A, 4C, and 4E. In later grades, students interpret more complex mathematical objects (like equations and graphs) in more sophisticated contexts.

General Task Model Expectations for Target 4D

- The student must solve a problem that results in a numerical answer and interpret the number in the context of the problem.

In Grades 3-5, Target 4D is assessed in conjunction with Target 4A, 4C, and 4E.

Target 4E: Analyze the adequacy of and make improvements to an existing model or develop a mathematical model of a real phenomenon.

General Task Model Expectations for Target 4E

- The student is presented with a problem arising in everyday life, society, or the workplace. The student either
    - chooses between competing mathematical models to solve the problem (which may depend on different interpretations of the problem), or
    - evaluates a partial or complete (possibly incorrect) solution to the problem, or
    - constructs a mathematical model to solve the problem

  It is not necessary that a student constructs a complete solution to the problem for this target.
- Tasks in this model can also assess Target 4B (Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem). Thus some tasks should plausibly entail a chain of reasoning to complete the task (not just a single step). For example, it might be necessary for the student to construct a two-step arithmetic expression to evaluate a model or solution, or to try out a geometric shape and then perform a calculation to see if it satisfies the requirements.
- The student is often required to draw upon knowledge from different domains, including knowledge from earlier grade-levels.
- Tasks have DOK Level 2, 3, or 4

Task Model 4E.1

Task Expectations:

- Students construct a geometric figure, a numerical expression, or a numerical equation that models a given problem.
- Students may or may not perform a multi-step numerical calculation to verify that the model solves the problem.
- The operations to be performed should not be explicitly given, but should be inferred from the situation.
- Students are expected to reason autonomously from a context to the figure, expression, or equation.
- Difficulty and grade level maybe be varied by varying the types of numbers used (whole numbers, fractions, decimals), the complexity of the geometric figure (square, rectangle, triangle, polygon), the complexity of the numerical expression or equation (number of steps to build it up), whether or not it is required to perform a numerical calculation to complete the task.

Example Item 4E.1a (Grade 3)
Primary Target 4E (Content Domain OA), Secondary Target 1A (CCSS 3.OA.A), Tertiary Target 4F

Tina has 4 packs of gum. Each pack has the same number of pieces of gum. Altogether there are 60 pieces of gum.

Part A
Make an equation to find the number of pieces of gum in each pack. Use *n* for the number of pieces in each pack.

Part B
How many pieces of gum are in each pack?

Rubric: (2 points) One point for a correct answer to each part. For Part A, the student enters a correct equation (e.g., $n=60\div4$, $4 \times n = 60$, $4 = 60 \div n$). For Part B, the student enters the correct number (15).

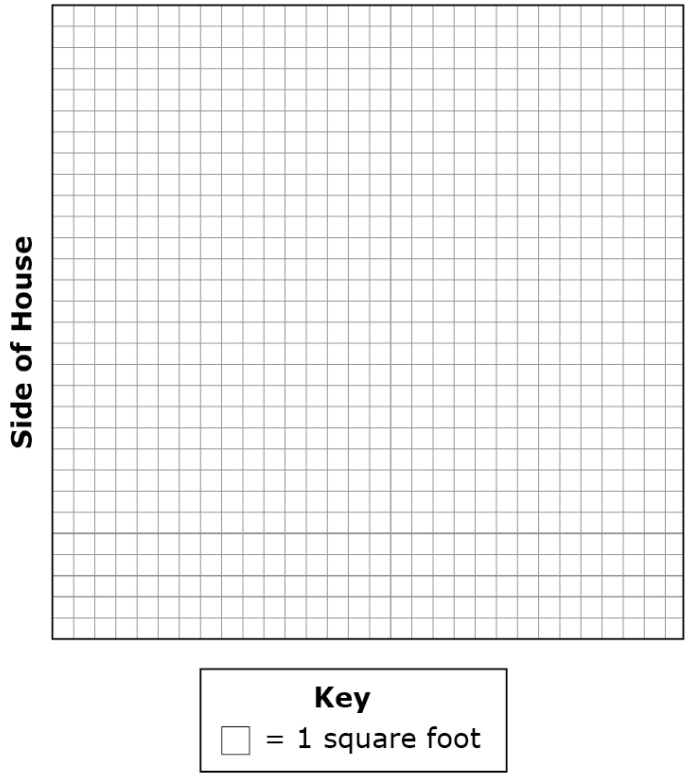
Response Type: Equation/Numeric (2 response boxes; label them *Part A* and *Part B*)

Example Item 4E.1b (Grade 4)
Primary Target 4E (Content Domain MD), Secondary Target 1I (CCSS 4.MD.3), Tertiary Target 4F

Tyra wants to enclose a section of her lawn for her dog to be able to have an outdoor play area. She knows that if she uses the side of her house as one side of the play area, her dog will have a larger outdoor play area. Tyra's plan for the play area includes the following:

- It will be in the shape of a rectangle.
- The side of the house will be used as one side of the rectangular area.
- She will use exactly 24 feet of fence material to enclose the play area.
- The length and width of the enclosure will be a whole number of feet.
- She wants the play area to be greater than 60 square feet.

Use the Connect Line tool to create a rectangular play area that meets Tyra's plan.



Rubric: (2 points) The student is able to construct a 4 by 16, 5 by 14, 6 by 12, 7 by 10, or 8 by 8 rectangle using the side of the house for the longer side.
(1 point) Partial credit is possible for constructing a rectangle that uses exactly 24 feet of fencing, but doesn't reflect using the side of the house as one of the sides, nor the area being greater than 60 square feet (e.g., 1 by 11, 2 by 10, 3 by 9, 4 by 8, 5 by 7, or 6 by 6).

Response Type: Graphing

Example Item 4E.1c (Grade 5)

Primary Target 4E (Content Domain OA), Secondary Target 1A (CCSS 5.NBT.B), Tertiary Target 4F

A school spends $2.40 on every lunch it serves in the cafeteria and $0.30 for each carton of milk.

- 250 people at the school get a lunch each day
- 120 people take a carton of milk

Create an expression using this information that shows how much the school spends altogether on lunches and milk each day.

Rubric: (1 point). Student constructs a correct numerical expression (250 x 2.40 + 120 x 0.30 or its equivalent).

Response Type: Equation/Numeric

An alternate (easier) version of the problem above:

A school spends $2.40 on every lunch it serves in the cafeteria and $0.30 for each carton of milk.

- 250 people at the school get a lunch each day
- 120 people take a carton of milk

Which expression represents the amount of money the school spends altogether on lunches and milk each day?

A. 250 x 2.40 + 120 x 0.30
B. 250 x 0.30 + 120 x 2.40
C. 250 x (2.40 + 0.30)
D. 120 x (2.40 + 0.30)

Rubric: (1 point). Student selects the correct numerical expression (A).

Response Type: Multiple Choice, multiple correct response

Task Model 4E.2

Task Expectations:

- Students choose between two or more different models to solve a given problem, between two or more problems that fit a given model, or between two or more different solutions to a given problem.
- Different models or solutions can depend on different (possibly incorrect) interpretations of the problem, but do not have to.

Example Item 4E.2a (Grade 3)
Primary Target 4E (Content Domain OA), Secondary Target 1D (CCSS 3.OA.D), Tertiary Target 4B

A large water jug holds 24 liters of water. Nan uses it for her animals.

- Nan fills her animals' water dish 2 times each day.
- She puts the same amount of water in the dish every time.
- She uses all of the water in 3 days.

Which equation can be solved to find the number of liters of water (n) she puts in the dish each time?

A. $3 \times 2 + n = 24$
B. $3 + 2 + n = 24$
C. $3 + 2 \times n = 24$
D. $3 \times 2 \times n = 24$

Rubric: (1 point) The student selects the correct equation (D).

Response Type: Multiple Choice, single correct response

Example Item 4E.2b (Grade 3)
Primary Target 4E (Content Domain OA), Secondary Target 1D (CCSS 3.OA.8), Tertiary Target 4B, Quaternary Target 4F

There are 123 girls and 135 boys in the third grade at a school. Today there are 9 third grade students absent.

Which equation can be used to find the total number of third grade students (s) in school today?

A. $123 + 135 = s$
B. $135 - 9 = s$
C. $123 + 135 + 9 = s$
D. $123 + 135 - 9 = s$

Rubric: (1 point) The student selects the correct equation (D).

Response Type: Multiple Choice, single correct response

Example Item 4E.2c (Grade 4)
Primary Target 4E (Content Domain OA), Secondary Target 1A (CCSS 4.OA.1), Tertiary Target 4B, Quaternary Target 4D

Which situation is represented by the equation $4 \times 3 = \square$?

A. A kitten weighs 4 pounds. A puppy weighs 3 times as much as the kitten.
How much does the puppy weigh?

B. A kitten weighs 4 pounds. A puppy weighs 3 pounds more than the kitten.
How much do they weigh altogether?

C. A kitten weighs 4 pounds. A puppy weighs 3 pounds more than the kitten.
How much does the puppy weigh?

D. A kitten weighs 4 pounds. A puppy weighs 3 times as much as the kitten.
How much do they weigh altogether?

Rubric: (1 point) The student correctly identifies the context that represents the multiplication equation as a multiplicative comparison (A).

Response Type: Multiple Choice, single correct response

Example Item 4E.2d (Grade 5)
Primary Target 4E (Content Domain NBT), Secondary Target 1 (CCSS 5.NBT.B), Tertiary Target 4D, Quaternary Target 4F

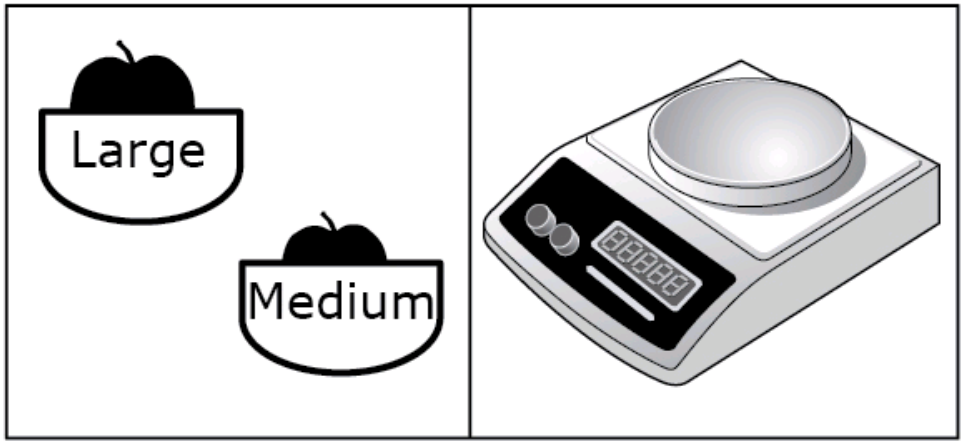
Molly and Sam need about 2 pounds of apples for a pie. Medium apples cost $0.45 each. Large apples cost $0.65 each.

Molly says: "Let's buy the medium apples, they are less expensive."

Sam says: "I think it's less expensive to buy large apples. They are more expensive but we won't have to buy as many of them."

Analyze both approaches. You can use the scale to weigh the apples.

Use the drop down menus to complete each statement.



Statement A:
Molly and Sam would need [1, 2, 3, 4, 5, 6, 7, 8] medium apples or [1, 2, 3, 4, 5, 6, 7, 8] large apples for the pie.

Statement B:
The number of medium apples that would be needed cost [more, less] than the number of large apples that would be needed. So [Molly, Sam] is correct.

Interaction: The student can drag apples one at a time onto the scale from bins labeled "Large" or "Medium" to get the weight in pounds, to the nearest $\frac{1}{8}$ pound. The scale should give weights as mixed numbers, in eighths of a pound. 6 medium apples should weight $2\frac{1}{8}$ pounds, 4 large apples should weigh $2\frac{1}{4}$ pounds. Reducing the number of apples by one should give a weight which is less than 2 pounds and not as not close to 2 pounds (e.g. $1\frac{3}{4}$ for 5 medium apples and $1\frac{5}{8}$ for 3 large apples).

Rubric: (2 points) The student selects the correct numbers and words in all of the drop-down menus (6, 4, more, Sam)

(1 point) Student identifies the correct number of each size of apple needed but does not compare their costs correctly or identify the right reasoning, or the numbers of apples are different but their cost is correctly compared and the correct conclusion is made about who is correct in their reasoning based on the numbers the student chose.

Response Type: Drop-Down Menu[7]

[7] Drop-Down Menu response type is not currently available, but is a planned enhancement to the test-authoring tool by 2017.

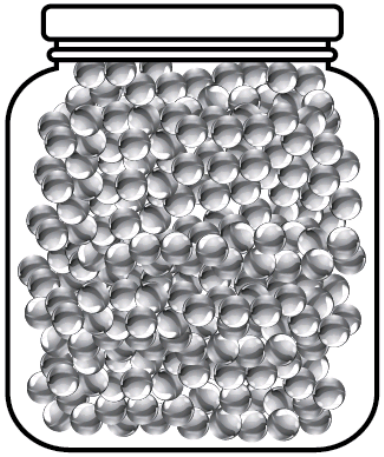
Task Model 4E.3

Task Expectations:

- The student makes estimates to solve a problem and then has a chance to improve the estimates.

Example Item 4E.3a (Grade 5)
Primary Target 4E (Content Domain OA), Secondary Target 1A (CCSS 4.OA.A), Tertiary Target 4B, Quaternary Target 4D

Lilian wants to estimate the number of marbles in a glass jar that has a mass of 2.3 kilograms when it is full.



Part A:
Make an estimate for the mass of a single marble, in grams.
Enter your estimate in the response box. [ ]

Make an estimate for the mass of the jar, in grams.
Enter your estimate in the response box. [ ]

Estimate the number of marbles in the jar based on the assumptions you made.
Enter your estimate in the response box. [ ]

Part B:
The jar has a mass of about 500 grams and there are about 600 marbles in the jar. Which of the following estimates is closest to the actual mass of a single marble?

A. 2 grams
B. 20 grams
C. 200 grams
D. 1200 grams

Interaction: The student enters values for the mass of a single marble and the mass of the jar. The student's choices do not factor into the score he or she receives as long as the estimate for the number of marbles is consistent with those estimates. The student has to make those estimates before moving on to Part B.

Rubric: (2 points) The student estimates the mass of a single marble *m* and the mass of the jar *b*, and makes an estimate of the number of marbles in the jar that is consistent with the initial estimates [e.g., (2300-*b*)/*m* +/-50, rounded to a whole number] and then selects the best estimate from the choices given (A).

(1 point) The student makes an estimate for the number of marbles that is consistent with his/her estimated masses in Part A or selects the best estimate from the choices given in Part B.

Response Type: Equation/Numeric and Multiple Choice, single correct response

Note: Functionality for this item type does not currently exist.

Commentary: This item type is new and may be unfamiliar to item writers and is designed to activate a particular practice which is important in mathematical modeling. In grades 3-5, students are learning how to make reasoned estimates by first developing the habit of making their best estimate (without penalty) and then revising their estimate when more information is known. The difference between items in this task model and Task Model 4C.1 is that the emphasis here is on making and revising the estimate and the emphasis in Task Model 4C.1 is on making and reflecting on the accuracy of the estimate.

Target 4F: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas).

Target 4F identifies a key step in the modeling cycle, and is thus present in the majority of modeling problems.

Task Model 4F.1

Task Model Expectations

- Students are presented with a mathematical problem in a real-world context where the quantities of interest are not named explicitly, are named but represented in different ways, or the relationship between the quantities is not immediately clear.
- The student is asked to solve a problem that may require the integration of concepts and skills from multiple domains.

Example Item 4F.1a (Grade 3)
Primary Target 4F (Content Domain MD), Secondary Target 1G (CCSS 3.MD.1), Tertiary Target 4A

The table shows the start and end times for runners in a race.

Racing Times		
Runner	**Start Time**	**End Time**
Mike	12:03 p.m.	12:26 p.m.
Ann	12:10 p.m.	12:17 p.m.
John	12:13 p.m.	12:19 p.m.
Patty	12:16 p.m.	12:25 p.m.

What is the difference, in minutes, between Patty's start time and Mike's start time?

Rubric: (1 point) The student enters the correct difference (13).

Response Type: Equation/Numeric (label the response box with minutes)